LIMITS OF THE VARIATION OF P_x IN UNIAXIAL STRAIN

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An isotropic elastic-plastic material is restrained by the following conditions:

I. The yield condition. This may take various forms. Whatever the form, it will not deviate greatly from the commonly-used Maxwell-Hinckey-Von Mises condition

$$(p_x - \bar{p})^2 + (p_y - \bar{p})^2 + (p_z - \bar{p})^2 \leqq 2Y^2/3 \qquad (1)^*$$

where Y may be a function of $\bar{p}$ and plastic work or plastic strain.

II. The incremental form of Hooke's law:

$$dp_{ij} = \lambda\delta_{ij}\, d\Theta \quad + 2\mu\, d\varepsilon^e_{ij} \qquad (2)$$

where $-d\Theta$ is increment in dilatation and $d\varepsilon_{ij}$ is the strain increment, taken positive in compression.

The mean pressure, $\bar{p}$, is given by

$$\bar{p} = (p_{11} + p_{22} + p_{33})/3 \qquad (3)$$

Eq. (1) is for principal axis coordinates. Represent such coordinates by the Cartesian axes shown in Fig. 1. OA is a line which makes equal angles with the three axes. Eq. (3) describes a plane with normal OA. On OA, $p_x = p_y = p_z$, so at the intersection of the plane of Eq. (3) with the line OA, say point Q, $\bar{p}$ is equal to the length of the intersected line.

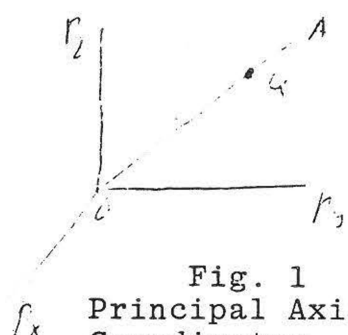

Fig. 1
Principal Axis Coordinates

In this case, Eq. (1) represents a sphere of radius $Y\sqrt{2/3}$ with Q as center. The intersection of this sphere with the plane of Eq. (3) is a circle of radius $Y\sqrt{2/3}$. The set of all such circles formed in a given deformation process produces a figure of rotation about the axis OA, with normal distance

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* Jaeger, Elasticity, Fracture and Flow, p. 92, Eqs. (22), (24)

from OA to the surface equal to $Y\sqrt{2/3}$. Since Y may depend on both $\bar{p}$ and plastic work or strain, the figure may not be the same for different deformation processes, but it always exists and Y is always finite. It is called the "yield surface".

Let the yield surface in Fig. 2 correspond to zero plastic work. Each deformation process is described by a locus of points in the (p_x, p_y, p_z) space. For example, the locus OD shown represents a particular process which remains elastic until it intersects the yield surface at D. If plastic work follows this contact, the surface will be altered. Consider for simplicity a material in which Y is constant. The yield surface is then a circular cylinder. It is conveniently represented by its projection on the plane $\bar{p}$ = const., Fig. 3. A process of uniaxial compression in the x-direction has a locus whose projection lies on the line AOB where $p_y = p_z$.

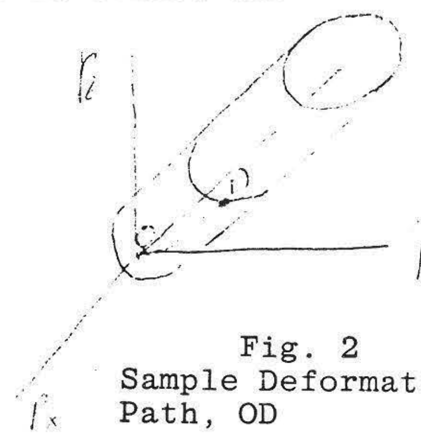


Fig. 2
Sample Deformation Path, OD

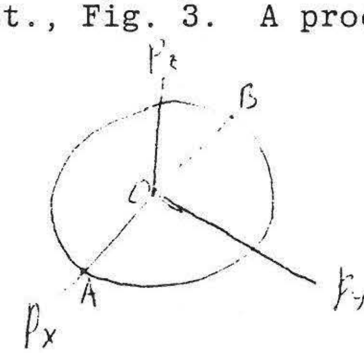


Fig. 3
Projection of Cylindrical Yield Surface on the Plane $\bar{p}$ = const.

For uniaxial compression, Eq. (1) reduces to

$$(p_x - p_y)^2 \leqq Y^2$$

For elastic uniaxial compression the inequality holds,

$$(p_x - p_y)^2 = \left(\frac{1-2\nu}{1-\nu}\right)^2 p_x^{\;2} < Y^2$$

$$= 4\tau^2, \text{ where } \tau = \text{maximum resolved shear stress.}$$

In the compression phase, starting from $p_x = p_y = 0$, $p_x > p_y$ and

$$p_x - p_y = \frac{1-2\nu}{1-\nu} p_x \equiv g p_x \qquad (4)$$

$$g = 1, \; \nu = 0$$

$$g = 1/2, \ \nu = 1/3$$

$$g = 0, \ \nu = 1/2$$

So usually $0 < g < 1$. Eq. (4) can also be written

$$p_y = p_x(1 - g)$$

also

$$p_z = p_x(1 - g)$$

This requires that the locus of states move from O to A as deformation proceeds. (Note that for a fluid the locus of states corresponds to the axis of the cylinder and the radius of the yield surface is identically zero.)

As compression proceeds, contact with the yield surface occurs at A. The material is now in the plastic state. If further compression occurs, the state point is constrained by the condition $p_y = p_z$ to move vertically on the yield surface, unless Y depends on $\bar{p}$, plastic work, or both. In that case, the projection of the state point in Fig. 3 may move along the radius OA, or its projection, according to changes which occur in Y.

In order to determine whether the state point remains at A in Fig. 3 or moves inward or outward, consider an increment of strain to be entirely elastic. Then

$$dp_x = (\lambda + 2\mu)d\varepsilon_x$$

$$dp_y = \lambda d\varepsilon_x$$

$$d(p_x - p_y) = 2\mu d\varepsilon_x$$

Since $d\varepsilon_x > 0$, $d(p_x - p_y) > 0$. If $d(p_x - p_y)/d\varepsilon_x = 2\mu > dY/d\varepsilon_x$ (which it usually is), the assumed strain increment, if entirely elastic, will carry the state point outside the yield surface. Since that is not allowed, all or part of $d\varepsilon_x$ is plastic and the state point remains at A (or moves outward slightly to correspond to the change in Y.). If, however, $d\varepsilon_x < 0$, corresponding to rarefaction, $d(p_x - p_y) < 0$, dY will not normally decrease--or will decrease more slowly than $d(p_x - p_y)$, therefore the state point moves inside the yield surface and the assumption of elastic deformation is correct. If rarefaction continues, the state point moves from A through O to B, where $p_y - p_x = Y$ and yield again occurs.

In the elastic process of uniaxial compression,

$$p_x - p_y = \frac{1-2\nu}{1-\nu} p_x$$

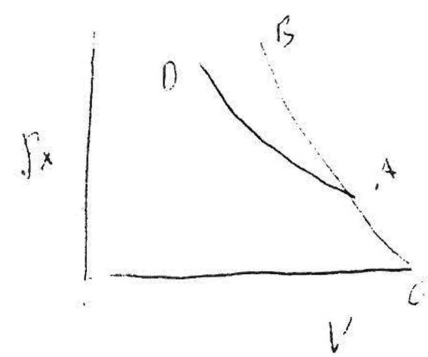
$$\bar{p} = \frac{1}{3} \frac{1+\nu}{1-\nu} p_x \qquad (6)$$

so

$$p_x - p_y = \frac{3(1-2\nu)}{1+\nu} \bar{p} \qquad (5)$$

i.e. the radius of the circle on which the state point lies increases linearly with distance along the axis of the yield surface. In other words, it lies on a cone with radius given by Eq. (5). (If ν depends on $\bar{p}$, the cone will be deformed, but the principle is unchanged.). When the rhs of Eq. (5) exceeds Y, this surface is metastable and, if relaxation to the plastic state requires a finite time, the state point lies between two circles: one of radius given by (5) and one of radius given by Y. For a given $\bar{p}$, i.e. a given density, it is not possible to exceed the value of p_x given by Eq. (6).

In the (p_x, V) plane, OAB represents stable and metastable elastic states, AD represents plastic states in compression. Any values of p_x, derived from experiments which lie above OAB are suspect.



Note

If p_x depends on strain rate, it may lie outside the cone of elastic compression, i.e. if

$$p_x = (\lambda + 2\mu)\varepsilon_x + \eta\, \dot{\varepsilon}_x$$

then $p_x > p_x$ (elastic). For example, if y = 1 poise and $\dot{\varepsilon} = 10^6$/sec, $\eta\dot{\varepsilon} = 10^6$ dyne/cm^2 = 1 bar. Since viscosity serves to keep the locus of states in the elastic shock on the Rayleigh line, the term is not completely negligible, but $\dot{\varepsilon}$ must considerably exceed its value in the shock for the term to be significant. See Discussion in Al paper by Arvidsson, Gupta, and Duvall.