WASHINGTON STATE UNIVERSITY
Pullman, Washington

WEAK SHOCK IMPEDANCE METHODS FOR THE QUARTZ GAUGE

J. W. Forbes, R. H. Mitchell

III. WEAK SHOCK IMPEDANCE METHODS FOR THE QUARTZ GAUGE

J. W. Forbes, R. H. Mitchell

A. INTRODUCTION

A major effort in the shock wave field is now focused on the low stress region of materials. The experimental effort in the low stress region is due in part to the development of the quartz gauge.[3.1] It is the purpose of this paper to describe the impedance matching technique for reducing the measured pressure and particle speed in the quartz gauge to the pressure and particle speed in the sample material. This work will be restricted to materials which exhibit two shock waves resulting from a violation of the stability criterion.

The typical shock experiment (Fig. 3.1) is one where a projectile is incident on the front surface of a sample. The back surface is in contact with a quartz gauge. Upon impact two shock waves go forward (+ x direction) in the sample while a shock wave goes backwards (- x direction) in the projectile. The first shock in the sample reaches the quartz face and a shock wave is transmitted into the quartz. A rarefaction wave is reflected back into the sample if its impedance is higher than that of quartz. A shock wave is reflected if its impedance is less. Assume the backward facing wave and the forward facing second shock do not interact. The second shock then proceeds unperturbed to the quartz boundary, a shock wave is transmitted into the quartz, and a rarefaction or shock wave is reflected, depending on relative impedances of sample and quartz.

The state ahead of a weak shock or rarefaction wave is related to the state behind the wave by the Rankine-Hugoniot jump conditions.[3.2,3.3] Let

the subscript "o" represent the state ahead of the wave and the subscript "1" represent the state behind the wave.

$$\rho_0(D_1 - u_0) = \rho_1(D_1 - u_1) \tag{3.1}$$

$$P_1 - P_0 = \rho_0(D_1 - u_0)(u_1 - u_0) \tag{3.2}$$

where ρ is density, D is wave speed, u is particle speed, and P is pressure.

Procedures for reducing quartz data to sample data are described here for three cases:

1) the sample is an elastic-plastic material with higher impedance than quartz,

2) it is a polymorphic material with higher impedance than quartz,

3) it is a material of lower impedance than quartz.

B. ELASTIC-PLASTIC MATERIAL OF HIGHER IMPEDANCE THAN QUARTZ

A simple solution to the impedance matching problem exists for this case. Assume that the rarefaction wave from the quartz boundary has the same speed as the elastic first shock wave in the material. The P-u plane describing the experiment is given in Fig. 3.2. A convenient but not necessary assumption is that the projectile material has a linear P-u relationship.

The first forward-facing elastic wave, D_1, shocks the material to state (P_1, u_1). The second forward-facing plastic shock wave, D_2, compresses the material to state (P_2, u_2). The backward-facing elastic wave, D_3, relieves the material from state (P_1, u_1) to the state (P_3, u_3). The second relief path from state (P_2, u_2) can be left unspecified; an arbitrary path is shown in Fig. 3.2. The state (P_2, u_2) is also defined by the backward-facing shock wave in the projectile.

2

Applying the Rankine-Hugoniot equations to the various states shown on the P-u diagram with $P_o=0$, $u_o=0$ results in the following set of equations.

$$\rho_o D_1 = \rho_1(D_1-u_1) \tag{3.3}$$

$$P_1 = \rho_o D_1 u_1 \tag{3.4}$$

$$P_2-P_1 = \rho_1(D_2-u_1)(u_2-u_1) \tag{3.5}$$

$$P_3-P_1 = \rho_1(D_1+u_1)(u_1-u_3) \tag{3.6}$$

$$P_2 = A + Bu_2 \tag{3.7}$$

The subscript indicates the material state behind the wave of the same subscript. The parameters known from the experiment are ρ_o, D_1, D_2, P_3, u_3, P_4, u_4, A and B. The constants A and B define the projectile's P-u relationship.

Solving these equations for the unknowns u_1 and P_1 results in

$$u_1 = \frac{D_1(P_3 + \rho_o D_1 u_3)}{P_3-\rho_o D_1 u_3 + 2\rho_o D_1^2} \tag{3.8}$$

$$P_1 = \frac{\rho_o D_1^2(P_3+\rho_o D_1 u_3)}{P_3-\rho_o D_1 u_3 + 2\rho_o D_1^2} \tag{3.9}$$

Using the results of Eqs. (3.8) and (3.9), P_2 becomes

$$P_2 = \frac{P_1 B-\rho_1(D_2-u_1)(A + Bu_1)}{B-\rho_1(D_2-u_1)} \tag{3.10}$$

The relief path for the second wave has not been used in the above results.

3

A test of relief modes for the second wave can in principle be made by assuming a model and comparing the results for P_2 with Eq. (3.10).

C. POLYMORPHIC MATERIAL WITH IMPEDANCE HIGHER THAN QUARTZ

The solution to this problem requires an assumption about the equation of state. The assumption made here is that the first phase of the material has a linear shock speed-particle speed, $(U_s - U_p)$, relation. It is also assumed that the bulk sound speed, c_o, at STP conditions is known.

The P-u plane is shown in Fig. 3.3. The state of the material behind the first shock, D_1, is (P_1, u_1). The state behind the second shock, D_2, is (P_2, u_2). The backward-facing rarefaction wave from the quartz face takes the material from state (P_1, u_1) to state (P_3, u_3). The second rarefaction fan takes the material from state (P_2, u_2) to state (P_4, u_4). The state (P_2, u_2) is also defined by the backward shock wave in the projectile.

The state $(P_3, 2u_1 - u_3)$ is symmetric to (P_3, u_3) and contains the same information as (P_3, u_3). A hypothetical forward facing shock D_3^+ would take the material from state $(P_3, 2u_1 - u_3)$ to state (P_1, u_1).

The Rankine-Hugoniot equations can be applied to the various states the P-u curve with $P_o = 0$, $u_o = 0$, resulting in the following set of equations:

$$\rho_o D_1 = \rho_1 (D_1 - u_1) \tag{3.3B}$$

$$P_1 = \rho_o D_1 u_1 \tag{3.4B}$$

$$P_2 - P_1 = \rho_1 (D_2 - u_1)(u_2 - u_1) \tag{3.5B}$$

$$P_2 = A + B u_2 \tag{3.7B}$$

$$P_3 - P_1 = \rho_1 (D_3^+ - u_1)(u_1 - u_3) \tag{3.11B}$$

4

$$D_1 = c_o + Su_1 \qquad\qquad (3.16)$$

$$D_3^+ = D_1 - S(u_3 - u_1) \qquad\qquad (3.17)$$

The subscript indicates the material state behind the wave of the same subscript. The parameters known from the experiment are ρ_o, D_1, D_2, P_3, u_3, P_4, u_4, A and B; the constants A and B define the projectile's P-u relationship.

The state symmetric to (P_3, u_3) lies on the first phase of the P-u branch. The symmetric state $(P_3, 2u_1 - u_3)$ has been used for Eqs. (3.11B) and (3.17). Eqs. (3.16) and (3.17) represent the linear U_s-U_p relation assumed for the first phase.

The analytical solution is difficult, but numerical solutions are readily found. A flow chart for numerical solution is given in Fig. 3.4. The procedure is started by choosing a value for u_1 and calculating all the unknown parameters related to the first shock wave in the material. The calculated pressure P_3 is compared to the measured P_3. The value of u_1 is increased by Δ if the two numbers disagree and the process is repeated until they agree; the solution for the first phase is then complete. The elimination of P_2 between Eqs. (3.5B) and (3.7B) allows calculation of u_2; calculation of P_2 from Eq. (3.7B) completes the solution.

The relief path for the second wave has not been used in the above results. A test of relief models for the second wave can in principle be made.

D. MATERIALS WITH IMPEDANCE LOWER THAN QUARTZ

The solution to this problem requires an assumption about the equation of state. The assumption made here is that the second phase of the material has a linear U_s-U_p relation.

The P-u plane for a material of lower impedance than quartz is given in Fig. 3.5. The state of the material behind the first shock, D_1, is defined by (P_1,u_1). The state behind the second shock, D_2, is defined by (P_2,u_2). The first backward-facing shock, D_3, takes the material from state (P_1,u_1) to (P_3,u_3). The second backward-facing shock from the quartz face takes the sample from state (P_2,u_2) to state (P_4,u_4). The state (P_2,u_2) is also defined by the backward shock wave in the projectile.

The state $(P_3,2u_1-u_3)$ is symmetric to (P_3,u_3) and contains the same information as (P_3,u_3). The state $(P_4,2u_2-u_4)$ is symmetric to (P_4,u_4) and contains the same information as (P_4,u_4).

The Rankine-Hugoniot equations can be applied to the various states on the P-u plane with $P_0=0$, $u_0=0$, resulting in the following set of equations.

$$\rho_0 D_1 = \rho_1(D_1-u_1) \tag{3.3A}$$

$$P_1 = \rho_0 D_1 u_1 \tag{3.4A}$$

$$P_2-P_1 = \rho_1(D_2-u_1)(u_2-u_1) \tag{3.5A}$$

$$P_2 = A + Bu_2 \tag{3.7A}$$

$$P_3-P_1 = \rho_1(D_3^+-u_1)(u_1-u_3) \tag{3.11}$$

$$P_4-P_2 = \rho_2(D_4^+-u_2)(u_2-u_4) \tag{3.12}$$

$$\rho_1(D_2-u_1) = \rho_2(D_2-u_2) \tag{3.13}$$

$$D_2 = D_3^+ + S(u_2+u_3-2u_1) \tag{3.14}$$

$$D_4^+ = D_2 + S(u_2-u_4) \tag{3.15}$$

6

The subscript indicates the state behind the wave of the same subscript. The states symmetric to (P_3, u_3) and (P_4, u_4) lie on the P-u branch for the second phase. The symmetric states have been used for Eqs. (3.11), (3.12), (3.13), and (3.14). A hypothetical forward-facing shock, D_3^+, would change the material from state $(P_3, 2u_1 - u_3)$ to (P_2, u_2). A hypothetical forward-facing shock, D_4^+, would change the material from state (P_2, u_2) to $(P_4, 2u_2 - u_4)$.

Equations (3.14) and (3.15) represent the assumed $U_s - U_p$ relation. Known parameters from the experiment are ρ_0, D_1, D_2, P_3, u_3, P_4, u_4, A and B. Parameters A and B define the linear P-u relationship for the projectile.

Analytical solution of the nine equations is difficult and possibly unattainable, but they can be solved numerically. A flow chart for the numerical solution is shown in Fig. 3.6.

The procedure described in Fig. 3.6 is started by choosing a value for u_1 and calculating all the other unknowns. The calculated pressure P_3 is compared to the measured value of P_3. If they disagree, u_1 is increased by Δ and the process repeated until they do agree. Choosing u_1 as the parameter to change makes it possible to use most of the equations of the set in their given form. The one exception to this is that Eqs. (3.5A) and (3.7A) need to be solved for u_2 by eliminating P_2 from the equation.

The above procedure is illustrated for Cadmium sulfide-lucite composites, which have lower impedance than quartz. CdS undergoes a polymorphic phase transition at about 30 kilobars, and two waves are formed if the driving pressure is in the right range. The data from two experiments with results of calculations for P_1 and P_2 are given in Table I. The projectile material was 6061-T6 aluminum.

REFERENCES

3.1 R. A. Graham, F. W. Neilson, and W. B. Benedick, J. Appl. Phys. 36, 1775 (1965)

3.2 M. H. Rice, R. G. McQueen, and J. M. Walsh, SOLID STATE PHYSICS, Vol. 6, 1-63 (1958)

3.3 G. E. Duvall and G. R. Fowles, HIGH PRESSURE PHYSICS AND CHEMISTRY, Vol. 2, 209-291 (1963)

TABLE I

DOUBLE SHOCK WAVE IN CdS

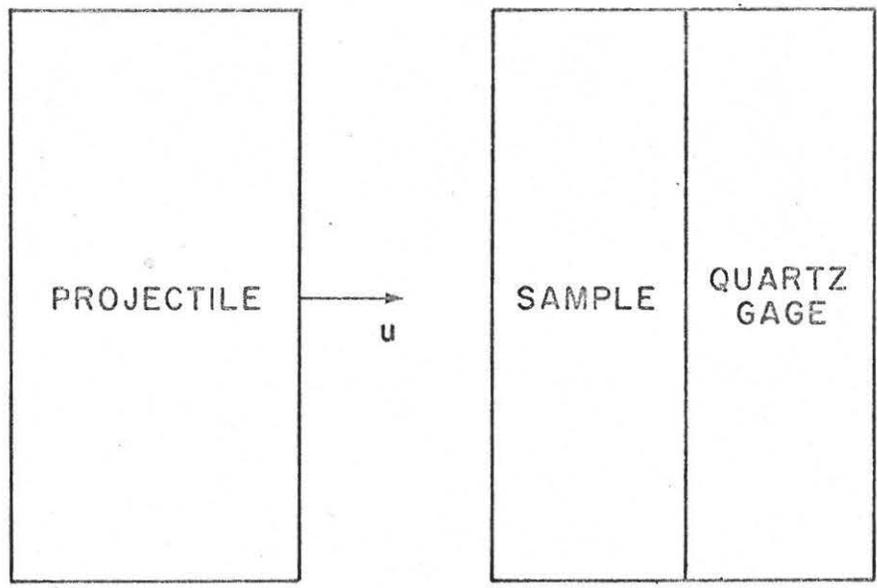| Shot Number	69-005	69-036
Sample Density (gm/cm^3)	1.785	2.025
D_1 (cm/μsec)	.2850	.2955
D_2 (cm/μsec)	.2780	.2938
P_3 (megabars)	.0365	.0332
u_3 (cm/μsec)	.0237	.0217
P_4 (megabars)	.0496	.0440
u_4 (cm/μsec)	.0322	.0285
Projectile Speed (cm/μsec)	.0869	.0777
A (megabars)	.1244	.1113
B (megabar cm/μsec)	-1.42	-1.42
Transition Pressure P_1 (megabars)	.0244	.0230
Final Pressure, P_2 (megabars)	.0327	.0330

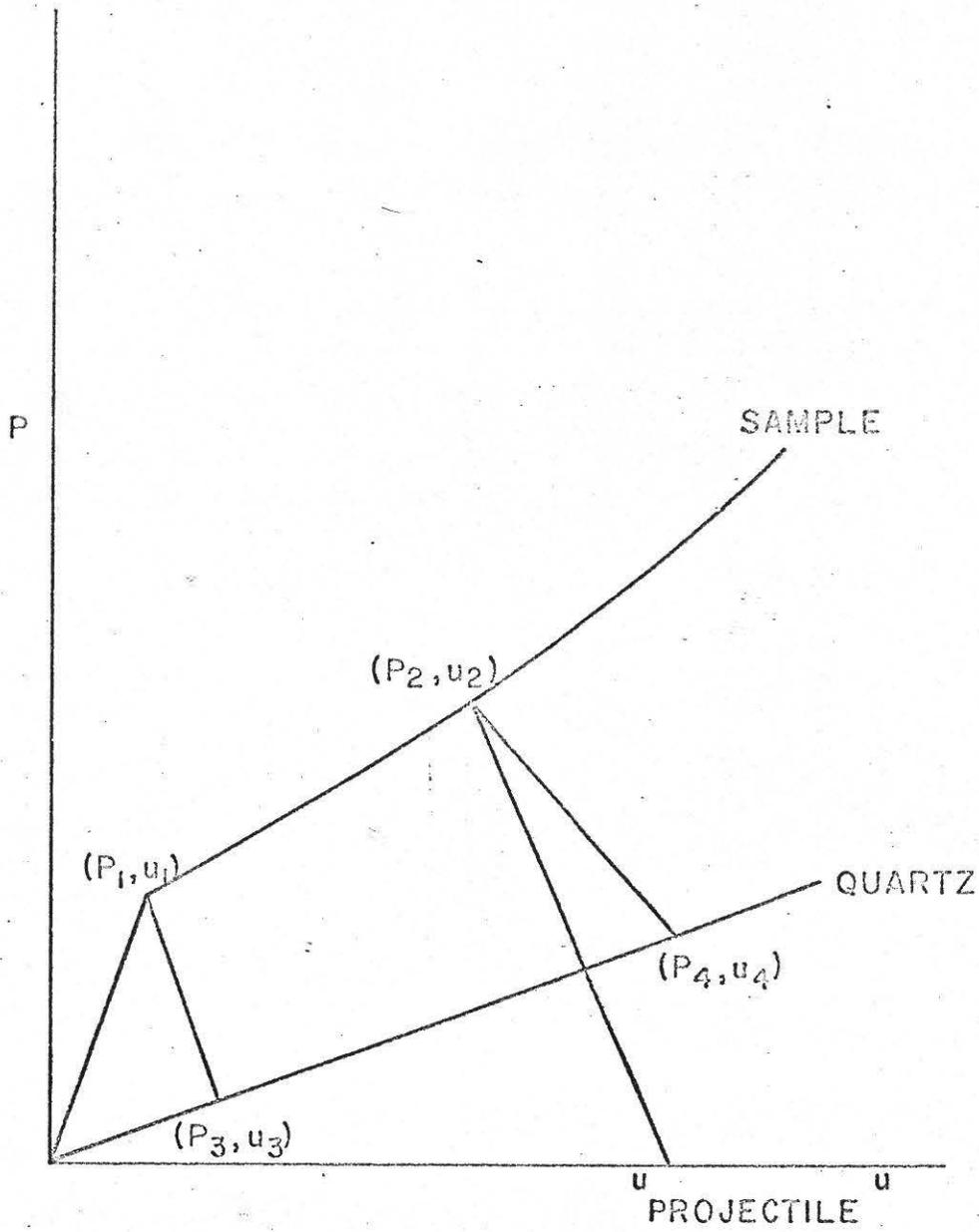Figure 3.1   Typical shockwave experiment

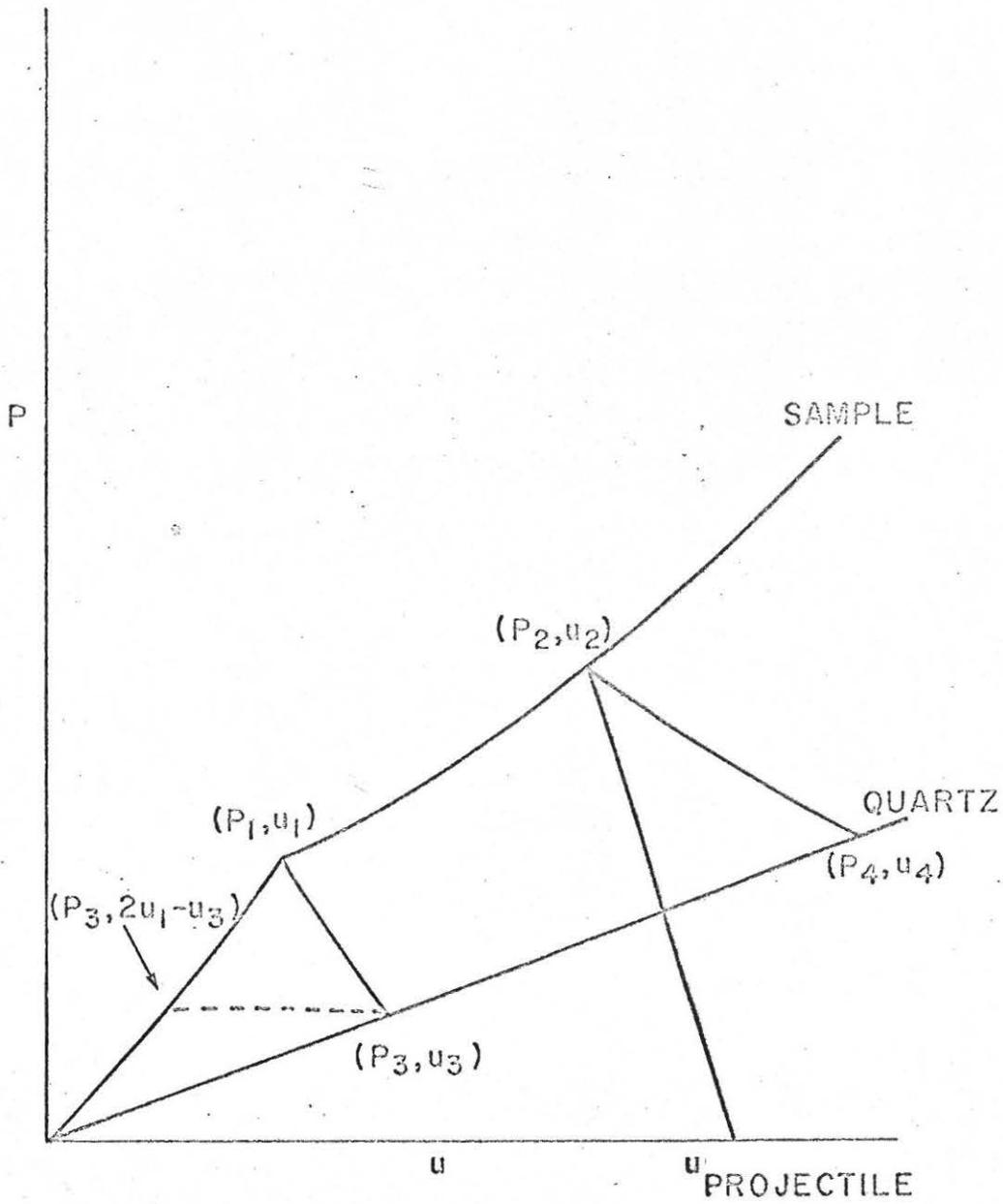Figure 3.2   P-u plane of elastic-plastic material of higher impedance than quartz.

Figure 3.3 P-u plane of polymorphic material with a higher impedance than quartz.

GIVEN U_1, Δ, P_3MEAS.,A, B

$\downarrow$ EQ. 3A

ρ_1
$\downarrow$ EQ. 4A

P_1
$\downarrow$ EQ. 15 $U_1 + \Delta$

S
$\downarrow$ EQ. 16

D_3^+
$\downarrow$ EQ. 11B

P_3
$\downarrow$

TEST IF
$(P_3$ CALC. $- P_3$ MEAS.$) \rightarrow$ NO
IS SMALL

$\downarrow$

YES
$\downarrow$ EQS. 5B AND 7B

U_2
$\downarrow$ EQ. 7B
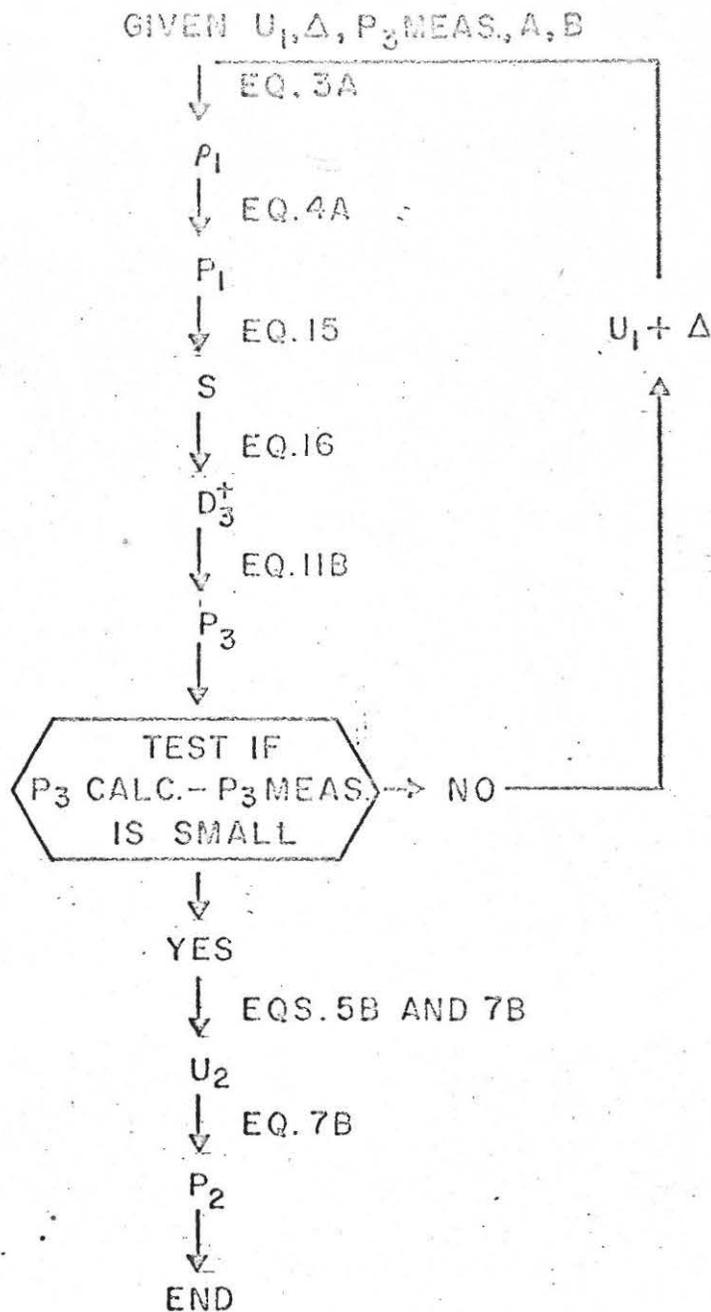
P_2
$\downarrow$

END

Figure 3.4   Flow of numerical impedance calculation for polymorphic material of higher impedance than quartz.

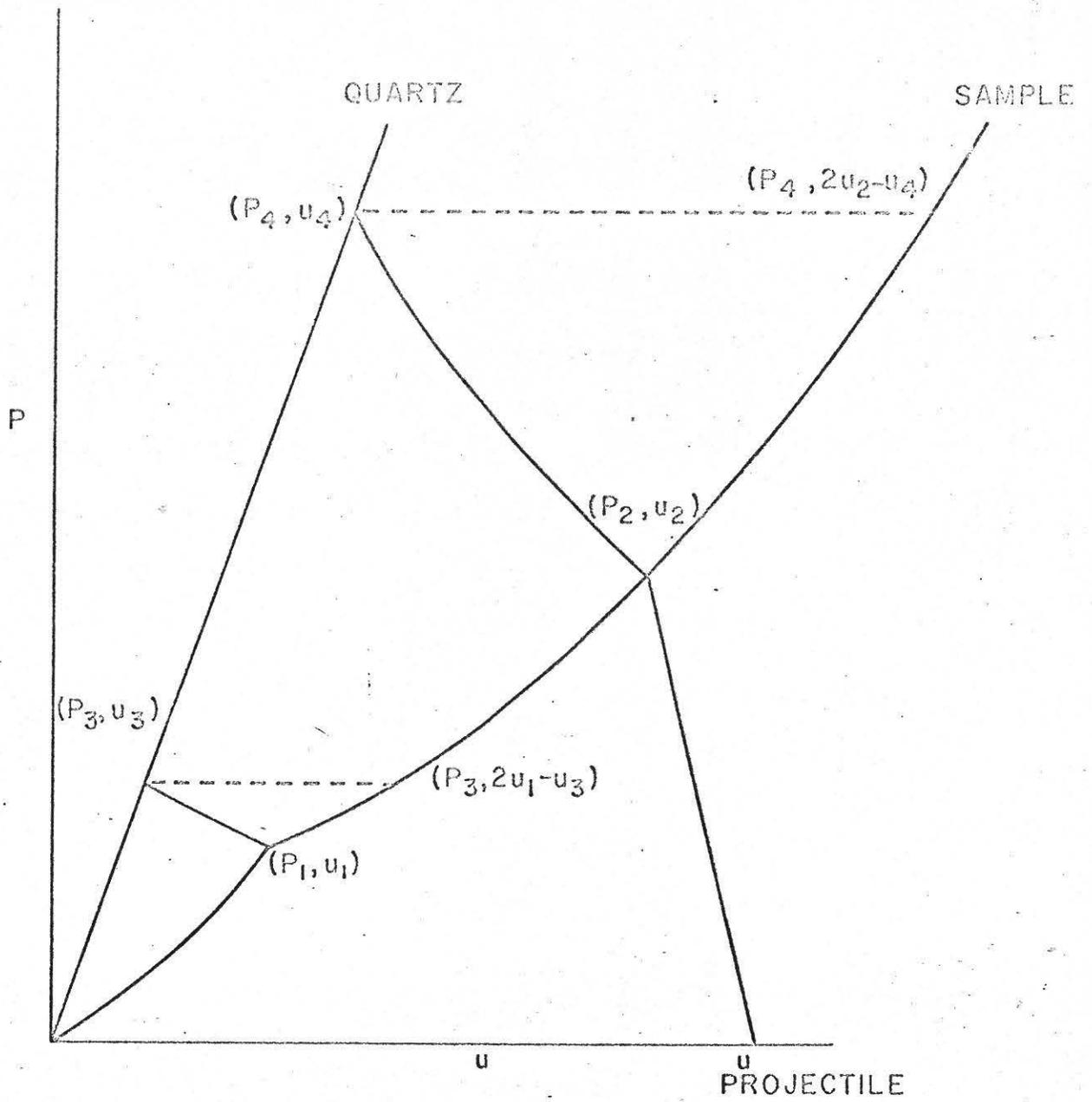Figure 3.5   P-u plane of material with a lower impedance than quartz.
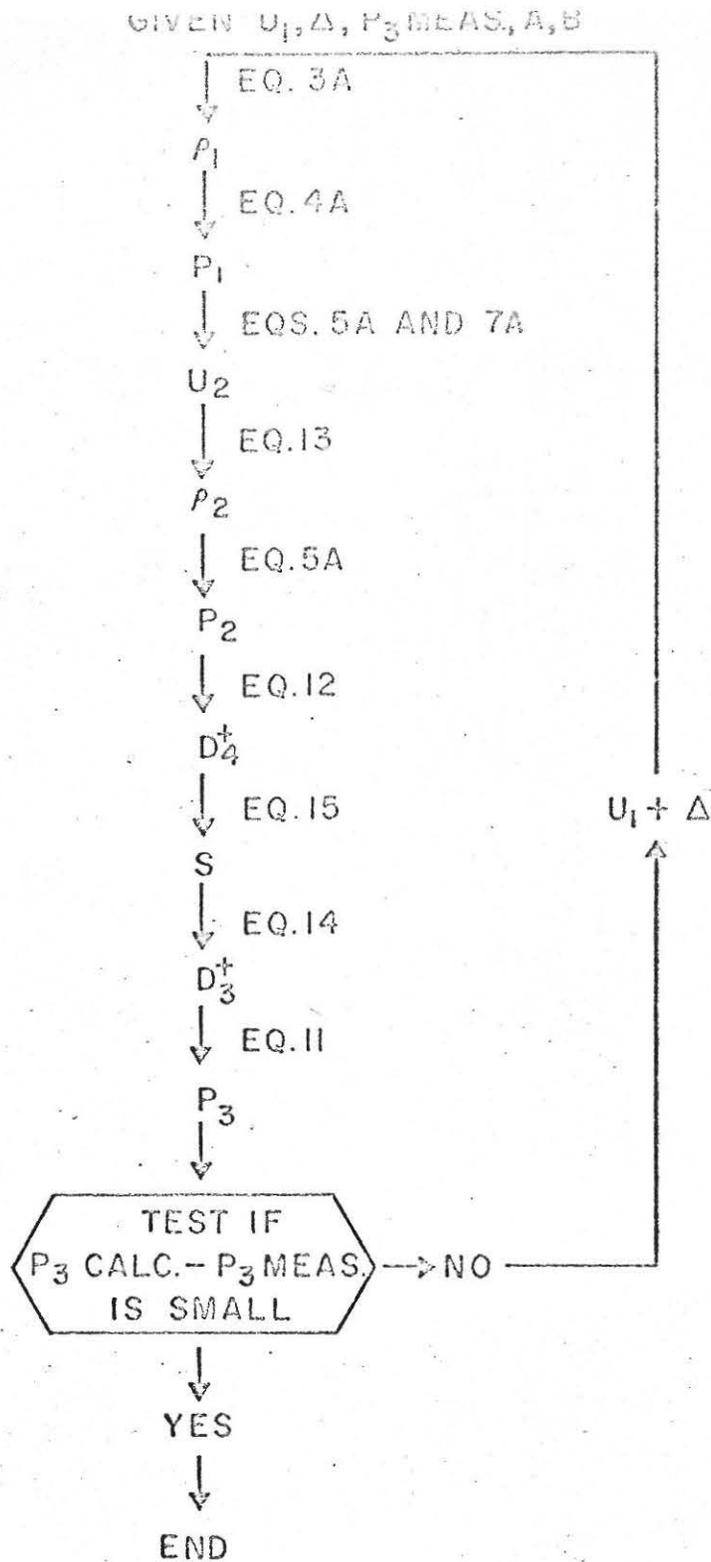
14

EQ. 3A

P_1

EQ. 4A

P_1

EQS. 5A AND 7A

U_2

EQ. 13

P_2

EQ. 5A

P_2

EQ. 12

D_4^+

EQ. 15

S

EQ. 14

D_3^+

EQ. 11

P_3

$U_1 + \Delta$

TEST IF
P_3 CALC. $-P_3$ MEAS.
IS SMALL → NO

YES

END

Figure 3.6   Flow of numerical impedance calculation
for materials of lower impedance than quartz.

15

INTERNAL REPORTS - 1972

1.   D.B. Hayes, "Computer Code Service Programs", Internal Report 01-72, June, 1972.