Q-METER AND A TERMINATED CABLE

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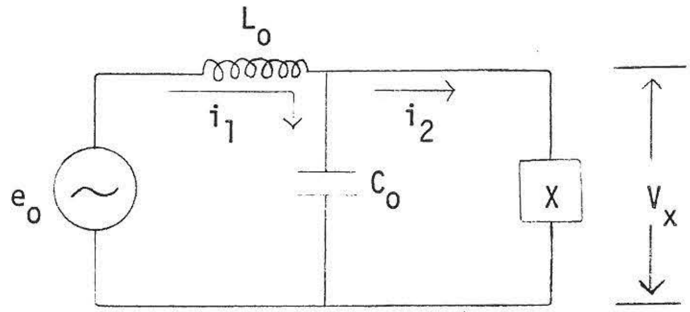
We sometimes have trouble with oscillatory signals from gages. The 1S2 provides some insight into the electrical character of the gage circuit. It would be nice to have more.

One possibility is the Q-meter, which measures resonant frequency. If a coil is connected between "LO" and "HIGH" and a transmission line to "GND" and "HIGH", is it possible to measure reactive deviations from a properly terminated line, an open circuit line or a shorted line? The analysis given here suggests that it may be possible, but some experiments and computations will be required to determine the utility of the procedure. The relevant equations are (17) and (18).



Circuit Equations

$$e_o = i_1 \cdot j\omega L_o + \frac{i_1 - i_2}{j\omega C_o} \qquad (1)$$

$$0 = \frac{i_2 - i_1}{j\omega C_o} + i_2 X \qquad (2)$$

X is impedance of the element connected between points marked "GND" and "HI".

$$j = -\sqrt{-1}$$

Elements C_o and X in parallel have a net impedance

$$Z_N = R_N + jX_N \qquad (2.1)$$

so Eqs. (1) and (2) can be combined to the form

$$i_1 (j\omega L_o + R_N + jX_N) = e_o \qquad (3)$$

At resonance the reactive components of the impedance vanish:

$$\omega L_o + X_N = 0 \qquad (4)$$

Since the capacitor C_o and the unknown X are in parallel, their admittances add:

$$j\omega C_o + \frac{1}{X} = \frac{1}{Z_N} \qquad (5)$$

With Eq. (2.1) and

$$X \equiv X_R + jX_x \qquad (6)$$

Eq. (5) becomes

$$\frac{Z_N^*}{|Z_N|^2} = j\omega C_o + \frac{X^*}{|X|^2}$$

$$Z_N^* = R_N - jX_N \; ; \qquad X^* = X_R - jX_x$$

$$Z_N^* = \left|\frac{Z_N}{X}\right|^2 X_R + j|Z_N|^2(\omega C_o - \frac{X_x}{|X|^2})$$

So the resonance condition becomes

$$\omega L_o - |Z_N|^2(\omega C_o - \frac{X_x}{|X|^2}) = 0 \tag{7}$$

$$|Z_N|^2 = \left[\frac{X_R^2}{|X|^4} + (\omega C_o - \frac{X_x}{|X|^2})^2\right]^{-1} \tag{8}$$

EXAMPLES

1. Lumped Constant Circuits

1.1 X is Infinite (Open Circuit)

$$\frac{X_R^2}{|X|^4} \rightarrow 0, \qquad \frac{X_x}{|X|^2} \rightarrow 0$$

$$|Z_N|^2 \rightarrow \frac{1}{\omega^2 C_o^2}$$

$$\text{Eq. (7)} \rightarrow \omega L_o - \frac{1}{\omega C_o} = 0$$

1.2

$$X = -j/\omega C_1$$

$$X_R = 0$$

$$X_x = -1/\omega C_1$$

$$\frac{X_x}{|X|^2} = -\omega C_1$$

$$|Z_N|^2 = (\omega C_o + \omega C_1)^{-2}$$

Eq. (7) becomes

$$\omega^2 = \frac{1}{L_o(C_o + C_1)}$$

The capacitance is additive.

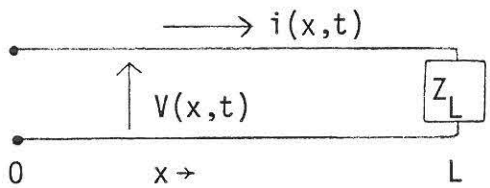
1.3 $$X = j\omega L$$

$$\omega L_o - \frac{1}{\omega C_o - (1/\omega L)} = 0$$

$$\omega^2 = 1/\bar{L}C_o$$

$$1/\bar{L} = \frac{1}{L} + \frac{1}{L_o}$$

2. X is a Loaded Transmission Line with Impedance R_o



$$i(x,t) = I_o e^{-jkx} + I_1 e^{jkx} \qquad (9.1)$$

$$V(x,t) = R_o I_o{}^{-jkx} - R_o I_1{}^{jkx} \qquad (9.2)$$

$k = \omega/c$; the time factor, $e^{j\omega t}$ is understood.

At $x = L$, $V = iZ_L$,

$$R_o I_o e^{-jkL} - R_o I_1 e^{jkL} = I_o Z_L e^{-jkL} + I_1 Z_L e^{jkL}$$

$$I_1 = I_o \frac{R_o - Z_L}{R_o + Z_L} e^{-2jkL} \equiv I_o B \qquad (10)$$

The impedance of the line is

$$X = \frac{V(0,t)}{i(0,t)} = R_o(1 - I_1/I_o)/(1 + I_1/I_o)$$

$$= R_o \frac{(1-B)}{(1+B)} = \frac{R_o}{|1+B|^2}(1-B)(1+B^*)$$

$$= \frac{R_o}{|1+B|^2}(1 - BB^* - 2jB_i) \quad (11)$$

where

$$B = B_R + jB_i \quad (12)$$

Eq. (10) gives

$$B_R = \frac{R_o^2 - |Z_L|^2}{|R_o + Z_L|^2}\cos 2kL - \frac{2R_oX_L}{|R_o + Z_L|^2}\sin 2kL \quad (13.1)$$

$$B_i = \frac{-2R_oX_L}{|R_o + Z_L|^2}\cos 2kL - \frac{R_o^2 - |Z_L|^2}{|R_o + Z_L|^2}\sin 2kL \quad (13.2)$$

where $Z_L = R_L + jX_L$

From Eq. (11) $X = X_R + jX_X$

$$X_R = \frac{R_o(1 - |B|^2)}{|1 + B|^2} \quad (14.1)$$

$$X_X = -\frac{2R_oB_i}{|1+B|^2} \quad (14.2)$$

2.1 $Z_L = R_o$, $X_L = 0$

From Eq. (13.1) $B_R = 0$

From Eq. (13.2) $B_i = 0$

From Eq. (14.1) $X_R = R_o$

From Eq. (14.2) $X_X = 0$

From Eq. (8) $$|Z_N|^2 = \frac{R_o^{\,2}}{1 + \omega^2 C_o^{\,2} R_o^{\,2}}$$

From Eq. (7) $$\omega L_o - \frac{R_o^{\,2}\omega C_o}{1 + \omega^2 C_o^{\,2} R_o^{\,2}} = 0$$

$$\frac{\omega^2}{\omega_o^{\,2}} = 1 - \frac{T_L}{T_c} \qquad (15)$$

where

$$\omega_o = 1/\sqrt{L_o C_o}$$

$$\left.\begin{aligned} T_L &= L_o/R_o \\ T_C &= R_o C_o \end{aligned}\right\}$$ time constants for inductor and capacitor

2.2 $Z_L \to \infty$ (open circuit)

From Eq. (13) $$B_R \to \cos 2kL$$

$$B_i \to \sin 2kL$$

$$|B|^2 = 1$$

$$|1+B|^2 = 2\sin^2 kL$$

From Eq. (14) $$X_R \to 0$$

$$X_x \to -R_o \cot kL \qquad (15.1)$$

From Eq. (8) $$|Z_N|^2 = \frac{1}{(\omega C_o - \frac{1}{X_x})^2}$$

From Eq. (7) $$\omega L_o - \frac{1}{(\omega C_o - \frac{1}{X_x})} = 0 \qquad (15.2)$$

$$\omega L_o - \frac{1}{\omega C_o + \frac{\tan kL}{R_o}} = 0$$

$$\omega^2 C_o L_o + \frac{\omega L_o}{R_o} \tan kL - 1 = 0$$

$$\frac{\omega^2}{\omega_o^2} + \frac{\omega}{\omega_o}\sqrt{\frac{T_L}{T_C}} \tan kL - 1 = 0 \qquad (16)$$

2.3 $R_L = 0, X_L \neq 0$

Eqs. (13) give

$$B_R = \frac{R_o^2 - X_L^2}{R_o^2 + X_L^2} \cos 2kL - \frac{2R_oX_L}{R_o^2 + X_L^2} \sin 2kL$$

$$B_i = \frac{-2R_oX_L}{R_o^2 + X_L^2} \cos 2kL - \frac{R_o^2 - X_L^2}{R_o^2 + X_L^2} \sin 2kL$$

$$|B|^2 = 1$$

$$|1+B|^2 = 2[R_o^2 + X_L^2 + (R_o \cos kL - X_L \sin kL)^2 - (R_o \sin kL + X_L \cos kL)^2]/(R_o^2 + X_L)^2$$

From Eq. (14)

$$X_R = 0$$

$$X_x = \frac{2R_o(R_o \cos kL - X_L \sin kL)(R_o \sin kL + X_L \cos kL)}{R_o^2 + X_L^2 + (R_o \cos kL - X_L \sin kL)^2 - (R_o \sin kL + X_L \cos kL)^2} \qquad (17)$$

From Eq. (8)

$$|Z_N|^2 = (\omega C_o - \frac{1}{X_x})^{-2}$$

From Eq. (7)

$$\omega L_o - \frac{1}{\omega C_o - \frac{1}{X_x}} = 0$$

$$X_x = \frac{\omega L_o}{\omega^2 L_o C_o - 1} \qquad (18)$$

X_x can be determined from Eq. (18) and X_L from Eq. (17). Note that Eq. (18) and Eq. (15.2) are identical and that Eq. (17) reduces to Eq. (15.1) when $X_L \to \infty$.

APPENDIX

One other case was considered; it doesn't seem to lead to much, but since the analysis is tedious, the details are included here.

2.4 Termination of the line differs but slightly from R_o. (It can be assumed that any reactive component at the terminated end may have either a very small or a very large value at some frequency. For the purposes of this example, suppose that the range of interest in far from resonance, so both ε and y are small.)

$$Z_L = R_o(1+\varepsilon) + jyR_o; \qquad y,\varepsilon \ll 1$$

$$|Z_L|^2 \simeq R_o^2 (1 + 2\varepsilon + y^2)$$

$$R_o^2 - |Z_L|^2 \simeq -R_o^2 (2\varepsilon + y^2)$$

$$|R_o + Z_L|^2 = |R_o(2+\varepsilon) + jyR_o|^2$$

$$\simeq 4R_o^2 (1 + \varepsilon + \frac{y^2}{4})$$

From Eq. (13)

$$B_R \simeq -\frac{1}{4}[(2\varepsilon + y^2)\cos 2kL + 2y\sin 2kL]$$

$$B_i \simeq \frac{1}{4}[(2\varepsilon + y^2)\sin 2kL - 2y\cos 2kL]$$

$$|B|^2 \simeq (\varepsilon^2 + y^2)/4$$

$$|1+B|^2 \simeq 1 - (\varepsilon + \frac{y^2}{2})\cos 2kL - y\sin 2kL$$

From Eq. (14)

$$X_R \simeq R_o[1 + (\varepsilon + \frac{y^2}{2})\cos 2kL + y\sin 2kL]$$

$$X_x \simeq -\frac{R_o}{2}[(2\varepsilon + y^2)\sin 2kL - 2y\cos 2kL]$$

$$|X|^2 \simeq X_R^2 \simeq R_o^2[1 + 2(\varepsilon + \frac{y^2}{2})\cos 2kL + 2y\sin 2kL]$$

From Eq. (8)

$$|Z_N|^2 \simeq [\frac{1}{X_R^2} + (\omega C_o - \frac{X_x}{X_R^2})^2]^{-1}$$

$$|Z_N|^2 (\omega C_o - \frac{X_x}{|X|^2}) \simeq \frac{X_R(\omega C_o X_R - \frac{X_x}{X_R})}{1 + (\omega C_o X_R - \frac{X_x}{X_R})^2}$$

If $\omega C_o R_o << 1$, which is commonly true,

$$|Z_N|^2(\omega C_o - \frac{X_x}{|X|^2}) \simeq X_R(\omega C_o X_R - \frac{X_x}{X_R})$$

$$\simeq R_o[\omega C_o R_o + (\varepsilon + \frac{y^2}{2})\sin 2kL - y\cos 2kL]$$

Eq. (7) becomes

$$\omega(T_L - T_C) - (\varepsilon + \frac{y^2}{2})\sin 2kL + y\cos 2kL = 0$$

INTERNAL REPORTS - 1980

1. G.E. Duvall, "Limits of the Variation of P_X in Uniaxial Strain", Internal Report 80-01, February, 1980.

2. G.E. Duvall, "Mechanical Strength, Dislocations, and Precursor Decay", Internal Report 80-02, November, 1980.