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PRESSURE BUILDUP AT A SHOCK FRONT WHICH INITIATES AN EXOTHERMIC REACTION

G.E. DUVALL

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Shock Dynamics Laboratory

Department of Physics

Washington State University

Pullman, WA 99164-2814

MODEL

The shock front is a discontinuous step. Derivatives are evaluated at and behind the shock front. The Hugoniot and equation of state of the unreacted material are known. Flow is one dimensional and Lagrangian coordinates are used with h denoting the undisturbed coordinate of a material element.

NOTATION

t = time

h = Lagrangian space coordinate $\left(=x(t=0)\right)$

x = position of a mass element at time t

$u = (\partial x/\partial t)_h$ = particle velocity

E = internal energy per unit mass

V = volume per unit mass

Γ = Grüneisen parameter

S = entropy per unit mass

T = temperature, ^oK

K_S = isentropic bulk modulus

p = pressure

C_V = specific heat at constant volume

c = sound velocity; $\rho c^2 = K_S$

Q = rate of release of energy of the reaction at constant volume

Vo = specific volume of undisturbed material

d/dt denotes time derivative along the path of the shock front

D = shock velocity

THEORY

If pressure is taken to be a function of E and V in an evolving system,

$$\frac{\partial p}{\partial t} = \left.\frac{\partial p}{\partial E}\right)_V \frac{\partial E}{\partial t} + \left.\frac{\partial p}{\partial V}\right)_E \frac{\partial V}{\partial t} \tag{1}$$

where $\partial/\partial t$ denotes rate of change for a particular mass element. With the thermodynamic parameters

$$\Gamma \equiv V\left.\frac{\partial p}{\partial E}\right)_V \tag{2}$$

$$C_V \equiv T\left.\frac{\partial S}{\partial T}\right)_V$$

$$K_S \equiv -V\left.\left(\frac{\partial p}{\partial V}\right)\right._S$$

$$c^2 \equiv VK_S$$

we obtain

$$\left.\frac{\partial p}{\partial V}\right)_E = -\frac{c^2}{V^2} + \frac{\Gamma p}{V} \tag{3}$$

Combining Eqns (1) - (3) gives

$$\frac{\partial p}{\partial t} = \frac{\Gamma}{V}\frac{\partial E}{\partial t} + \left(\frac{\Gamma p}{V} - \frac{c^2}{V^2}\right)\frac{\partial V}{\partial t} \tag{4}$$

The first law can be written

$$\frac{\partial E}{\partial t} = -p\frac{\partial V}{\partial t} + \frac{\partial Q}{\partial t} \tag{5}$$

Combining Eqns (4) and (5) yields

$$\frac{\partial p}{\partial t} = \frac{\Gamma}{V}\frac{\partial Q}{\partial t} - \frac{c^2}{V^2}\frac{\partial V}{\partial t} \tag{6}$$

or

$$\frac{\partial V}{\partial t} = \frac{\Gamma V}{c^2}\frac{\partial Q}{\partial t} - \frac{V^2}{c^2}\frac{\partial p}{\partial t} \tag{7}$$

The hydrodynamic equations for plane one-dimensional flow in Lagrangian form are

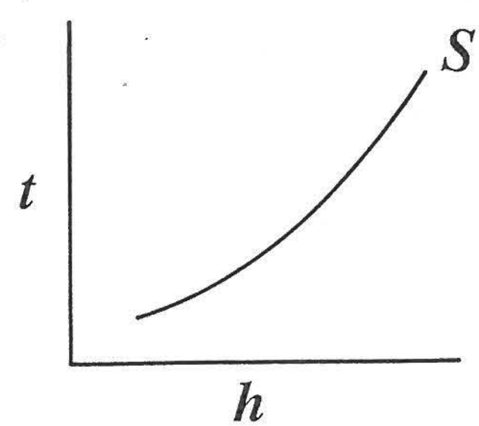
$$\frac{\partial V}{\partial t} = V_o\frac{\partial u}{\partial h} \tag{8}$$

$$\frac{\partial u}{\partial t} = -V_o\frac{\partial p}{\partial h} \tag{9}$$

Elimination of $\partial V/\partial T$ between Eqs (7) and (8) yields

$$V_o\frac{\partial u}{\partial h} + \frac{V^2}{c^2}\frac{\partial p}{\partial t} = \frac{\Gamma V}{c^2}\frac{\partial Q}{\partial t} \tag{10}$$

The path of a shock discontinuity in the (h,t) plane is shown as the curve S in the adjacent figure. Denote it's slope at any point by



$$\frac{dt}{dh} = \frac{1}{D}$$

The directional derivative of any function, f(t,h), along S can be written as

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + D\frac{\partial f}{\partial h}$$

or

$$\frac{\partial f}{\partial t} = \frac{df}{dt} - D\frac{\partial f}{\partial h} \tag{A}$$

Substitute u for f in Eqn (A) and combine with Eqn (9) to eliminate $\partial u/\partial t$. The result is

$$\frac{du}{dt} - D\frac{\partial u}{\partial h} = -V_o\frac{\partial p}{\partial h} \tag{11}$$

Next substitute p for f in Eqn (A) and combine with Eqn (10) to eliminate $\partial p/\partial t$:

$$V_o\frac{\partial u}{\partial h} + \frac{V^2}{c^2}\,\frac{dp}{dt} - \frac{V^2 D}{c^2}\,\frac{\partial p}{\partial h} = \frac{\Gamma V}{c^2}\,\frac{\partial Q}{\partial t} \tag{12}$$

Eliminate $\partial u/\partial h$ between Eqs (11) and (12):

$$V_o\frac{du}{dt} + \frac{V^2 D}{c^2}\,\frac{dp}{dt} = \left(\frac{V^2 D^2}{c^2} - V_o^2\right)\frac{\partial p}{\partial h} + \frac{\Gamma VD}{c^2}\,\frac{\partial Q}{\partial t} \tag{13}$$

The momentum jump condition, $p = \rho_o uD$ can be differentiated along S to obtain

$$\frac{du}{dt} = \frac{1}{\rho_o D}\left(1-\rho_o u\frac{dD}{dp}\right)\frac{dp}{dt} \tag{14}$$

Eqn (14) can be substituted into (13) to yield the equation for pressure buildup in the shock:

$$\frac{dp}{dt} = \frac{D\left(\dfrac{V^2D^2}{V_o{}^2c^2}-1\right)\dfrac{\partial p}{\partial h}+\dfrac{\Gamma VD^2}{V_o{}^2C^2}\dfrac{\partial Q}{\partial t}}{1+\dfrac{V^2D^2}{V_o{}^2c^2}-\dfrac{uD}{V_o{}^3}\dfrac{dD}{dp}}$$

In applying this equation to an experimental situation, knowledge of the Hugoniot for unreacted material gives V, D and u as functions of p ; c^2 and Γ are obtained from the equation of state. Temperature and pressure at the shock front are known, and these, in principal, determine $\partial Q/\partial t$. This leaves dp/dt and $\partial p/\partial h$ undetermined, and if one is measured, the other can be calculated. For a steady, Chapman-Jouguet detonation, $D=U+C$ and $dp/dt=0$, so a relation is obtained between $\partial p/\partial h$ and $\partial Q/\partial t$. This might be of some use in connection with measurements of the von Neumann spike.

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