What is the limiting performance of PDV (really)?

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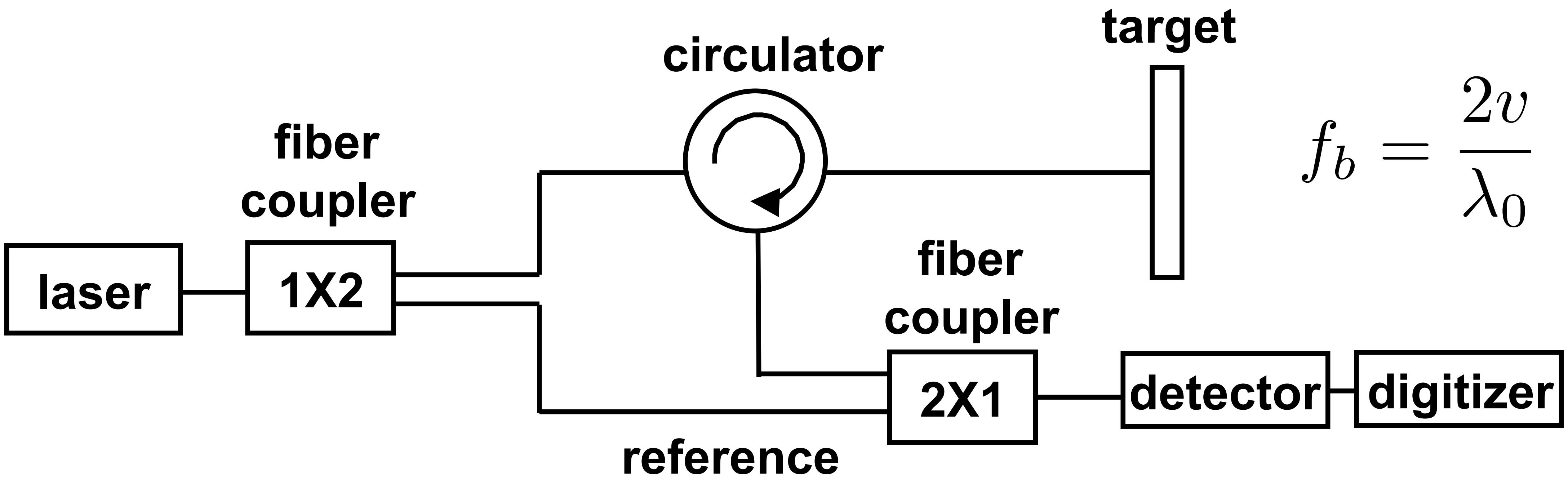
Overview

- Background
  - What does performance mean?
  - Uncertainty principle vs peak fitting
- Monte-Carlo **simulations**
  - Accuracy/precision trends as a function of:
    - Frequency
    - Noise fraction
    - FFT window selection
  - Consistency with **analytic** estimate
- Frequency-conversion PDV
- VISAR comparison

A quick review



$$f_b = \frac{2v}{\lambda_0}$$

"Conventional" PDV

- PDV sensitive to target displacement
  - One fringe for every half wavelength of motion
  - Beat frequency proportional to velocity
    - 1 km/s: 1.29 GHz
    - 1 GHz: 775 m/s

What is performance?

- Quantitatively, how good can PDV be?
  - **Accuracy**: being right (on average)
  - **Precision**: variability about the average
  - Small numbers are “high” performance

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- Things that are neglected
  - Window corrections (0.2-1%)
  - Probe effects/cosine corrections (<0.1%-?)
  - Absolute wavelength errors (<10-100 ppm)
  - Digitizer clock errors (<10 ppm)

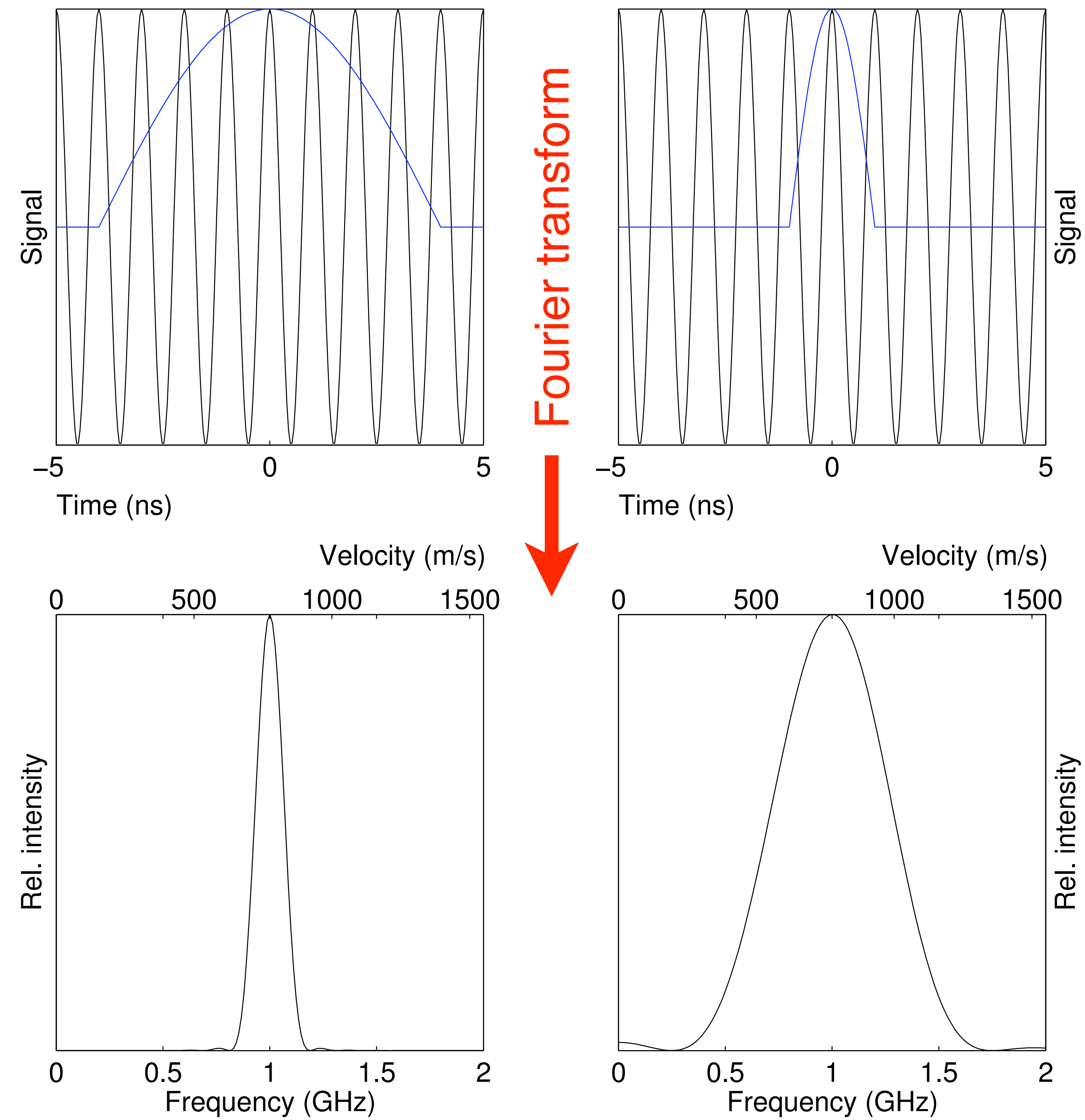
- Looking for an equivalent to the “1-2% of a fringe” rule [VISAR]

Uncertainty principle

- Time-velocity uncertainty product always exceeds a constant

$$\sigma_v \ \sigma_t \geq \frac{\lambda_0}{8\pi}$$

- 1 m/s velocity width requires >62 ns! (1550 nm)
- This is really a separability criterion, not location uncertainty
- No reference to noise or sampling rate



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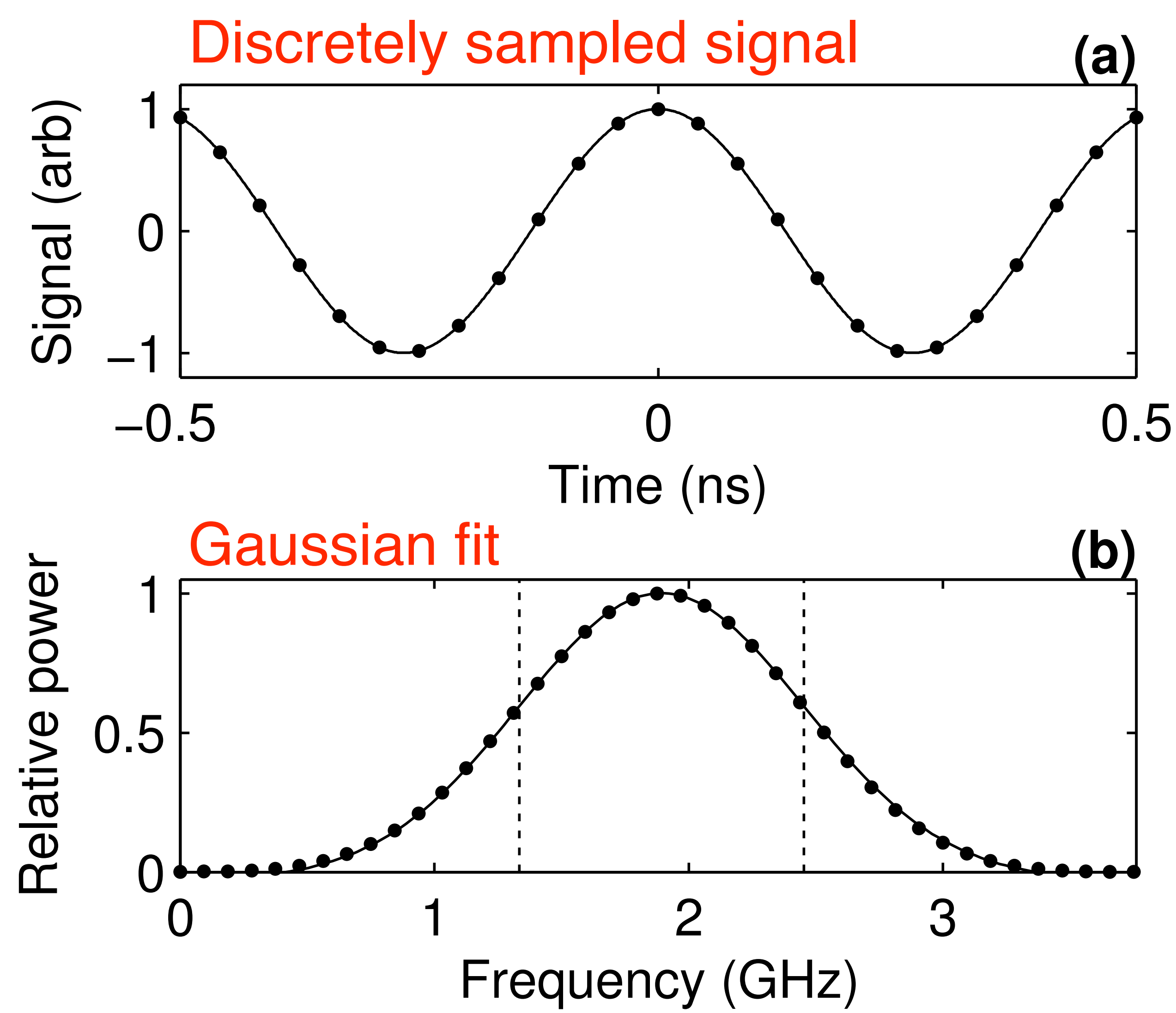
Peak fitting

- Fit the PDV power spectrum with curve
  - Estimate resolution from how quickly residual increases away from the minimum

- Peak location can be determined more narrowly than uncertainty principle bound
  - However, the results may be systematically wrong!
  - Error depends on many factors

1.880 GHz signal sampled 25 times in 1 ns



Peak location: 1.894 GHz
Peak width: 1 GHz
Fit uncertainty: 0.003 GHz (2 m/s)
Error: 0.014 GHz

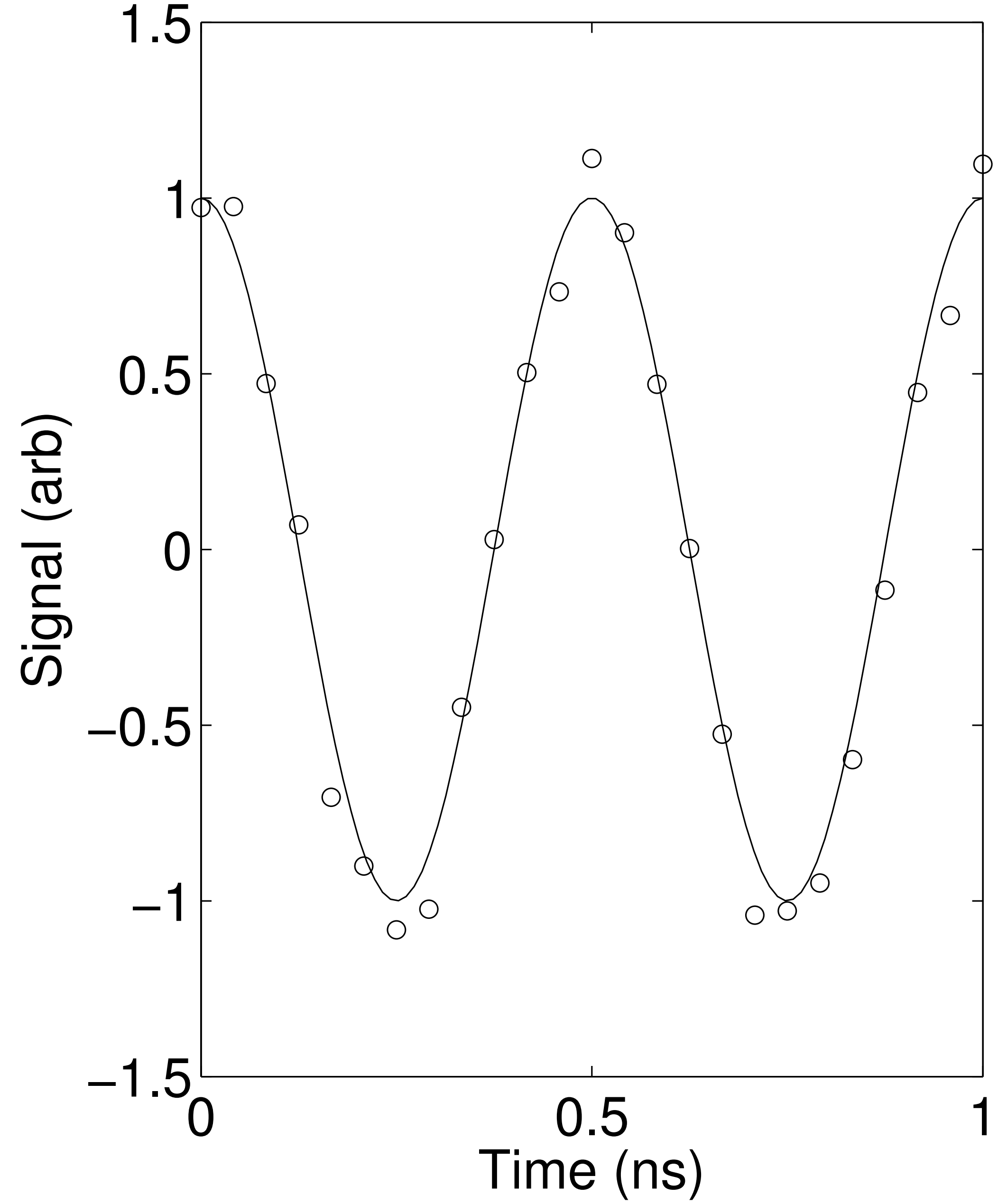
Monte Carlo simulations

$$s_k = \cos\left(2\pi f_0 T k + \delta\right) + \sigma R_k$$

- Generate discretely sampled signal
  - Specified frequency f_0
  - k=-M..M (2M+1 points)
  - Sampling interval T
  - Noise fraction σ
  - Random phase δ
  - Random noise array R
- Extract frequency from signal (FFT analysis, etc.)
- Compare the result to input frequency
- Repeat many times with different phases and noise arrays

One iteration

25 samples over 1 ns, 10% noise



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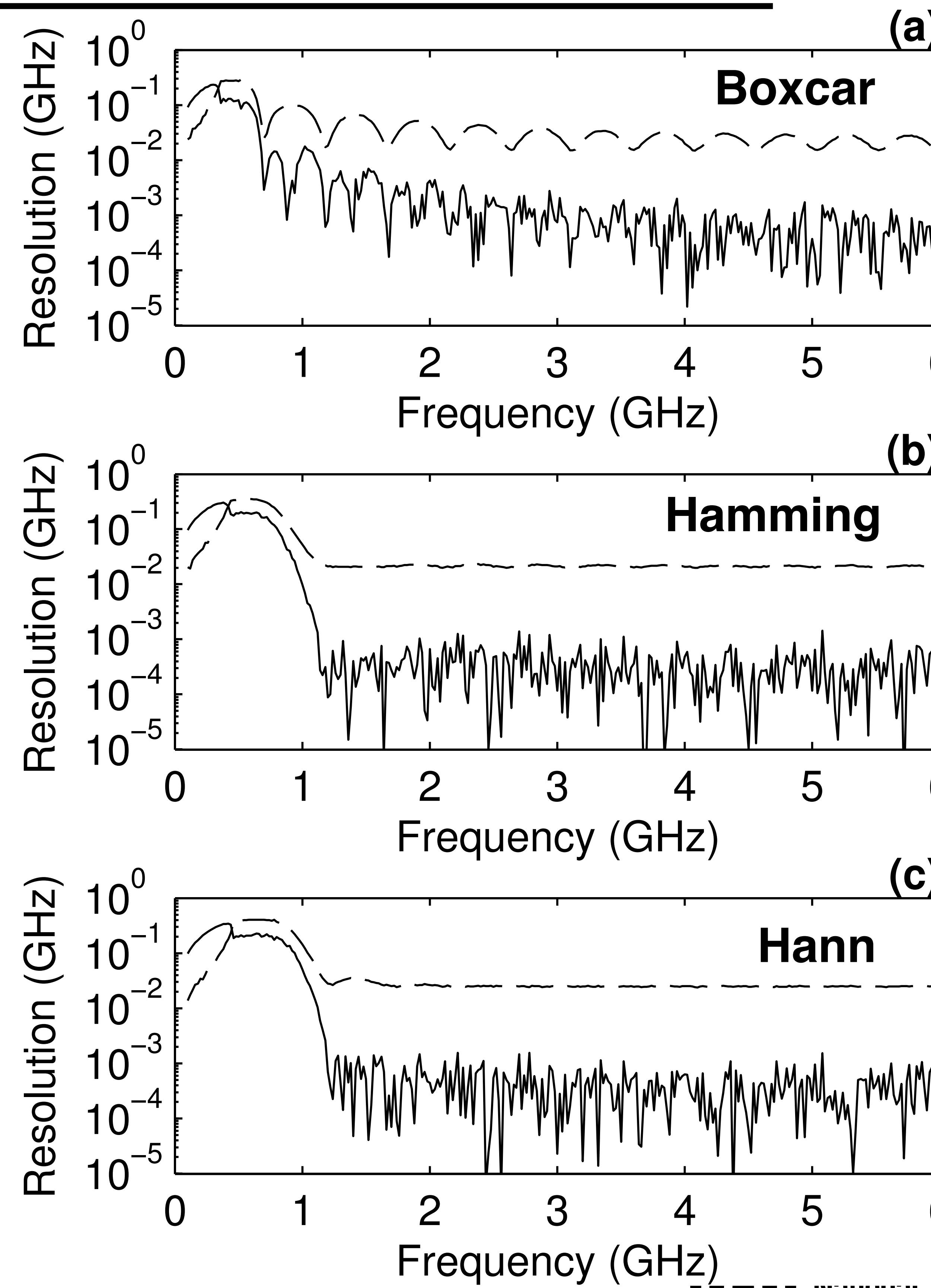
Resolution example

- Inputs
  - 1 ns duration, 25 GS/s
  - 10% noise fraction
  - 0.1-6 GHz (0.02 GHz steps)
  - ~590 million iterations total

- Analyze many random signals
  - Ensemble of frequency results from a set of inputs
  - **Accuracy**: difference between ensemble mean and input (solid)
  - **Precision**: ensemble std. deviation (dashed)

- Poor accuracy and resolution below 1 GHz (partial fringe)

- Measurements are precision-limited above 1 GHz

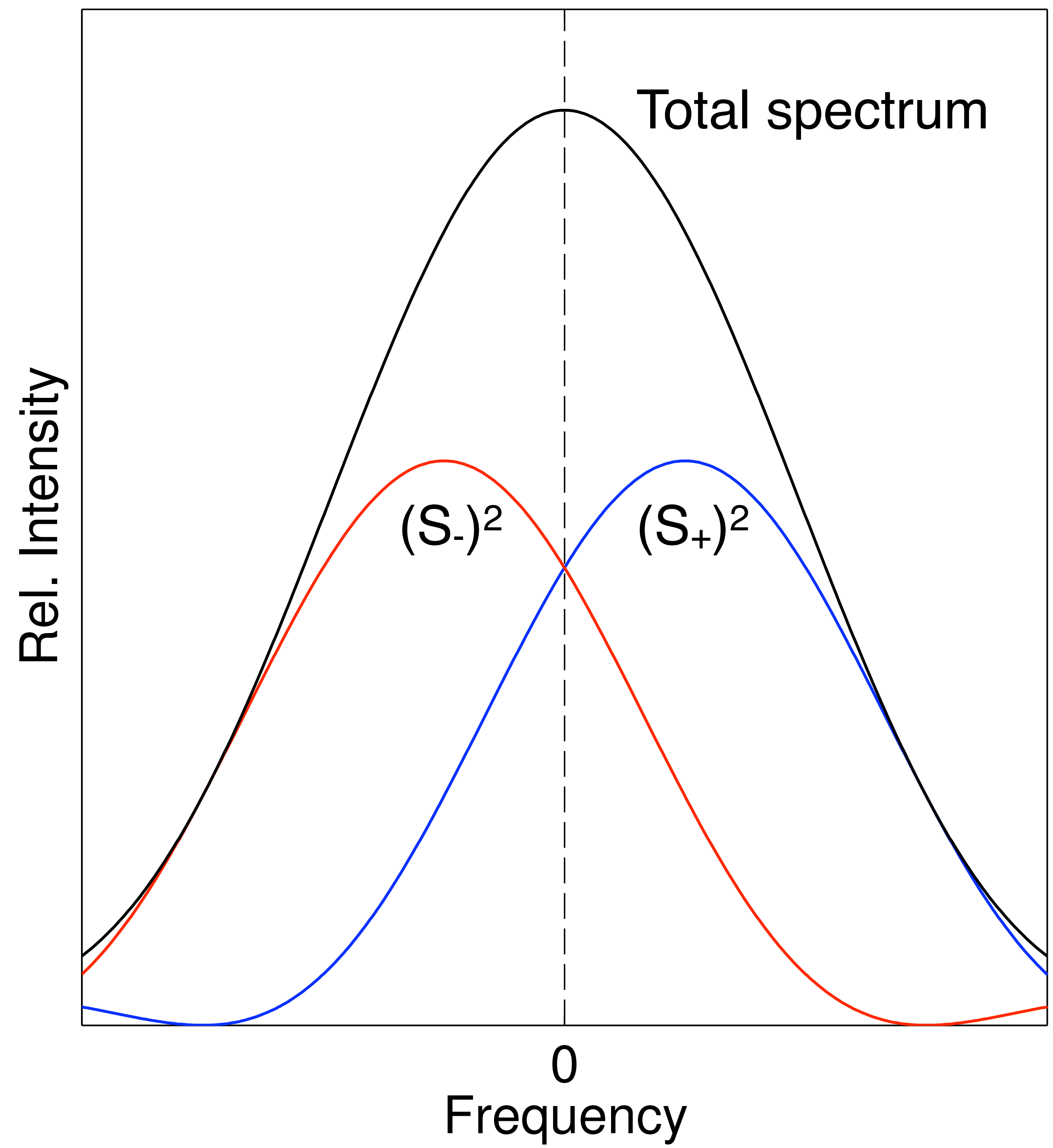


Power spectrum subtlety #1

$$S^2(f) \propto S_+^2(f) + S_-^2(f) + 2\cos(2\delta)S_+(f)S_-(f)$$ (real signal)

- Negative frequency components affect the positive frequency domain!
  - Significant overlap for narrow digital windows
  - Power spectrum biased away from the S_+ peak
  - Reduces **accuracy**

- Cross-term creates time variation
  - Reduces **precision**



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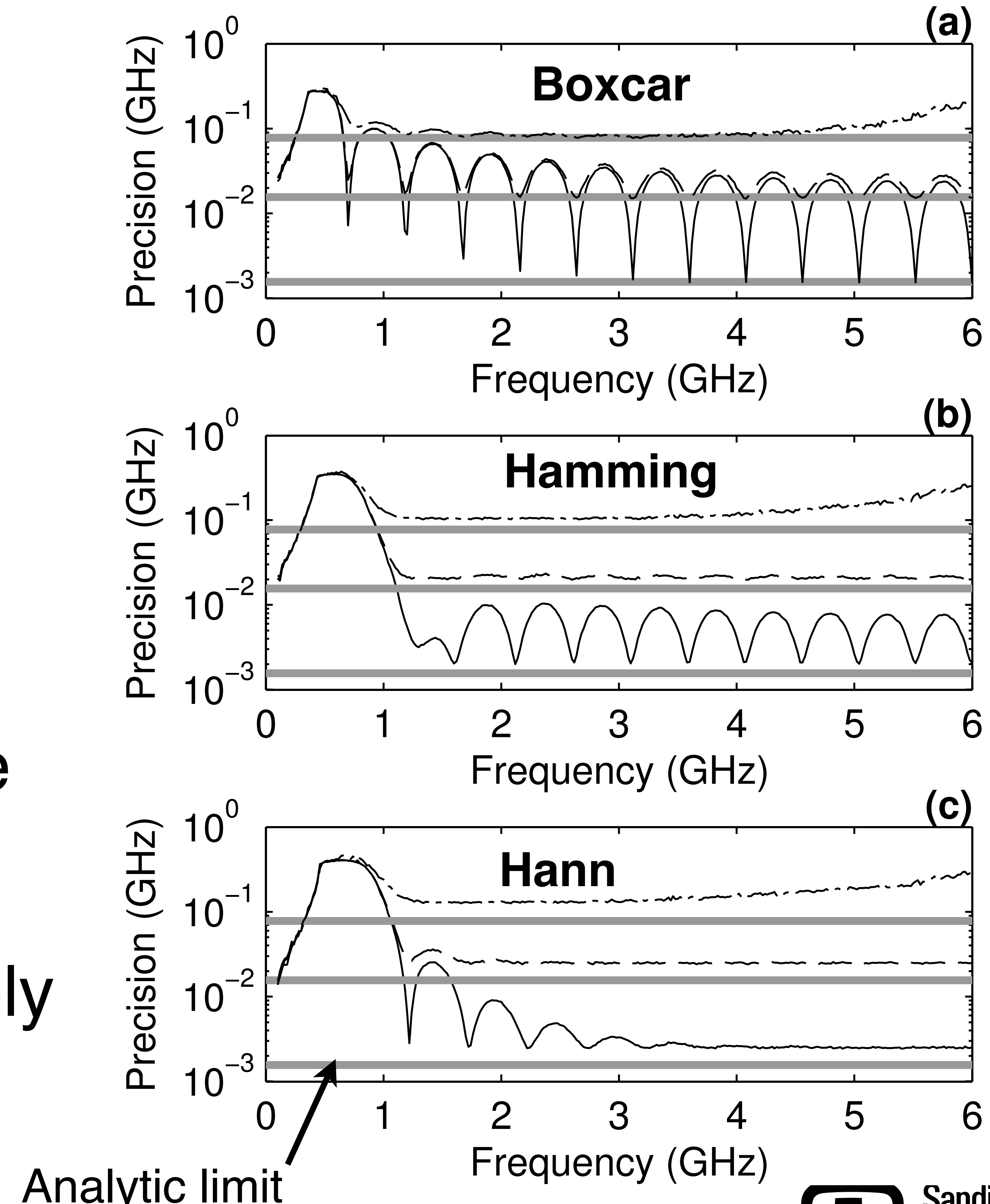
Precision and noise

- Simulations performed at various noise fractions
  - Solid line: 1%
  - Dashed line: 10%
  - Dot-dash line: 50%
- Window performance
  - Hann window best a low noise fractions
  - Boxcar best at high noise fractions
- Precision benefits are largely constant above the low frequency “shoulder”



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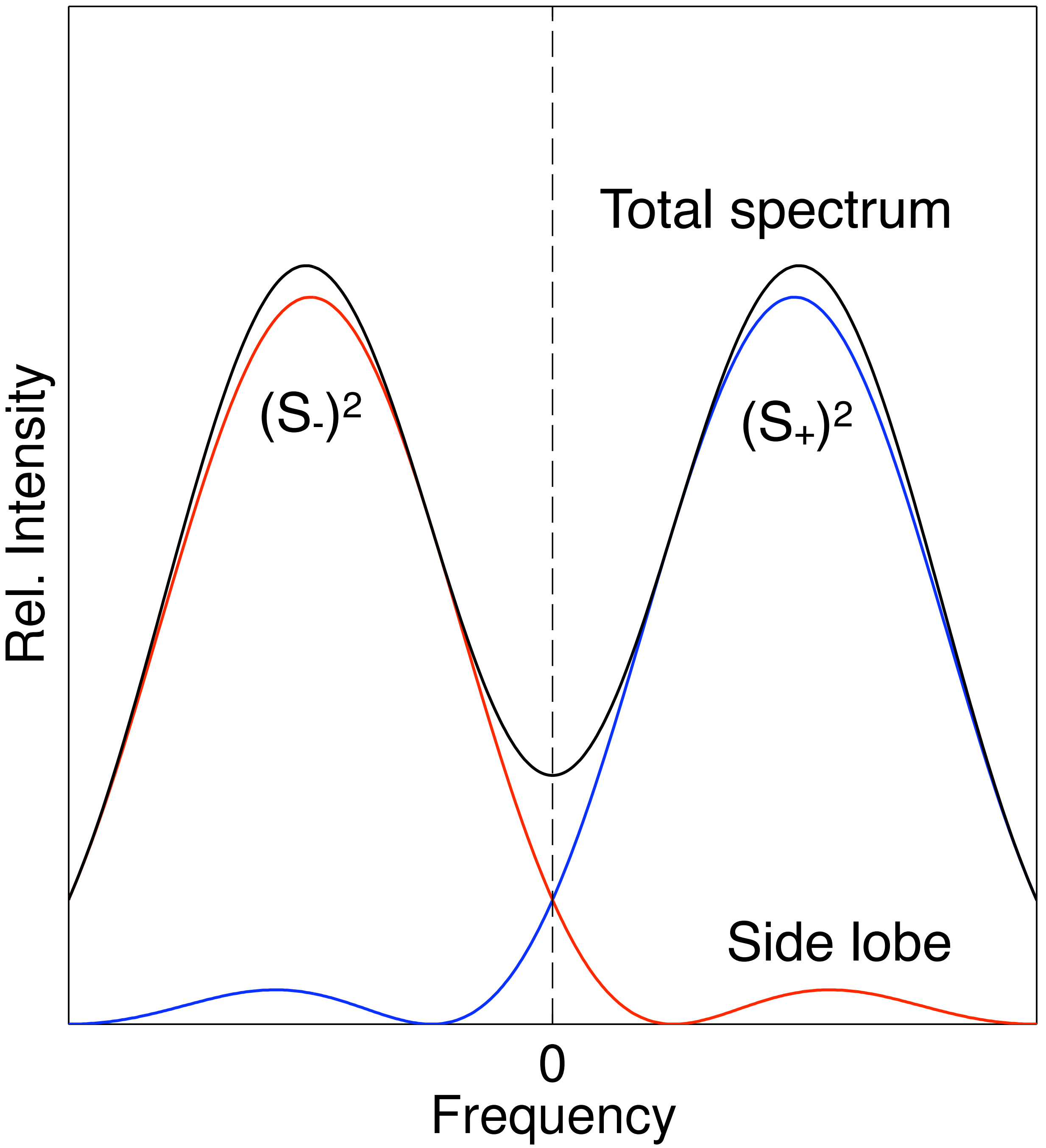
Power spectrum subtlety #2

$$S^2(f) \propto S_+^2(f) + S_-^2(f) + 2\cos(2\delta) S_+(f) S_-(f)$$ (real signal)

- Side lobes have an effect
  - Net spectrum can be biased toward or away from DC (accuracy)
  - Cross term reduces precision (dominant effect)
- Resonances due to overlap of a primary peak with a side lobe



Limiting performance

There is a limit to how well frequency can be determined from a discretely sampled signal

$$\Delta f = \sqrt{\frac{6}{N}}\frac{\sigma}{\pi\tau} = \sqrt{\frac{6}{f_s}}\frac{\sigma}{\pi}\tau^{-3/2}$$

- Fixed parameters:
  - Sampling rate
  - Noise fraction (digital filtering does NOT help)
- Adjustable parameters:
  - Analysis duration
  - Improvement faster than uncertainty principle predicts
- FFT windows alter the effective duration
  - This expression assumes all data is treated equally (boxcar window)

Some typical values

1550 nm, 10% noise, boxcar window

Duration (ns)	Rise time (ns)	25 GS/s Resolution (m/s)	50 GS/s Resolution (m/s)
0.1	0.06	380	270
0.5	0.29	34	24
1.0	0.58	12	8.5
5.0	2.9	1.1	0.76
10	5.8	0.38	0.27
50	29	0.034	0.024

This resolution is only attainable above low-frequency shoulder!

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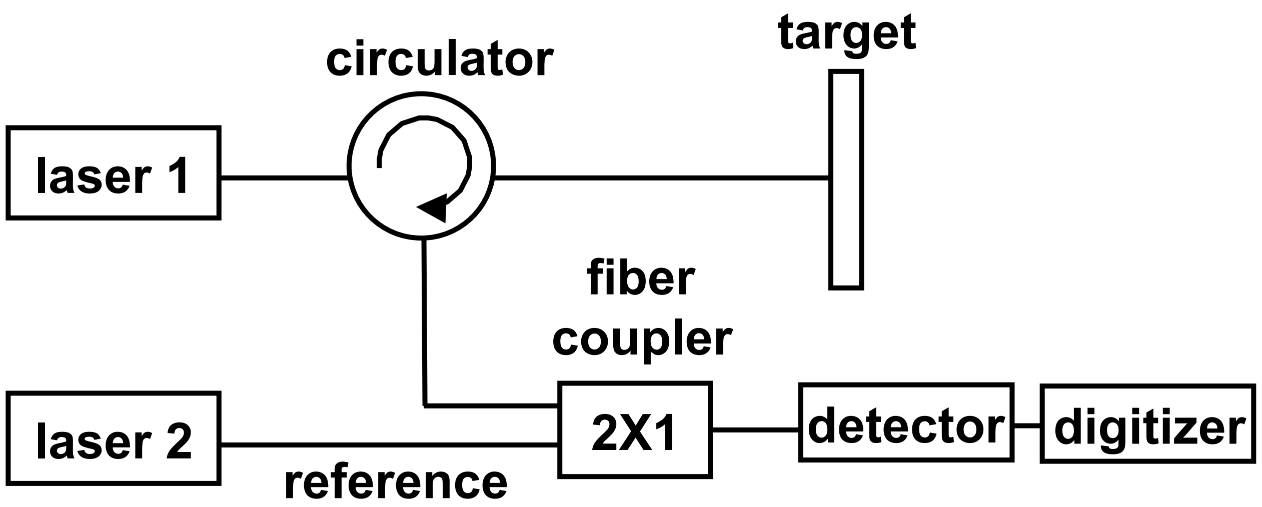
Frequency-conversion PDV

- Two wavelengths
  - One illuminates target
  - One serves a reference

Tune wavelengths to get any desired beat frequency

- Advantages
  - Always beating
  - Provides direction information
  - Utilizes the power of the FFT
  - Avoids low frequency shoulder



Limiting performance can be achieved at any (measurable) velocity!

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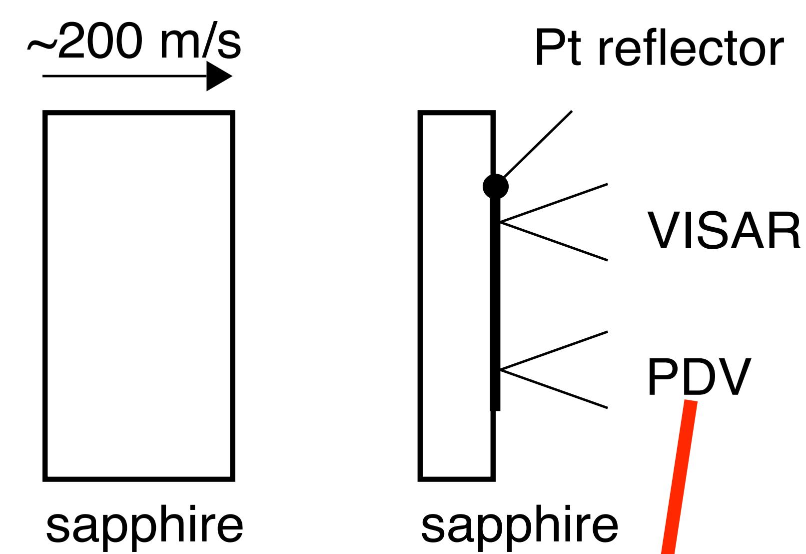
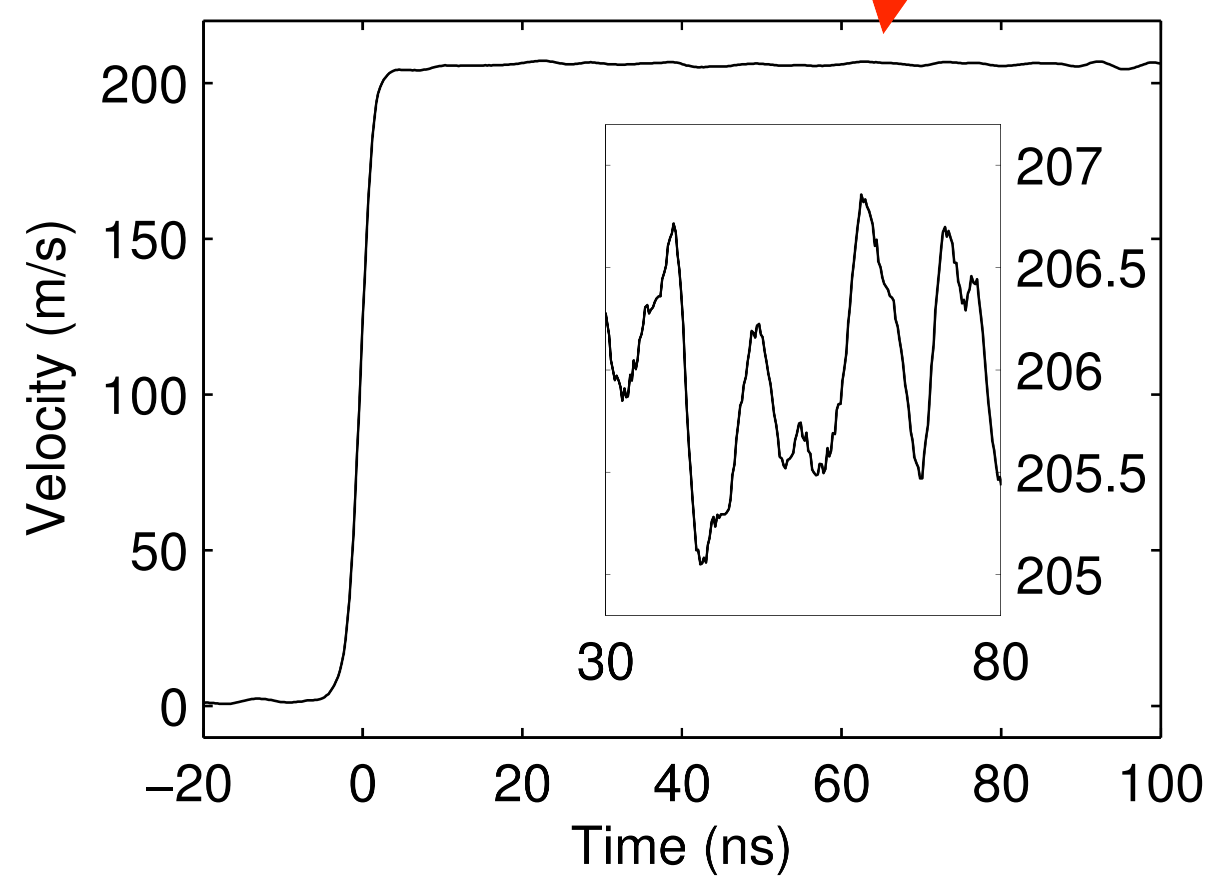
Experimental test

- Symmetric impact with velocimetry at the free surface
- Compared PDV with air-delay VISAR (14 m/s VPF) and shorting pins
- PDV precision trends consistent with analytic limit





Diagnostic	Time scale (ns)	Velocity change (m/s)
Pins	>1000	206.6 (1.1)
VISAR	19	206.11 (0.28)
PDV	19	205.95 (0.32)
PDV	10	205.93 (0.64)
PDV	1	206 (21)

Comparison with VISAR

- Resolution ratio (PDV/VISAR)
  - PDV noise fraction σ~10%
  - VISAR fringe resolution ε~2%
  - Common delay/analysis time

$$\rho = \frac{1550\ \mathrm{nm}}{532\ \mathrm{nm}} \sqrt{\frac{6}{f_s \tau}} \frac{\sigma}{\pi \epsilon}$$

- For 1 ns analysis, ratio is 2.3: VISAR better (maybe)
- At 10 ns analysis, ratio is 0.72: PDV better
- PDV time scale is adjustable, VISAR fixed by hardware

- Mitigating factors
  - PDV rises faster than VISAR for common delay/analysis time (improvement depends on window)
  - PDV uses one signal, VISAR uses 4-8 signals (2-2.8 x improvement)
  - VISAR requires good characterization (5-10% fringe resolution not unusual)

Summary

- The limiting performance of PDV is determined by power spectrum location resolution
  - The uncertainty principle overestimates error
  - Peak fit confidences underestimates error

- Simulations indicate that PDV is:
  - Inaccurate and imprecise at low frequencies
  - Accurate and (potentially) precise otherwise
  - Limiting performance can be tied to sampling rate, noise fraction, and analysis duration

- **Frequency conversion is a good thing!**

- PDV is competitive with VISAR, despite wavelength difference

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