Shock Dynamics Center Internal Report 94-04

Material Model for PMMA for Use in Shock Wave Experiments and 1-D Calculations

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Introduction:

This internal report describes a material model for polymethyl methacrylate (PMMA) for use in the design of shock wave experiments and 1-D wave code calculations. The particular PMMA modeled here is Rohm and Haas Type II UVA Plexiglas. The following discussion presents the equations and figures as needed. It is well known that the shock response of PMMA at relatively low stresses can be characterized as viscoelastic and the material undergoes thermal softening at high stresses until losing strength completely[1,2]. Further, the existing experimental data[1,3,4] show that the equilibrium response of PMMA displays a cusp at about 7.4 *kbar* as the Hugoniot elastic limit and a second one at about 240 *kbar*. In this report, however, we are seeking a simple model that can be directly used in the current version of the COPS code and reflects the equilibrium response only. Therefore, we will limit our material model for PMMA below the second cusp (up to 240 *kbar*) and ignore the first cusp as if the material is nonlinear elastic (as for designing shock wave experiments) or fluid (as for the COPS calculations).

Note: The shock response of PMMA is complicated by influences of confinement, rate-dependence and temperature effects. The model presented here is a simplified approximation and those interested in a more detailed understanding of PMMA should come by and see one of us (YMG) for related references. Also, the paper by Gupta[2] regarding compression and shear wave propagation in PMMA has the following error: for this material it is not possible to integrate the bulk modulus-density results to obtain the mean stress-density relation as was done in the paper. It may be possible to obtain this relation but further analysis is required.

A. Data Sources:

The Hugoniot data for PMMA was quoted from the work by Barker & Hollenbach[1] (up to 18.3 *kbar*) and the work by Schuler & Nunziato[5] (from 19.4 to 59.5 *kbar*), and from "LASL Shock Hugoniot Data"[3] (from 61.1 to 237.3 *kbar*). The initial density varies from 1.180 to 1.189 g/cm^3 for the data quoted here. We use $\rho_0 = 1.184\ g/cm^3$ for the model (the same as used in Barker & Hollenbach's work). The initial equilibrium longitudinal modulus, L_E (= 88.24 *kbar*) was quoted from the result of Nunziato & Sutherland's stress relaxation study[6].

B. Procedure to determine the parameters:

1) Fit the measured P_x - μ (longitudinal stress vs. volume compression) results with a cubic function fixing the coefficient of the first term as the initial equilibrium longitudinal modulus.

Note, that the imposed constraint is not required in general since the change in the material response at the Hugoniot elastic limit has been neglected. In this particular case, however, we used this constraint because it does not alter the fitting accuracy.

2) Deduce P_x - u_p and U_s - u_p relationships for the given P_x - μ relationship through the jump conditions.

3) Fit the P_x - u_p relationship with a quadratic function and the U_s - u_p relationship with a linear function.

C. Results

i) Representation for shock wave experiments:

1) Longitudinal stress, P_x $(kbar)$ vs. particle velocity, u_p $(mm/\mu s)$:

$$P_x = 31.33u_p + 17.80u_p^2.$$

The result is presented in Fig. 1 and in the P_x - u_p plot attached. The coefficient of the leading term does not reflect the initial equilibrium longitudinal modulus.

2) Longitudinal stress vs. volume compression, μ $(= V_0 / V - 1)$:

$$P_x = 88.20\mu + 80.15\mu^2 + 383.8\mu^3.$$

The result is shown in Fig. 2 and in the P_x - μ plot attached. The coefficient of the leading term in this case is the initial equilibrium longitudinal modulus for the reason stated previously.

3) Shock wave speed, U_s $(mm/\mu s)$ vs. particle velocity:

$$U_s = 2.729 + 1.457u_p$$

As shown in Fig. 3, the difference between the model and the measurements is noticeable at low particle velocities due to ignoring rate dependence in this simplified model. The model prediction becomes reasonably good when the particle velocity is beyond 1.2 $mm/\mu s$ (corresponding to a stress above 60 $kbar$). The constant term is consistent with the initial equilibrium modulus.

As a comparison, we note that the following parameters are used in the SHOCKUP program for PMMA[7]:

$$\rho_0 = 1.186\ g/cm^3$$

and

$$U_s = 2.72 + 1.454u_p.$$

ii) Representation for COPS code calculations:

In the COPS calculations, PMMA is treated as the following: The shear modulus is artificially set to zero (G = 0). The longitudinal response of PMMA is fed into the code as if it is a mean stress response by letting

$$P_H = 88.20\mu + 80.15\mu^2 + 383.8\mu^3 \quad (kbar).$$

This approach provides a resulting longitudinal response prescribed by the model in a format consistent with that of the COPS code. Note, that this technical treatment works when *only longitudinal response is of interest*. The approximation used here is not appropriate for two-dimensional calculations and at low stresses.

Again, the thermodynamic stress calculated using the Mie-Gruneisen equation,

$$(P - P_H) = \frac{\Gamma(V)}{V}(E - E_H)$$

is actually the longitudinal stress. Here the subscript "H" denotes the quantities defined on the Hugoniot curve, E is the internal energy per unit mass, and $\Gamma(V)$ is the Gruneisen parameter. For PMMA, $\Gamma(V) = 0.742V/V_0$ (quoted from Ref. 7). This treatment is reasonable considering the negligible strength of PMMA.

iii) Results of the COPS code calculation:

A series of 1-D wave propagation calculations have been performed using the COPS code and the above computational model and assuming that a 4 *mm* thick PMMA plate impacts a 20.5 *mm* thick target of the same material for several impact velocities. The results are listed in Table 1 and compared with those calculated directly from the P_x - u_p and U_s - u_p relationships. The shock wave speed was measured at the middle point of the computed wave front. The result may involve errors due to the nature of such measurements. However, the overall agreement is good as expected. The cell layout, the input data file and the output plots of stress and particle velocity histories for a sample calculation are attached.

Impact velocity (*mm/μs*)	Particle velocity (*mm/μs*)	P_x calculated from P_x - u_p (kbar)	U_s calculated from U_s - u_p (*mm/μs*)	P_x computed using COPS (*kbar*)	U_s determined from numerical results (*mm/μs*)
0.55	0.275	9.962	3.1297	9.918	3.046
3.8	1.9	123.8	5.4973	124.1	5.515
5.9	2.95	247.3	7.0272	246.6	7.061

Table 1, Comparison between the analytic solutions and the numerical results for the material model of PMMA

References:

[1] L. M. Barker and R. E. Hollenbach, *Journal of Applied Physics*, **41** 4208-4226, 1970.

[2] Y. M. Gupta, *Journal of Applied Physics*, **51**, 5352-5361, 1980.

[3] "*LASL Shock Hugoniot Data*", edited by S. P. Marsh, University of California Press, Berkeley, 1980.

[4] "*Compendium of Shock Wave Data*", edited by M. van Thiel, Lawrence Livermore Laboratory, University of California, 1977.

[5] K. W. Schuler and J. W. Nunziato, *Rheologica Acta*, **13**, 265-273, 1974.

[6] J. W. Nunziato and H. J. Sutherland, *Journal of Applied Physics*, **44**, 184-187, 1973.

[7] R. L. Gustavsen, *Ph.D. Thesis*, Washington State University, 1989.

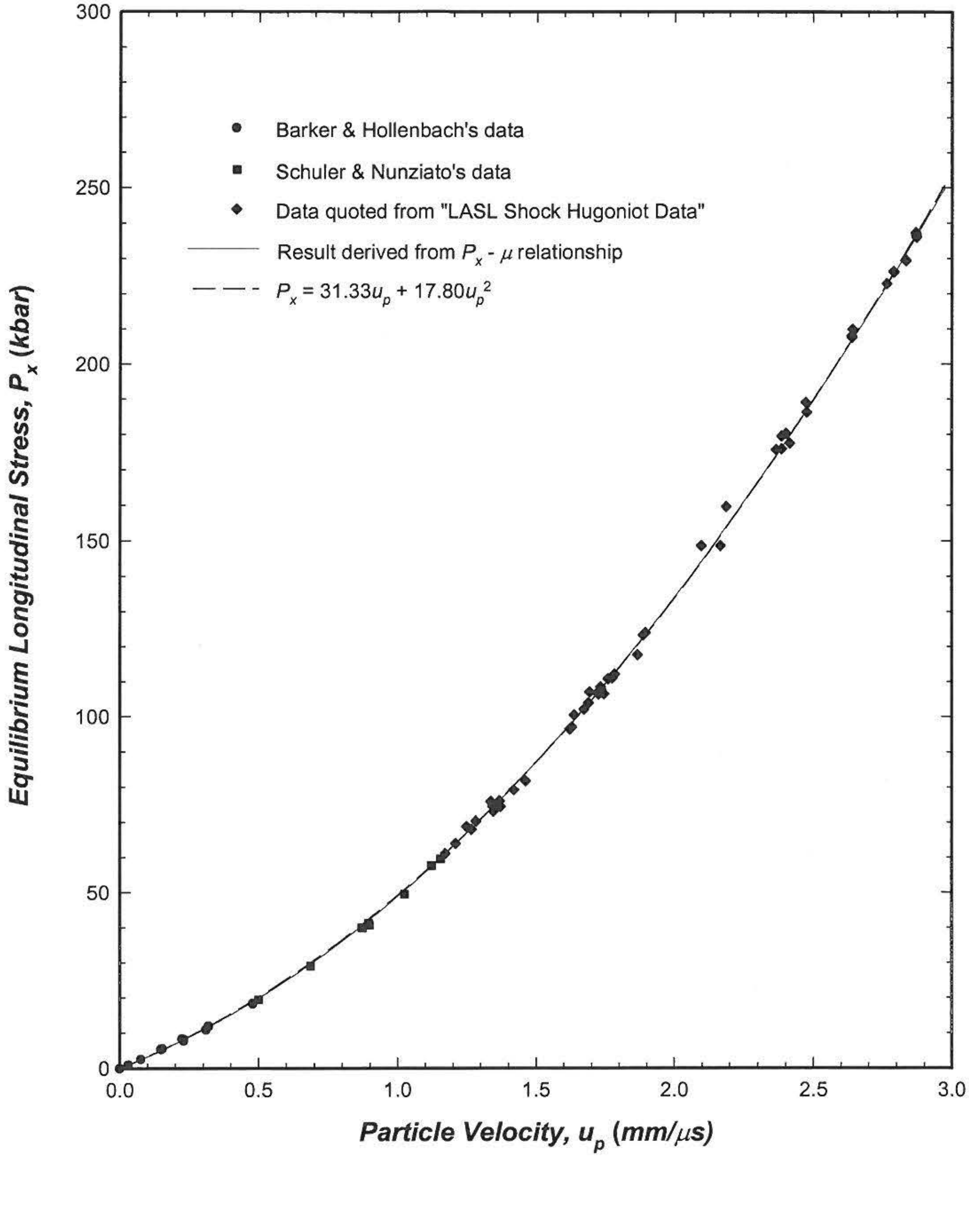


Fig. 1 P_x - u_p relationship for PMMA

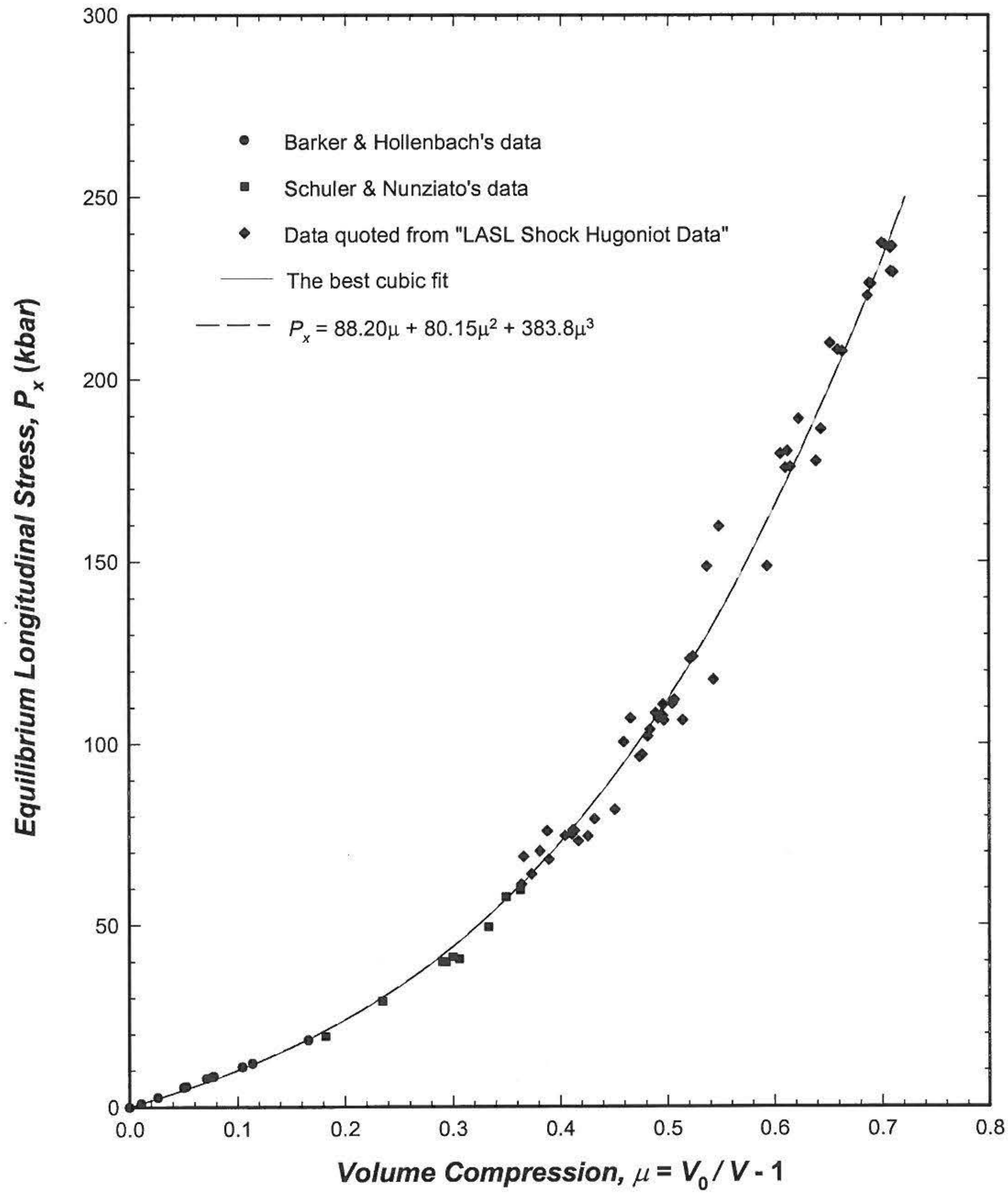


Fig. 2, P_x - μ relationship for PMMA

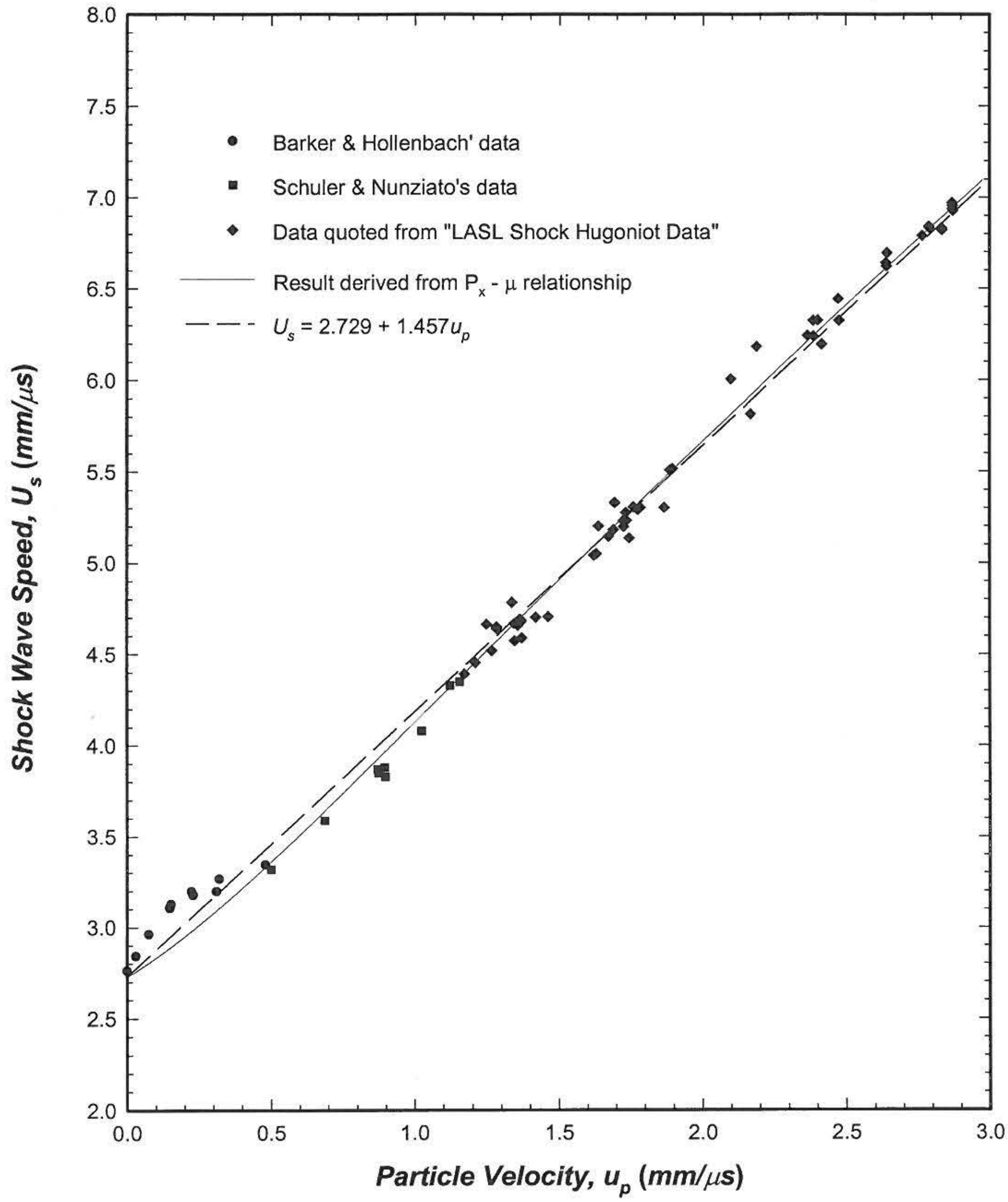
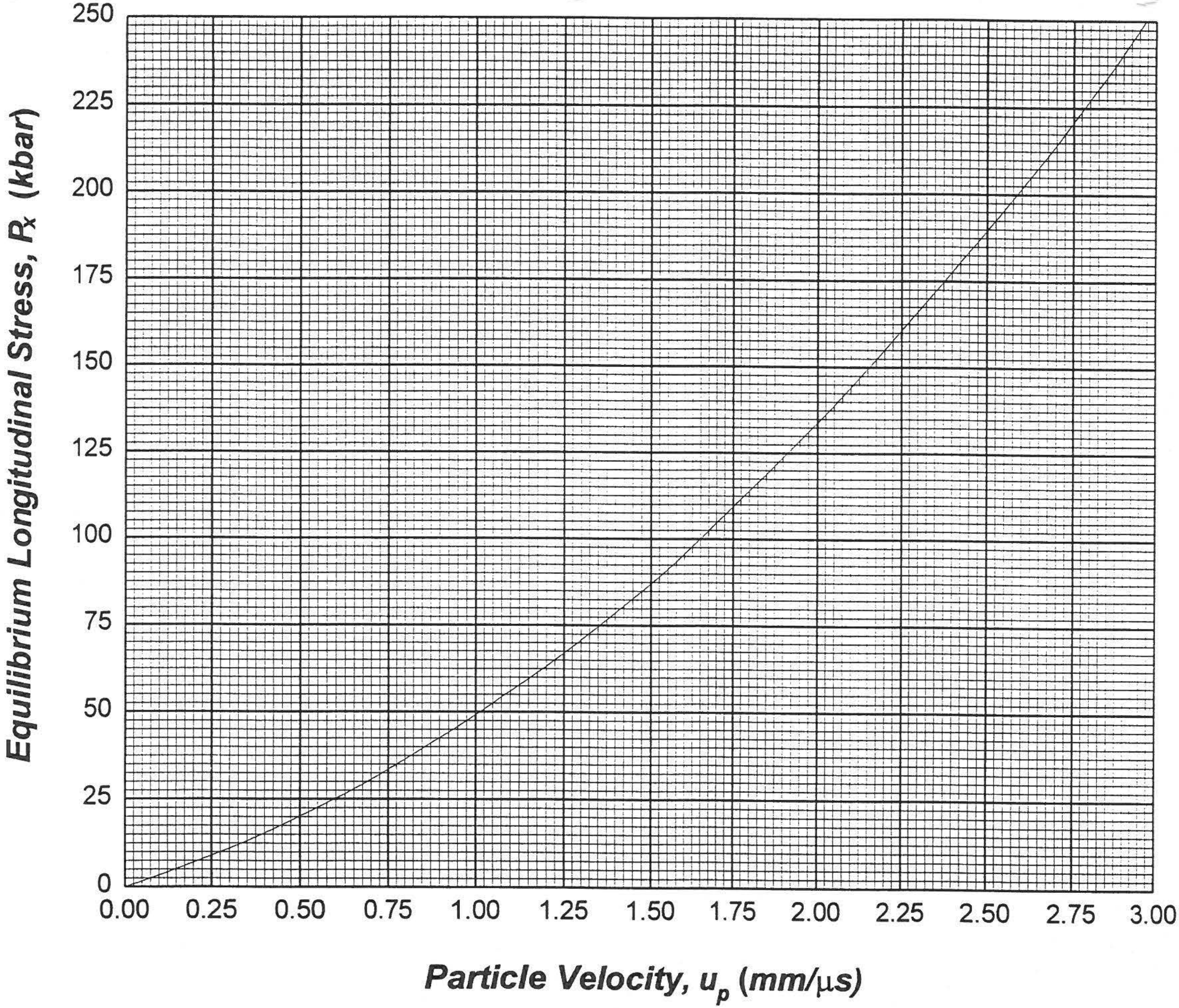
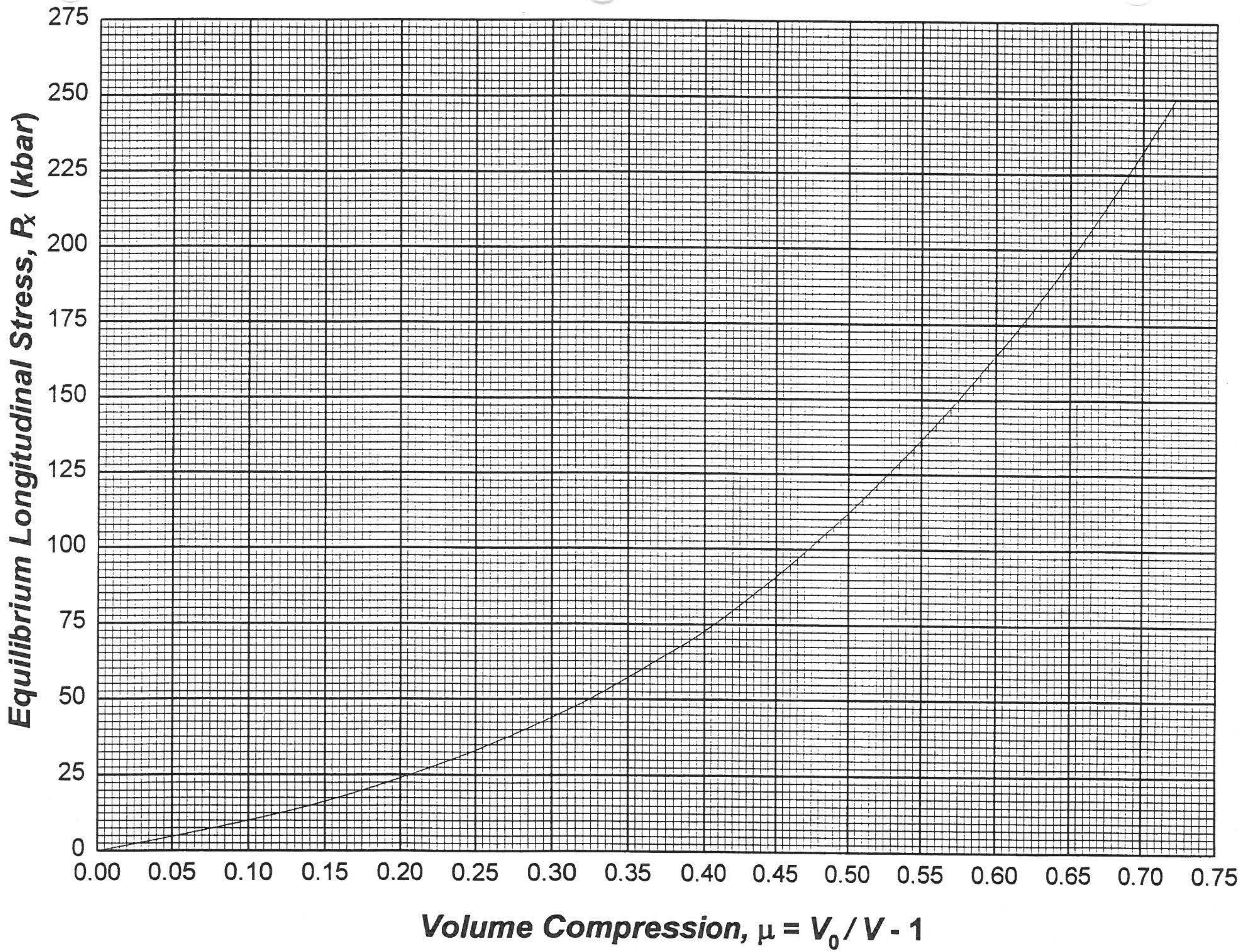


Fig. 3, U_s - u_p relationship for PMMA

P_x - u_p Plot for PMMA



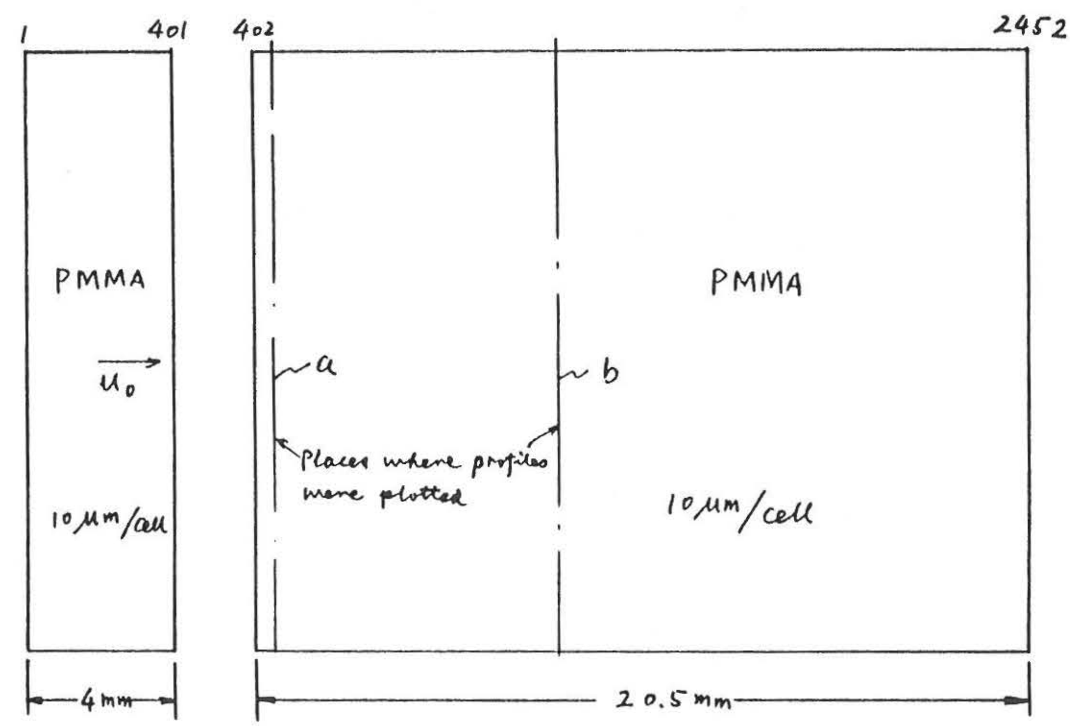
P_x - μ Plot for PMMA

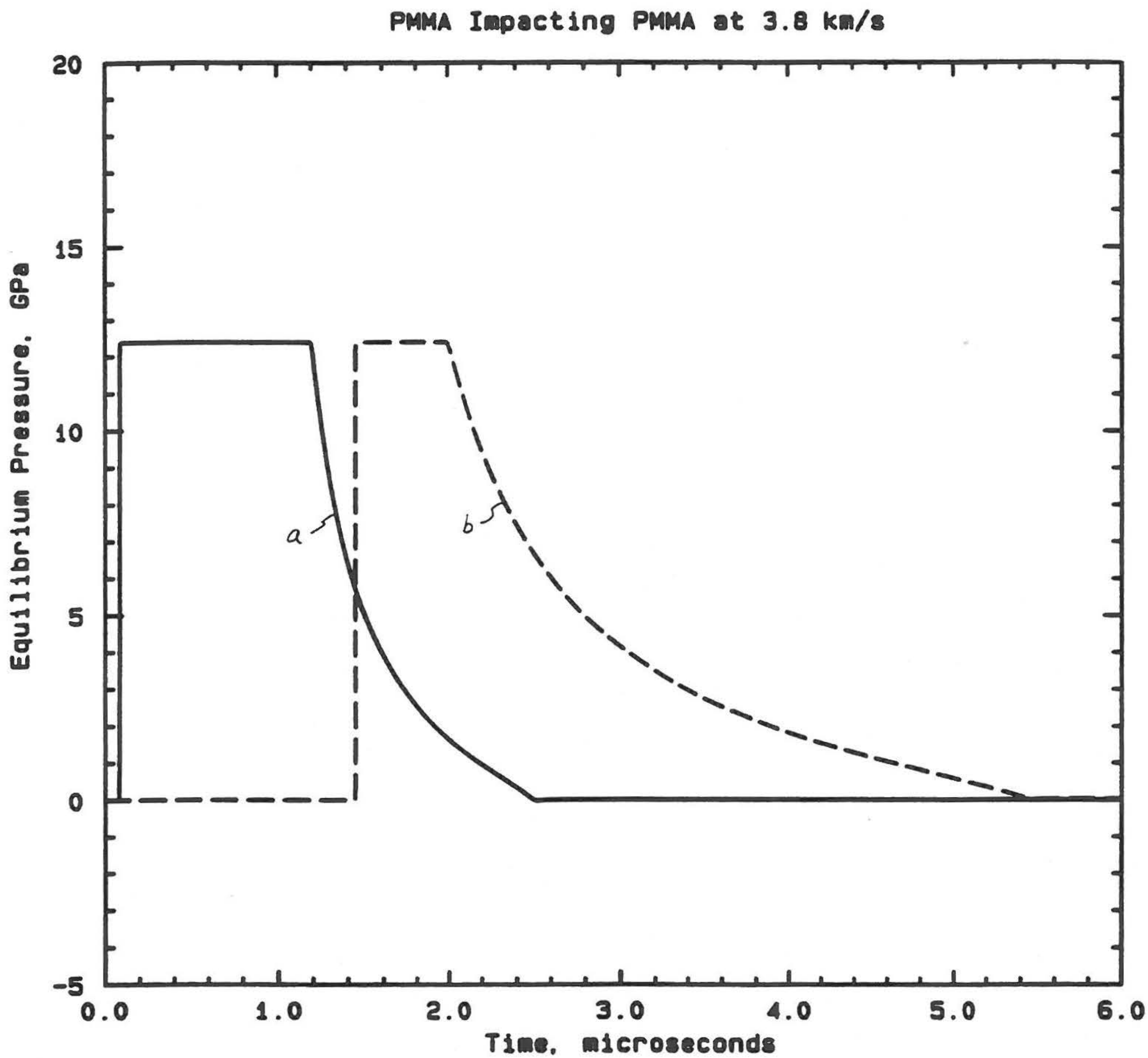


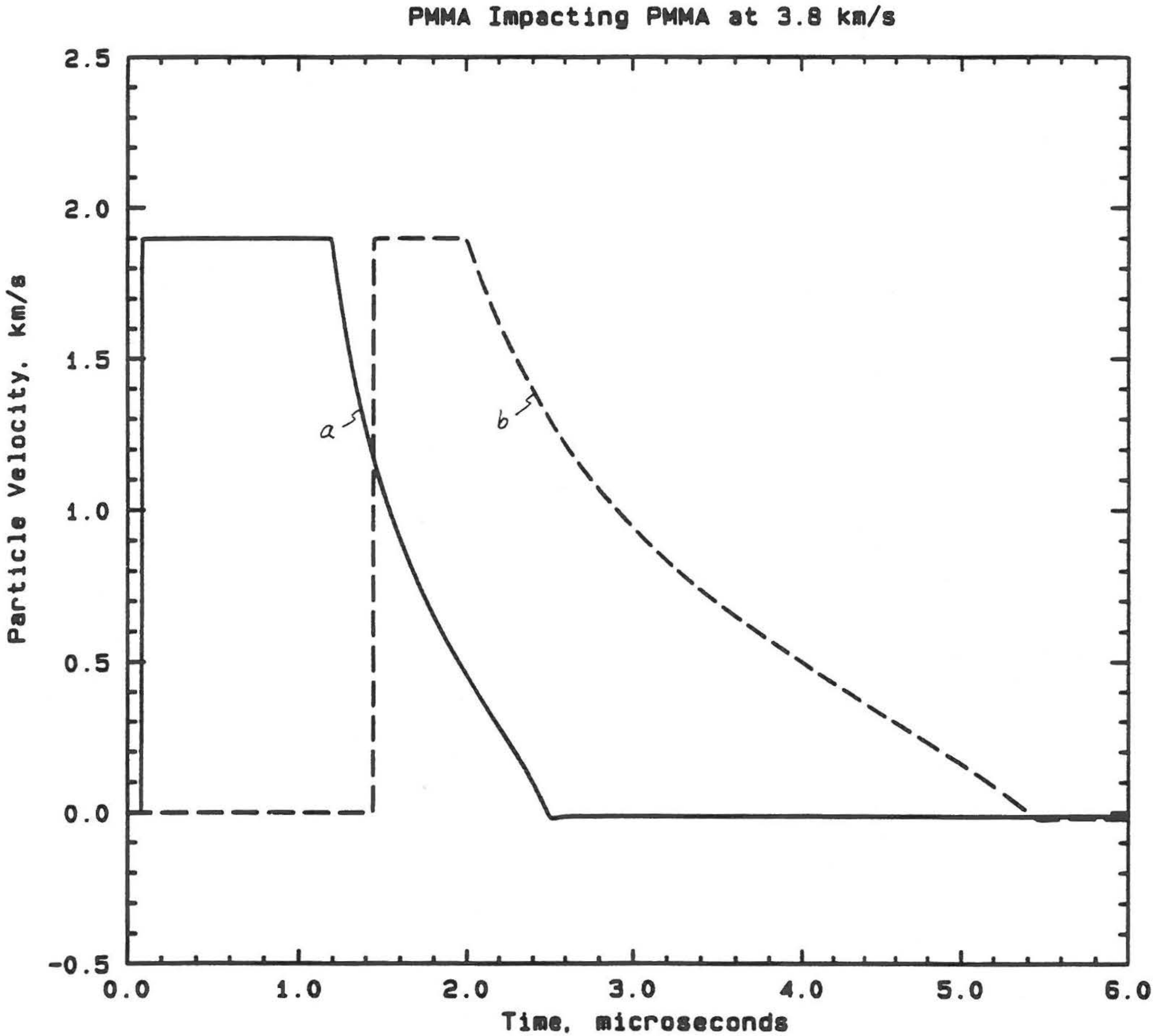
```
Simulation of 1-D Wave Propagation in PMMA
Max Cycle=    0010000 Max Time= 6.000e-06 nedit    =  02000000 iplot=  1 mtrls=  1
ntedt     =         0 njedt    =         2 nlayers =         2 layfl    =        1
islide    =         0
jedit     =   452 1202
jmat      =     1    1
proj      = 3.800e+05 angle    =      90.0
cl1       =      0.40 cq1      =       0.0 cl2     =       0.0 cq2      =      0.0
ism       =     0
dtmult    =1.00000000
ncell 1   =       400 thick 1 = 4.000e-01 PMMA
ncell 2   =      2050 thick 2 = 2.050e+00 PMMA
PMMA                  density = 1.184e+00 indp =  1 indd =  1 indc =  0
mu1       = 8.820e+10 mu2     = 8.015e+10 mu3     = 3.838e+11 gamma/V = 8.785e-01
Yo        = 1.000e+30
G0        = 0.000e+00
plots     =         2
vars      =     1   12
ifreq     =     1
```

↑ "input. dat"

CELL LAYOUT




PMMA Impacting PMMA at 3.8 km/s
Equilibrium Pressure, GPa
Time, microseconds
20
15
10
5
0
-5
0.0
1.0
2.0
3.0
4.0
5.0
6.0
a
b



PMMA Impacting PMMA at 3.8 km/s
Particle Velocity, km/s
Time, microseconds
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
0.0
1.0
2.0
3.0
4.0
5.0
6.0
a
b