IMPACT RESPONSE OF THE SHORTED QUARTZ GAGE TO 40 KB

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ABSTRACT

A new fit to the piezoelectric constant, k, for the shorted quartz gage is reported. Two hitherto unreported measurements by Y. M. Gupta at about 30 kb and two points by G. Rosenberg at 42 kb are included. The result is

$$k \times 10^8 = 1.893 \pm .031 + (9.625 \pm 1.09) \times 10^{-3} p_x \quad \text{coulombs/cm}^2\text{kb}$$

The standard deviation of the measurements from the above line is 0.028×10^{-8} coulombs/cm^{2}kb.

A 60 kb point reported by Rosenberg has an abnormally small value of k. This is assumed to result from internal conduction and current relaxation, as reported by Graham for the shunted gage.

It is concluded that the shorted gage can be used, with caution, to 40 kb.

Hayes and Gupta in 1974 described the response of a 12 mm x 3 mm quartz gage in the shorted mode to uniform shock pressures between 20 and 26 kilobars (kb).[1] They recommended that the active area of the gage be calculated as if its diameter were the arithmetic mean of the inner and outer diameters of the gap separating sensitive area from the outer guard ring. The calibration constant inferred from their experiments was

$$k = (1.89 + 0.0107p_x) \times 10^{-8} \text{ coulombs/cm}^2\text{kb} \qquad (1)$$

$$20 < p_x < 26 \text{ kb.}$$

Since that time Gupta has made two additional calibration shots at about 30 kb for the same gage configuration,[2] and Rosenberg has made two shots near 40 kb and one at 58 kb.[3] The experiments reported by Hayes and Gupta and the later ones made by Gupta involved symmetric impact in which an x-cut quartz crystal mounted in the projectile struck a quartz gage mounted in a target holder. Consequently the particle velocity induced in the quartz gage was equal in each case to one-half the projectile velocity. Since the latter velocity is readily measured, the particle velocity was accurately known in each case. Gupta's data are entered in Table II, along with those reported in Ref. 1.

Rosenberg mounted two aluminum and one copper impactor in a single projectile and struck three gages simultaneously. In these experiments, pressure and particle velocity in the quartz are determined by intersection of the Rankine-Hugoniot (R-H) curves for impactor and target in the pressure, particle velocity space. Uncertainties in the R-H curves of the impactors are thus added to other uncertainties in the experiment. Rosenberg's projectile velocity was 0.54 mm/μsec. The R-H curve for x-cut quartz, according to Graham[4] is

$$p_x = 148.76U_p + 23.56U_p^2 \ . \qquad (2)$$

The aluminum impactor was 6061-T6 alloy, and the only Hugoniot data for this are given by Lundergan and Herrmann.[5] Asay[6] has expressed their result as

$$p_x = 6.4 + 150(U_p - .038) + 7.2(U_p - .038)^2 \quad (3)$$

McQueen and coworkers at LASL have measured the R-H curve for copper:[7]

$$U_s = 3.940 + 1.489U_p \quad , \text{mm}/\mu\text{sec} \quad (4)$$

with ρ_o = 8.93 g/cc, this gives

$$p_H = 351.8U_p + 133.0U_p^2 \quad (5)$$

In the second edition of the LLL Compendium (p.142), the same data are fitted by least squares to yield

$$U_s = (3.94 \pm .03) + (1.498 \pm .021)U_p \quad (6)$$

which gives

$$p_x = 351.84U_p + 133.7U_p^2 \quad (7)$$

Table II of the Compendium gives for $80 < p_x < 1800$ kb

$$U_s = (3.91 \pm .012) + (1.5 \pm .011)U_p \quad (8)$$

or

$$p_x = 349.16U_p + 134.84U_p^2 \quad (9)$$

McQueen et al.[7] found no significant elastic precursor in their experiments, so Eqs. (5), (7), and (9) should be comparable to hydrostatic data. Bridgman's measurements to 30 kb give, at 25°C, (Clark, p. 108),

$$1 - V/V_o = 0.727p - 1.46p^2$$

where p is in megabars. In order to correct this to the Hugoniot curve, assume a a Mie-Gruneisen equation of state:

$$p_H = p(V,T_o) + \frac{\Gamma C_V}{V}(T_H - T_o)$$

The rate of increase of temperature with pressure on the R-H curve is, at $T = T_o$,[8]

$$\frac{dT}{dp} = \left.\frac{\alpha V T/C_V}{1 + T\alpha\Gamma}\right]_{V=V_o, p=p_o, T=T_o}$$

where

$$\alpha_o = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = 0.531 \times 10^{-4}/°C \qquad \text{(Clark)}$$

$$\Gamma_o = 1.96 \quad \text{(Mott and Jones, Appendix II)}$$

Then

$$p_H - p(V,T_o) = \frac{\Gamma C_V}{V}(T_H - T_o) \simeq \frac{\Gamma C_V}{V}\left(\frac{dT}{dp}\right)_o$$

$$= \frac{\alpha_o \Gamma_o T_o}{1 + \alpha_o \Gamma_o T_o} p_H$$

$$= 0.030 p_H$$

and

$$1 - V/V_o = 0.7052 p_H - 1.374 p_H^2$$

This gives

$$p_H = 3.559 U_p (1 - 1.9485 p_H)^{-1/2}$$

Since the second term in brackets is small, it can be expanded as a continued fraction to give

$$p_H = 355.9 U_p + 123.4 U_p^2 \qquad (10)$$

Pressure and particle velocity in Eqs. (2), (3), (5), (7), (9), and (10) are kilobars and mm/μsec, respectively. For copper, the greatest difference in the R-H curves is between Eqs. (9) and (10). When calculating pressure for the impactor, particle velocity, U_p, must be replaced by the difference between projectile and particle velocity, $(W-U_p)$. From Eqs. (9) and (2), particle velocity and pressure are found to be, for W = .54 mm/μsec,

$$U_p = .37891 \text{ mm/}\mu\text{sec}, \qquad p_x = 59.74 \text{ kb.}$$

From Eqs. (10) and (2)

$$U_p = .38037 \text{ mm/}\mu\text{sec}, \qquad p_x = 59.95 \text{ kb.}$$

The error due to uncertainty in the copper R-H curve is negligible. Combining Eqs. (2) and (5) gives for the aluminum impactor

$$U_p = .2691 \text{ mm/}\mu\text{sec}, \qquad p_x = 41.73 \text{ kb.}$$

In its simplest form, wherein electric polarization is assumed to be proportional to shock pressure and elastic wave velocity in quartz is assumed constant, the theory of the quartz gage gives for the output current into a low resistance load

$$i(t) = \frac{kU_s A p_x(t)}{\ell} \qquad (11)$$

where k is the polarization constant, U_s is elastic wave velocity, A is effective gage area and ℓ is gage thickness. Equation (11) applies until the elastic wave reaches the back surface of the gage. $P_x(t)$ is pressure at the shocked interface. Since U_s is not constant, Eq. (11) applies strictly in this approximation only if $p_x(t) = p_1$ = constant.

In simplest terms, the calibration process consists of measuring pairs of values $(p_x, i\ell/A)$ for various uniform shock states and determining the form of the resulting curve, $p_x(i\ell/A)$. If quartz were a precisely linear elastic medium, the ratio $i\ell/p_xA = kU_s$ would be constant. It is not, and this is acknowledged my making kU_s a function of p_x. The problem is further simplified by setting $kU_s = k'U_s'$ where U_s' is set equal to the elastic longitudinal wave velocity, and the variation of U_s with pressure is absorbed in k'. This leads to no error, since kU_s is used as a product. A further simplification which does lead to an error results from the way p_x is determined in the calibration. The measured quantity, directly or indirectly, as indicated in the preceding paragraphs, is particle velocity, U_p. The relation between p_x and U_p is quadratic, as given in Eq. (2). The convention is adopted that p_x and U_p are linearly related, and the value of p_x in Eq. (11) is replaced by $p_x'' \equiv A'U_p$. The result is that k' is replaced by k" where

$$k''U_s'p_x'' = i\ell/A$$

and

$$k'' = i\ell/AU_s'p_x'' \tag{12}$$

In other words, a relation is established between $i\ell/A$ and p_x'' in place of the required relation between $i\ell/A$ and p_x. With k" determined as in Eq. (12), an experiment in which the measured gage current is I is interpreted as corresponding to a pressure

$$p'''_{(x)} = \frac{I\ell}{Ak''U_s'} \tag{13}$$

But if p_x'' in Eq. (12) is less than the true value, p_x, then k" is greater than the true value, k, and p_x''' is less than the true value, $p_x^{(t)}$. In order to correct for this error, k" must be multiplied by p_x''/p_x, and true pressure is given by

$$p_x^{(t)} = \frac{I\ell p_x}{AU_s'k''p_x''} \tag{14}$$

Since p_x is given by Eq. (2) and $p''_x \equiv A'U_p$, it follows that

$$p_x/p''_x = (A/A') + Bp''_x/(A')^2 \tag{15}$$

$$k'/k'' = p''_x/p_x = (A'/A)/(1 + Bp''_x/AA') \tag{16}$$

If k" has been determined to be a linear function of p''_x,

$$10^8k'' = k''_o + k''_1p''_x$$

Then

$$10^8k' \simeq ak''_o + a^2(k'_1 - bk''_o)p_x + 2a^3b(bk''_o - k''_1)p_x^2 \tag{17}$$

where a = A'/A, b = B/AA'. With A' = 151.6, A = 148.76, B = 23.56, a = 1.019, b = 1.045×10^{-3}, Eq. (17) becomes

$$10^8k' = 1.019\ k''_o + 1.038(k''_1 - 1.045 \times 10^{-3}k''_o)p_x$$
$$+ 2.211 \times 10^{-3}\ (1.045 \times 10^{-3}k''_o - k''_1)p_x^2 \tag{18}$$

Values of k'_o and k'_1 obtained from Eq. (18) will approximate the relation between k' and p_x, but they will not be the same as those obtained by calculating p_x from Eq. (15) and k' from Eq. (16) and fitting the resulting set of data by a straight line. This is illustrated in Table III.

The third term in Eq. (18) contributes 0.8% to k' at 30 kb, 1.3% at 40 kb, and 2.8% at 60 kb. At or below 40 kb it can be neglected. The ratio p_x/p''_x is given in Table I for several values of p''_x.

TABLE I

p''_x =	10	20	30	40	50	60 kb
p_x/p''_x =	.992	1.002	1.012	1.022	1.033	1.043

Since current relaxation and internal conduction occur at pressures greater than 40 kb,[9] one should not normally plan to use quartz gages above 40 kb. Graham's remarks apply to the shunted gage. Reproducibility of the shorted gage has been questioned at even lower pressures.

Values of k" and p_x'' reported by Hayes and Gupta,[(1)] by Gupta[(2)] and calculated from Rosenberg's data are given in Table II. Values of p_x and k' calculated from Eq. (15) and (16) with A' = 151.6, A = 148.76, B = 23.56 are also given in Table II. This value of A' is the same value used by Hayes and Gupta and by Gupta, but it differs slightly from that used by Rosenberg.

TABLE II

Shot No.*	U_p (mm/μsec)	p_x'' (kb)	$k''\times10^8$ (coulombs/cm^{2}kb)	p_x (kb)	$k'\times10^8$ (coulombs/cm^{2}kb)
HG1	.1305	19.78	2.095	19.81	2.092
HG2	.1315	19.93	2.060	19.97	2.056
HG3	.1315	19.93	2.085	19.97	2.081
HG4	.1362	20.69	2.134	20.74	2.129
HG5	.1695	25.70	2.168	25.90	2.151
HG6	.174	26.38	2.160	26.60	2.142
G1		29.11	2.213	29.44	2.188
G2		31.66	2.193	32.10	2.163
R1	.269	40.78	2.324	41.73	2.271
R2	.269	40.78	2.388	41.73	2.334
R3	.379	57.46	2.373	59.77	2.281

*HG refers to Hayes and Gupta, Ref. 1; G to Gupta, Ref. 3; R to Rosenberg, Ref. 3. $p_x'' = 151.6U_p$.

Values of k", p_x'', k', p_x are plotted in Fig. 1. Four sets of values of k_o'', k_1'', k_o', k_1' obtained by linear least squares fitted to the data of Table II are given in Table III.

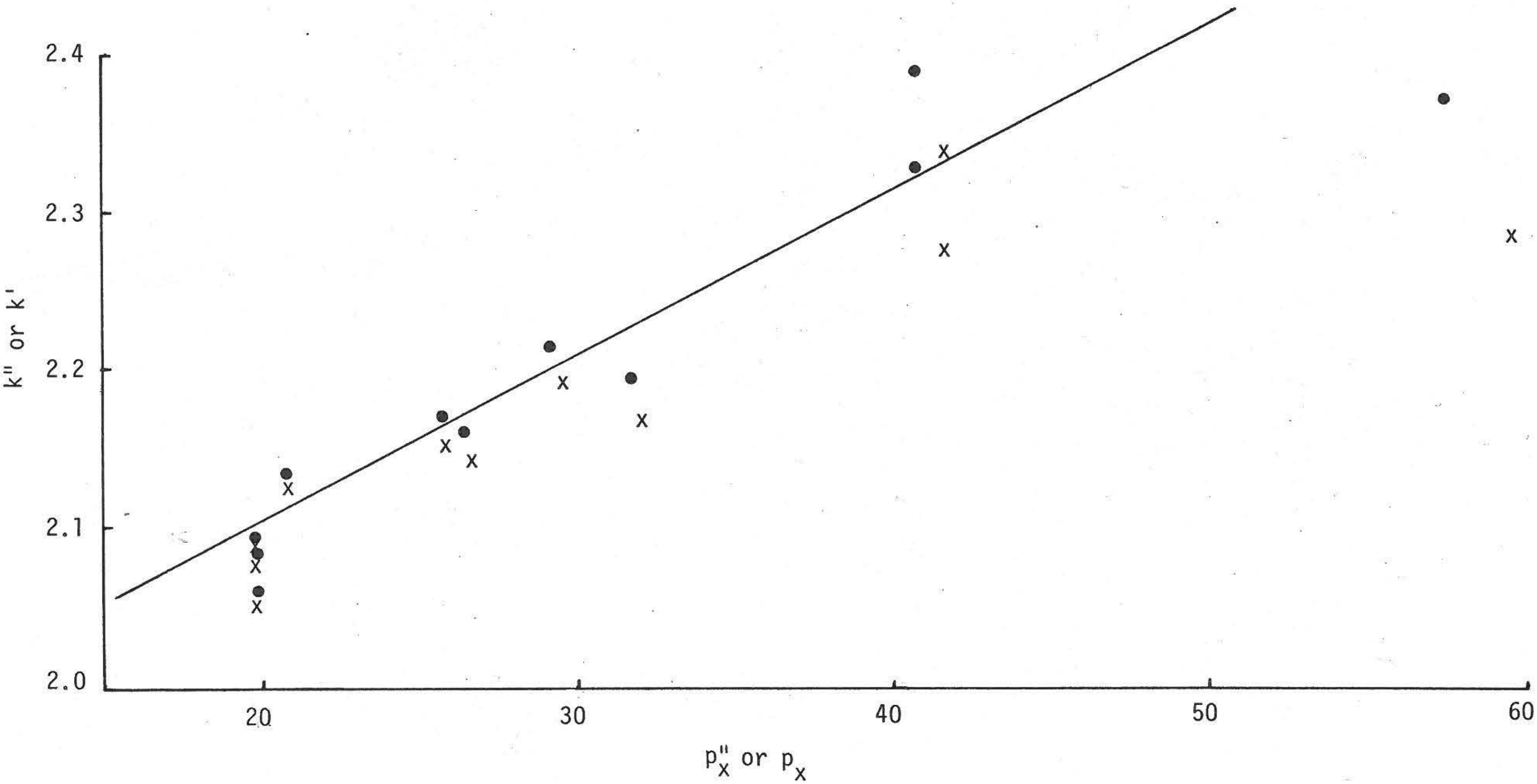

Figure 1. Pressure dependence of the quartz gage constant. Solid line is linear least squares with point R3 omitted. x - k'; ● - k".

TABLE III

Constants determined by least squares fitting of the data in Table II: $k = k_o + k_1 p_x$. Underlined values are assumed at present to best represent gage performance in the interval $20 \le p_x \le 40$ kb. Standard deviations for these best values are 0.031 for k_o' and 1.09×10^{-3} for k_1'.

Source	$k_o'' \times 10^8$	$k_1'' \times 10^8$	$k_o' \times 10^8$	$k_1' \times 10^8$
HG	1.847	12.23×10^{-3}	1.894	9.690×10^{-3}
HG + G	1.884	10.55×10^{-3}	1.931	8.004×10^{-3}
HG + G + R1 + R2	1.841	12.42×10^{-3}	$\underline{1.893}$	$\underline{9.625 \times 10^{-3}}$
HG + G + R	1.930	8.926×10^{-3}	1.982	6.177×10^{-3}

The first set, labeled HG, are obtained from the data of Ref. 1. These values of k_o'', k_1'' differ slightly from those given in Ref. 1 because of differences in rounding off. The second set include Gupta's data, the third set includes the two 40 kb points of Rosenberg's data, the fourth set includes Rosenberg's 60 kb point. It is apparent from both Fig. 1 and Table III that the 60 kb point represents a deviation from the linear dependence of k on p_x, which is otherwise evident to 40 kb. Without further calibrations between 40 and 60 kb, the reproducibility of such nonlinear effects cannot be determined. It is reasonable to suppose, however, that the shorted gage, whose performance is reported here, suffers the same deviant behavior as reported by Graham above 40 kb[9], or worse. Consequently the third set of values of k_o' and k_1' given in Table III are assumed, at present, to represent the best available values of the calibration constant for the 1/2" x 1/8" x 3 mm shorted gage; it appears to be reliable to 40 kb for measuring the initial rise in shock pressure.

A set of self-consistent constants for quartz is

$$\rho_o = 2.650 \text{ g/cc}, \quad U_s' = 5.72 \times 10^5 \text{ cm/sec}, \quad Z_Q = 151.6 \text{ kb } \mu\text{sec/mm}$$

$$k' \times 10^8 = 1.893 \pm .031 + (9.625 \pm 1.09) \times 10^{-3} p_x, \text{ coulombs/cm}^2\text{kb} \quad (19)$$

This appears to be the best value of k' to use at this time for the 1/2" x 1/8" x 3 mm shorted gage, though it hardly differs significantly from earlier values.

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