# SHOCK WAVE PRECURSOR DECAY

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## I. Introduction

The relation of shock wave precursor decay to material properties has proved a useful tool in the study of various dynamic mechanical and thermal processes in solids.[1,2,3] The first appearance of such a relation, insofar as I know, is in a report by Harris.[4] He obtained, specifically for a polytropic gas, an equation relating rate of decrease of shock wave amplitude to an amplitude gradient immediately behind the shock front. He assumed the shock to be a discontinuity in field variables and neglected transfer of heat among mass elements behind the shock.

The Harris relation, or a variation, was used in various applications during the years of World War II and after.[5,6] Shortly after my introduction into the study of shock waves in solids in 1953, I generalized the Harris relation to an arbitrary equation of state. Then in 1960, having become interested in elastic-plastic material models, I extended it to include what I have called "stress-relaxing solids" and which have been labelled by others "viscoelastic solids." The better and historical label is "Maxwell solid."[7]

The first occasion I had to report the use of the decay equation externally was in a report of progress made on an Air Force Office of Scientific Research contract in 1968. In 1963 a related calculation was reported at an IUTAM Symposium on stress waves at Brown University, the Proceedings of this Symposium, edited by W. Prager and H. Kolsky, were published in 1965.[9] In the last 10 years variations on this relation, developed independently by Coleman and Gurtin,[10] have appeared frequently in the mathematical literature.[11,12,13] The purpose of this note is to show that each of the relaxation forms used is obtained from a more general expression. Whenever the pressure component normal to the shock front depends upon a variable other than specific volume of the material, V, Maxwell-like terms appear in the decay equation. Considerations are limited to plane waves and uniaxial strain.

## II. The Decay Equation

The calculation will be restricted to plane waves travelling in the +x direction. The flow equations are

Conservation of mass:

$$\frac{d\rho}{dt} + \rho\frac{\partial u}{\partial x} = 0 \tag{1}$$

Equation of motion:

$$\rho\frac{du}{dt} + \frac{\partial p^*}{\partial x} = 0 \tag{2}$$

Conservation of energy:

$$\frac{de}{dt} = \frac{dw}{dt} + \frac{dQ}{dt} \tag{3}$$

Here $\rho = 1/V$ is material density, u is material velocity, p* is the x-component of pressure including dissipation terms, e is specific internal energy; dw/dt is rate at which work is done on unit mass; dQ/dt is rate at which heat is delivered to unit mass; d/dt denotes the convective derivative.

Internal energy and entropy do not normally play a significant role in precursor decay, and their inclusion in the decay relation complicates the problem conceptually, so Eq. (3) will at first be ignored. Suppose then, that

$$p^* = p^*(V,\xi) \tag{4}$$

so that

$$\frac{dp^*}{dt} = \frac{\partial p^*}{\partial V}\frac{dV}{dt} + \frac{\partial p^*}{\partial \xi}\frac{d\xi}{dt}$$

$$= a^2 \frac{d\rho}{dt} + \alpha\frac{d\xi}{dt} \qquad (5)$$

where a is frozen sound speed, i.e., sound speed with $\xi$ = constant. Elimination of $d\rho/dt$ between Eqs. (1) and (5) gives

$$\frac{dp^*}{dt} + a^2\rho\,\frac{\partial u}{\partial x} - \alpha\frac{d\xi}{dt} = 0 \qquad (6)$$

Denote path of the shock front by $x = X(t)$ and shock velocity by $R = DX/Dt$. Derivative of any field variable $f(x,t)$ along a path parallel to the shock front is denoted $Df/Dt$:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + R\frac{\partial f}{\partial x}$$

$$= \frac{df}{dt} + (R-u)\,\frac{\partial f}{\partial x} \qquad (7)$$

since $df/dt = \partial f/\partial t + u\partial f/\partial x$. Substitution of Eq. (7) into Eqs. (2) and (6) gives the following pair:

$$\frac{Du}{Dt} - (R - u)\,\frac{\partial u}{\partial x} = -\,\frac{1}{\rho}\,\frac{\partial p^*}{\partial x} \qquad (8)$$

$$\frac{Dp^*}{Dt} + a^2\rho\,\frac{\partial u}{\partial x} = (R - u)\,\frac{\partial p^*}{\partial x} + \frac{d\xi}{dt} \qquad (9)$$

Now apply Eqs. (8) and (9) to the region just behind the discontinuity representing the shock. The shock jump condition which represents the equation of motion is

$$p^* = \rho_o Ru \tag{10}$$

where pressure in the unshocked state is assumed to be negligible and $\rho_o$ denotes unshocked mass density. Any change in shock pressure p* is accompanied by changes in R and u:

$$\frac{1}{p^*}\frac{Dp^*}{Dt} = \frac{1}{R}\frac{DR}{Dt} + \frac{1}{u}\frac{Du}{Dt} \tag{11}$$

$$= \frac{A}{R}\frac{Dp^*}{Dt} + \frac{B}{R}\frac{D\xi}{Dt} + \frac{1}{u}\frac{Du}{Dt} \tag{12}$$

where

$$A = \frac{\partial R}{\partial p^*},\ B = \frac{\partial R}{\partial \xi} \tag{13}$$

Eq. (12) can be used to eliminate Du/Dt from Eq. (8). The result is

$$\left(\frac{u}{p^*} - \frac{uA}{R}\right)\frac{Dp^*}{Dt} - (R - u)\frac{\partial u}{\partial x} = \frac{uB}{R}\frac{D\xi}{Dt} - \frac{1}{\rho}\frac{\partial p^*}{\partial x} \tag{14}$$

It is now possible to eliminate $\partial u/\partial x$ between Eqs. (9) and (14):

$$\left[(R - u) + \frac{a^2\rho u}{p^*} - \frac{a^2\rho Au}{R}\right]\frac{Dp^*}{Dt} = \left[(R - u)^2 - a^2\right]\frac{\partial p^*}{\partial x}$$

$$+ \alpha(R - u)\frac{d\xi}{dt} + \frac{a^2\rho Bu}{R}\frac{D\xi}{Dt} \tag{15}$$

With $D\xi/Dt = d\xi/dt + (R - u)\ \partial\xi/\partial x$, Eq. (15) becomes

$$\frac{Dp^*}{Dt} = M\frac{\partial p^*}{\partial x} + L\frac{d\xi}{dt} + N\frac{\partial \xi}{\partial x} \tag{16}$$

where

$$M = (R - u)\ \left[(R - u)^2 - a^2\right]/Q$$

$$L = \left[\alpha(R - u)^2 + a^2\rho_o Bu\right]/Q$$

$$N = a^2\rho_0 Bu(R - u)/Q$$

$$Q = (R - u)^2 + a^2(1 - \rho_0 Au)$$

Examples

In an elastic-plastic-relaxing solid, outside the yield surface, p* depends on both V and plastic strain, $\varepsilon_x^p$. If stresses are supported by elastic strains alone, and if plastic dilatation vanishes,

$$\begin{aligned} \dot{p}^* &= a^2\dot{\rho} - 2\mu\dot{\varepsilon}_x^p \\ &\equiv a^2\dot{\rho} - F \end{aligned} \qquad (17)$$

where $a^2$ is independent of $\varepsilon_x^p$ and F is the relaxation function. The dot over a symbol indicates the convective derivative. With $\xi = \varepsilon_x^p$, $\alpha = -2\mu$, B = 0, Eq. (16) becomes

$$\frac{Dp^*}{Dt} = M\frac{\partial p^*}{\partial x} - \frac{2\mu(R - u)^2}{(R - u)^2 + a^2(1 - \rho_0 Au)} F \qquad (18)$$

For piezoelectric material with electric field, E, and displacement, D, in the direction of wave propagation, p* depends on V and either E or D. For $\xi$ = D, D = D(x,t), both $\partial D/\partial t$ and $\partial D/\partial x$ vanish, so Eq. (16) reduces to

$$Dp^*/Dt = M\, \partial p^*/\partial x \qquad (19)$$

For $\xi$ = E, $\alpha = \partial p^*/\partial E$ is a piezoelectric coupling coefficient. Shock velocity depends on E, but the dependence is negligible except for very large fields, since electrostatic stress depends on $E^2$.

For a fluid with shear viscosity $\nu$,

$$p^* = p(V) + 4\nu\dot{\varepsilon}_x/3$$

$\xi = \dot{\varepsilon}_x$, $\alpha = 4\nu/3$, and B = 0. The last condition follows from the requirement that a shock be formed. Then

$$\frac{Dp^*}{Dt} = M\frac{\partial p^*}{\partial x} + \frac{4\nu(R-u)^2}{3[(R-u)^2 + a^2(1-\rho_0 Au)}\frac{d\dot{\varepsilon}_x}{dt} \tag{20}$$

For a plastic, shear-yielding solid in which a shear stress, $\tau$, proportional to plastic strain rate exists and there is no plastic dilatation,

$$\tau = 2\nu(\dot{\varepsilon}_x^p - \dot{\varepsilon}_y^p)/2$$

$$p^* = p_S^*(V) + 2\nu\dot{\varepsilon}_x^p$$

where $p_S^*(V)$ represents the quasistatic dependence of $p^*$ on V. Now $\xi = \dot{\varepsilon}_x^p$, $\alpha = 2\nu$, and $B = 0$.

Combinations of the above effects are possible. Extension of the analysis to include two or more variables, $\xi_1$, $\xi_2$, etc., is straightforward.

## III. Thermal Effects

Suppose that $\xi = S$, the specific entropy. Eq. (16) follows, as before, but it now contains derivatives of S. These can be eliminated through use of Eq. (3) and some assumptions about the underlying thermodynamics of the material.

Let

$$p^* = p^*(V,S,\xi).$$

Then

$$\frac{dp^*}{dt} = a^{*2}\frac{d\rho}{dt} + \frac{\Gamma^* T}{V}\frac{dS}{st} + \alpha^*\frac{d\xi}{dt} \tag{21}$$

Suppose the rate at which work is done on unit mass is

$$\frac{dw}{dt} = =p^* \frac{dV}{dt} + \eta^* \frac{d\xi}{dt} + \frac{dQ}{dt} \tag{22}$$

Substitution of Eq. (22) into (3) gives

$$\frac{de}{dt} = -p^* \frac{dV}{dt} + \eta^* \frac{d\xi}{dt} + \frac{dQ}{dt} \tag{23}$$

If internal energy is a state function of $V,S,\xi$, then

$$\frac{de}{dt} = \frac{\partial e}{\partial V}\bigg|_{S,\xi} \frac{dV}{dt} + \frac{\partial e}{\partial S}\bigg|_{V,\xi} \frac{dS}{dt} + \frac{\partial e}{\partial \xi}\bigg|_{V,S} \frac{d\xi}{dt} \tag{24}$$

$$\equiv -p \frac{dV}{dt} + T \frac{dS}{dt} + \eta \frac{d\xi}{dt} \tag{25}$$

If $\xi$ is a dissipative variable, e may still be a state function of V and S and $\eta \equiv 0$. Eliminating de/dt between Eqs. (23) and (25) yields

$$T \frac{dS}{dt} = (p - p^*) \frac{dV}{dt} + (\eta^* - \eta) \frac{d\xi}{dt} + \frac{dQ}{dt} \tag{26}$$

Entropy, S, can now be eliminated between Eqs. (21) and (26):

$$[a^{*2} + (p^* - p)V\Gamma^*] \frac{d\rho}{dt} = \frac{dp^*}{dt} - [\alpha^* + \rho\Gamma^*(\eta^* - \eta)] \frac{d\xi}{dt} - \rho\Gamma^* \frac{dQ}{dt} \tag{27}$$

If the system is reversible with $p^* = p$, $\eta^* = \eta$, and if $dQ/dt = 0$, $\alpha^* = \alpha$, Eq. (27) reduces to Eq. (5). The calculation proceeds as before from this point; the difference is that the extra terms of Eq. (27) must be carried along.

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