PLASTIC STRAIN IN LiF COMPRESSED UNIAXIALLY IN A <100> DIRECTION

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Define T = total strain

P = plastic strain

E = elastic strain

Assume that

(1) increments in elastic and plastic strain add to yield the increment in total strain:

$$dT = dE + dP \qquad (1)$$

(2) increments in stress, S, are linearly related to increments in elastic strain:

$$dS = C \, dE \qquad (2)$$

Let strain increments occur in time dt. Define $\dot{S} = dS/dt$, etc. and combine (1) and (2):

$$\dot{S} = C\dot{E} = C\dot{T} - C\dot{P} \qquad (3)$$

For <100> orientation, crystallographic coordinates coincide with laboratory coordinates, so S, C, T, and P are expressed in crystallographic coordinates. Elastic constants, C, depend on elastic strain.

For plane wave propagation in an x_1-direction (<100>), only the (11) component of stress-rate is required:

$$\dot{S}_{11} = C_{11jk} \dot{T}_{jk} - C_{11jk} \dot{P}_{jk} \qquad (4)$$

For uniaxial strain only T_{11} differs from zero, therefore

$$\dot{S}_{11} = C_{1111} \dot{T}_{11} - C_{11jk} \dot{P}_{jk} \qquad (5)$$

In order to apply Eq. (5), an explicit expression is required for $\dot{P}_{jk}$. Proceed as follows.
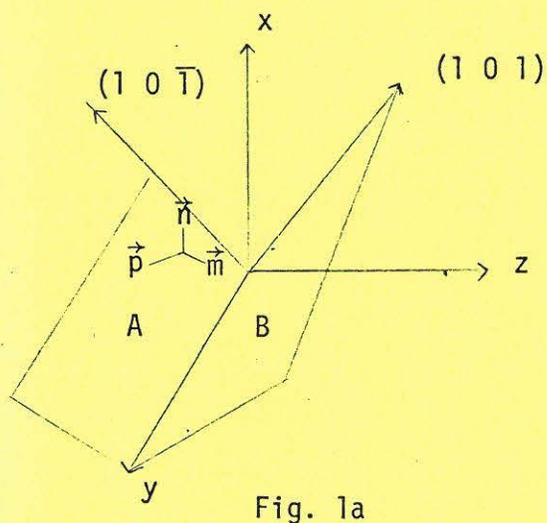

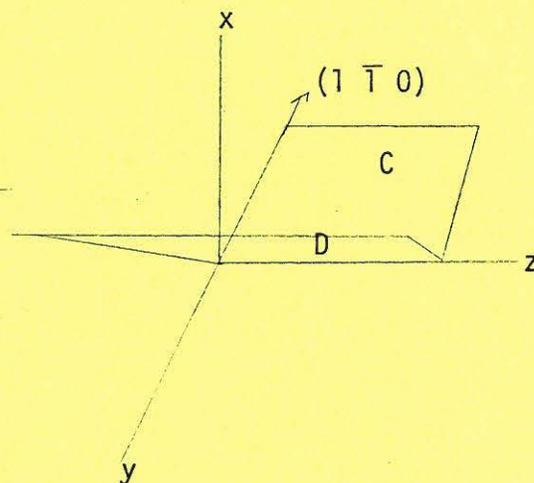(1 0 $\bar{1}$)   (1 0 1)

Fig. 1a

(1 $\bar{1}$ 0)

Fig. 1b

In Fig. 1 are shown the four glide planes of LiF which are active in this situation. There is but a single slip direction in each glide plane, so there are four active slip systems. These are

$$
\begin{aligned}
&\text{A:} \quad [1\ 0\ 1]\ (\bar{1}\ 0\ 1) \\
&\text{B:} \quad [\bar{1}\ 0\ 1]\ (1\ 0\ 1) \\
&\text{C:} \quad [\bar{1}\ \bar{1}\ 0]\ (1\ \bar{1}\ 0) \\
&\text{D:} \quad [1\ \bar{1}\ 0]\ (\bar{1}\ \bar{1}\ 0)
\end{aligned}
\tag{I}
$$

For each slip system define a unit normal to the glide plane,

$$
\vec{n} = \vec{i}_2,
$$

and a unit vector in the direction of slip,

$$
\vec{m} = \vec{i}_1
$$

These in turn define a third unit vector of a right-handed coordinate

system,

$$\vec{p} = \vec{m} \times \vec{n} = \vec{i}_3^{\,\prime}$$

The components of these unit vectors in crystallographic and laboratory coordinates are obtained from (I):

A: $\vec{m} = (-1, 0, 1)/\sqrt{2}$

   $\vec{n} = (1, 0, 1)/\sqrt{2}$

   $\vec{p} = (0, 1, 0)$

B: $\vec{m} = (1, 0, 1)/\sqrt{2}$

   $\vec{n} = (-1, 0, 1)/\sqrt{2}$

   $\vec{p} = (0, -1, 0)$

C: $\vec{m} = (1, -1, 0)/\sqrt{2}$ (II)

   $\vec{n} = (-1, -1, 0)/\sqrt{2}$

   $\vec{p} = (0, 0, -1)$

D: $\vec{m} = (-1, -1, 0)/\sqrt{2}$

   $\vec{n} = (1, -1, 0)/\sqrt{2}$

   $\vec{p} = (0, 0, 1)$

We wish now to relate plastic strain in crystal coordinates to slip on the individual glide planes. Consider a single slip system, Fig. 2.
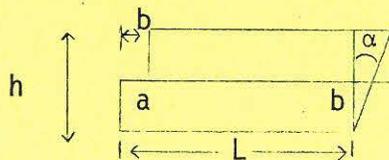

Fig. 2

Here ab is the intersection of the glide plane, assumed normal to the plane of the paper, with the plane of the paper. The upper half of the block has

slipped a distance b relative to the lower half, producing an angular
displacement α:

$$\alpha = b/h$$

If this slip has occurred as a consequence of a dislocation moving
through the distance L at speed v in time t, then

$$\alpha = vbt/Lh.$$

If the width of the block is w, normal to the paper, and the dislocation
line is also normal to the paper, its length is w, so

$$\dot{\alpha} \equiv \alpha/t = vbw/Lhw \quad .$$

But w/Lhw is line length per unit volume, defined as N_m^k, the density
of mobile dislocations active on glide plane k. More generally,

$$\dot{\alpha}^k = \phi vb N_m^k \tag{6}$$

where ϕ is a geometrical factor which accounts for curvature and incli-
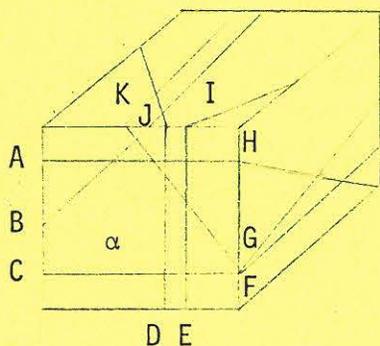nation of the dislocations.

Fig. 3

In Fig. 3 is represented a cube of LiF with {100} faces. Face α is
criss-crossed by traces of the six primary slip systems. Mobile disloca-
tions, both screw and edge, lie in these slip systems. If the dislocations

are evenly distributed among the six systems, one-sixth of the etch pits counted on a particular face should lie on each slip system. Then $N_m^k = N_m/6$, where N_m is the total number of etch pits per unit area on a {100} face. (For large applied stresses it is reasonable to suppose that all dislocations on the glide planes are mobile.)

The strain rate tensor corresponding to Eq. (6) is

$$\dot{P}^{k'} = \frac{\dot{\alpha}^k}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{7}$$

where superscript "k" denotes the particular glide plane represented in Fig. 2.

Plastic strain rate given by Eq. (7) in coordinates attached to the slip plane is transformed to crystal coordinates by a rotation $A = ||a_{ij}||$. The corresponding strain rate is

$$\dot{P}_{ij}^k = a_{im} a_{jn} \dot{P}_{mn}^{k'} \tag{8}$$

Applying this to Eq. (7) gives

$$\dot{P}_{ij}^k = (\dot{\alpha}^k/2) \cdot (a_{i1}a_{j2} + a_{j1}a_{i2}) \tag{9}$$

The total plastic strain rate to be used in Eq. (3) is obtained by adding contributions from the individual slip systems

$$\dot{P}_{ij} = \sum_k \dot{P}_{ij}^k \tag{10}$$

The transformation components a_{ij} in Eq. (8) can be expressed in terms of the unit vectors of the slip system coordinates, $\vec{i}_j'$

$$a_{kj} = \vec{i}_k \cdot \vec{i}_j' \equiv i_{jk}'$$

or

$$A = \begin{pmatrix} i_{11}' & i_{21}' & i_{31}' \\ i_{12}' & i_{22}' & i_{32}' \\ i_{13}' & i_{23}' & i_{33}' \end{pmatrix} \tag{11}$$

In terms of the unit vectors $\vec{m}$, $\vec{n}$, $\vec{p}$.

$$A = \begin{pmatrix} m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \\ m_3 & n_3 & p_3 \end{pmatrix} \tag{12}$$

In Eq. (11), i_{12}' is the x_2-component of $\vec{i}_1'$, etc.

Consider now a shear rate $\dot{\alpha}^A$ on slip system A. From (II) and Eq. (12):

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & 1 & 0 \end{pmatrix}$$

and, from Eq. (9),

$$\dot{P}_{11}^A = (\dot{\alpha}^A/2) \cdot (2a_{11}a_{12}) = (\dot{\alpha}^A/2)(-2/2) = -\dot{\alpha}^A/2 \tag{13a}$$

$$\dot{P}_{12}^A = (\dot{\alpha}^A/2) \cdot (a_{11}a_{22} + a_{12}a_{21}) = 0$$

$$\dot{P}_{13}^A = (\dot{\alpha}^A/2) \ (a_{11}a_{32} + a_{12}a_{31}) = 0$$

$$\dot{P}_{22}^A = (\dot{\alpha}^A/2) \ (2a_{21}a_{22}) = 0$$

$$\dot{P}_{23}^A = (\dot{\alpha}^A/2) \ (a_{21}a_{32} + a_{22}a_{31}) = 0$$

$$\dot{P}_{33}^A = (\dot{\alpha}^A/2) \cdot (2a_{31}a_{32}) = (\dot{\alpha}^A/2) \cdot (2/2) = \dot{\alpha}^A/2 \qquad \text{(13b)}$$

For slip system B, from (II) and Eq. (12):

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & -\sqrt{2} \\ 1 & 1 & 0 \end{pmatrix}$$

From Eq. (9), with k = B,

$$\dot{P}_{11}^B = -\dot{\alpha}^B/2, \quad \dot{P}_{33}^B = \dot{\alpha}^B/2 \qquad \text{(14)}$$

and all other components vanish.

For slip system C,

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -\sqrt{2} \end{pmatrix}$$

$$\dot{P}_{11}^C = -\dot{\alpha}^C/2, \quad \dot{P}_{22}^C = \dot{\alpha}^C/2.$$

All other components vanish.

For slip system D,

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

$$\dot{P}_{11}^{D} = -\dot{\alpha}^{D}/2, \quad \dot{P}_{22}^{D} = \dot{\alpha}^{D}/2$$

From symmetry of the crystal and the slip systems, one would expect $\dot{\alpha}^{A} = \dot{\alpha}^{B} = \dot{\alpha}^{C} = \dot{\alpha}^{D} = \dot{\alpha}$, so the total strain rate tensor is

$$\dot{P} = \dot{\alpha} \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{15}$$

Substituting this into Eq. (5) gives

$$\dot{S}_{11} = C_{1111}\dot{T}_{11} - C_{1111}(-2\dot{\alpha}) - C_{1122}\dot{\alpha} - C_{1133}\dot{\alpha}$$

Using reduced notation (Internal Report No. 01-69) and cubic symmetry $(C_{13} = C_{12})$,

$$\dot{S}_{11} = C_{11}\dot{T}_{11} + 2(C_{11} - C_{12})\dot{\alpha} \tag{16}$$

or

$$\dot{S}_{11} = C_{11}\dot{T}_{11} - (C_{11} - C_{12})\dot{P}_{11} \tag{17}$$

or, with Eq. (6),

$$\dot{S}_{11} = C_{11}\dot{T}_{11} - 2(C_{11} - C_{12})\phi vb(N_m/6) \tag{18}$$

In more familiar notation

$$\dot{\sigma}_x = C_{11}\dot{\epsilon}_x + 2(C_{11} - C_{12})\dot{\alpha} \qquad (19)$$

$$= C_{11}\dot{\epsilon}_x - (C_{11} - C_{12})\dot{\epsilon}_x^p \qquad (20)$$

$$= C_{11}\dot{\epsilon}_x - 2(C_{11} - C_{12})\phi vb(N_m/6) \qquad (21)$$

so $\dot{\epsilon}_x^p = \phi vbN_m/3$.

If maximum resolved plastic strain rate is defined as

$$\dot{\gamma}_p = (\dot{\epsilon}_x^p - \dot{\epsilon}_y^p)/2,$$

then from Eq. (15)

$$\dot{\gamma}_p = 3\dot{\epsilon}_x^p/4 \qquad (22)$$

Substituting Eq. (22) into (20) gives

$$\dot{\sigma}_x = C_{11}\dot{\epsilon}_x - \frac{4}{3}(C_{11} - C_{12})\dot{\gamma}_p \qquad (23)$$

Eq. (22) holds precisely for isotropic elastic materials and Eq. (23) translates to the isotropic form if $C_{11} \equiv \lambda + 2\mu$, $C_{11} - \dot{C}_{12} = 2\mu$.

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