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Phase Errors and the Capture Effect

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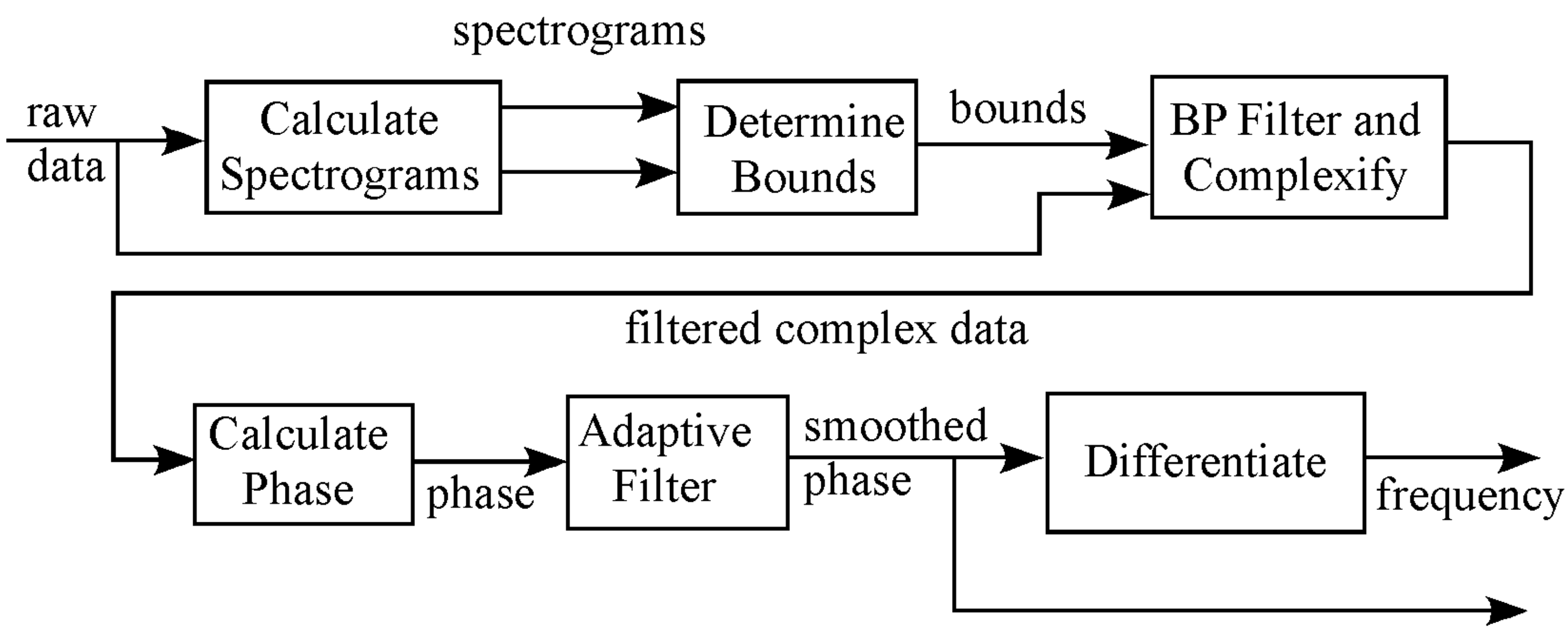
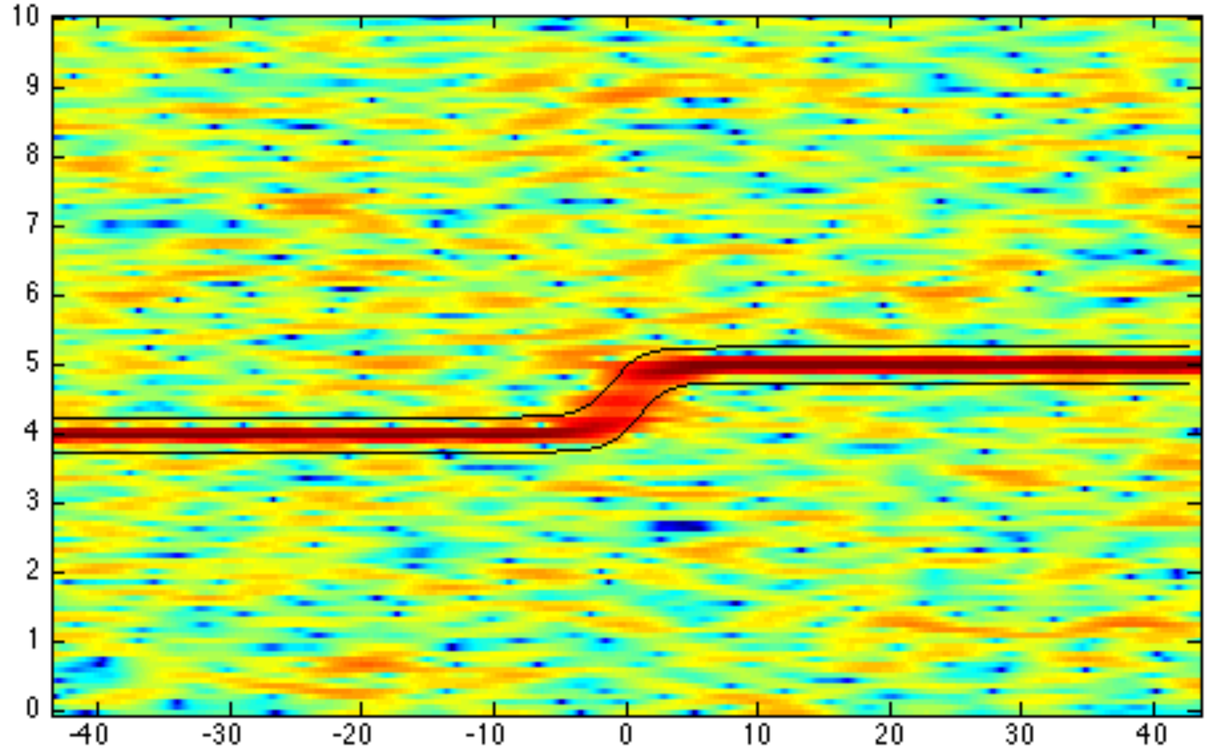
Analysis Algorithm

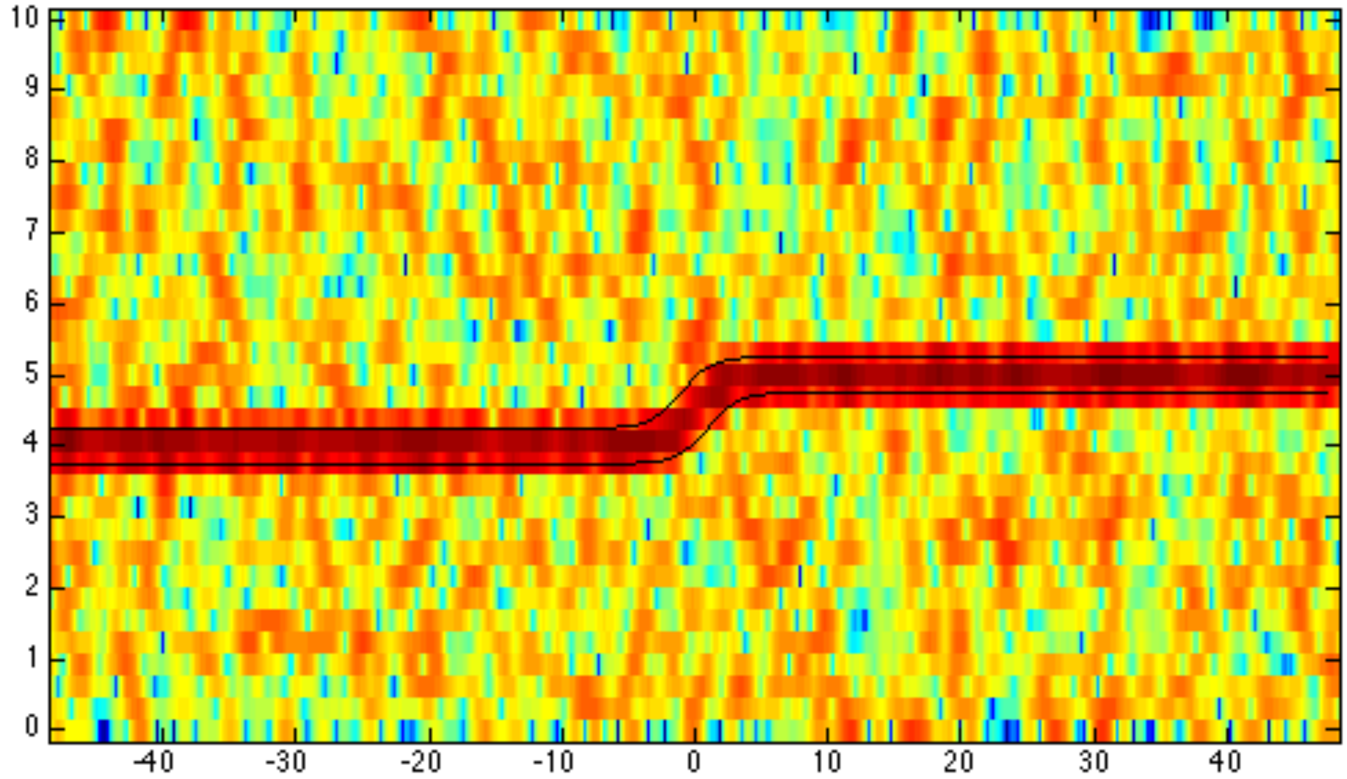


Spectrograms and Bounds



This SGM does not justify the bounds at the transition. 256 point window width.

Spectrograms and Bounds



But this one does. 64 point window width.

The Filtered Signal

$$z_F(t) = a(t)e^{j\phi(t)} + n_F(t)$$ ← The filtered noise

phase → $$\phi_F(t) = \mathrm{imag}(\log(z_F(t)))$$

frequency → $$f_F(t) = \frac{1}{2\pi}\frac{d\phi_F}{dt}.$$

Rewriting:

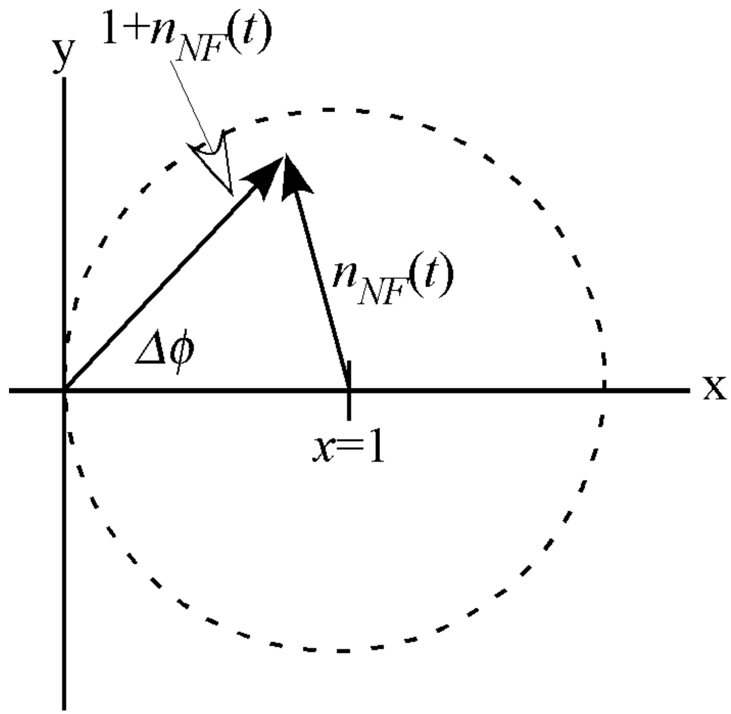
Normalized filtered noise

$$z_F(t) = a(t)e^{j\phi(t)}(1 + e^{-j\phi(t)}n_F(t)/(a(t)) = (a(t)e^{j\phi(t)})(1 + n_{NF}(t))$$

The Phase Error

$$z_F(t) = (a(t)e^{j\phi(t)})(1 + n_{NF}(t))$$

- Phase error is phase of the second factor
  - Because phase of product is sum of phases.
- $n_{NF}(t)$ is filtered noise divided by signal amplitude.



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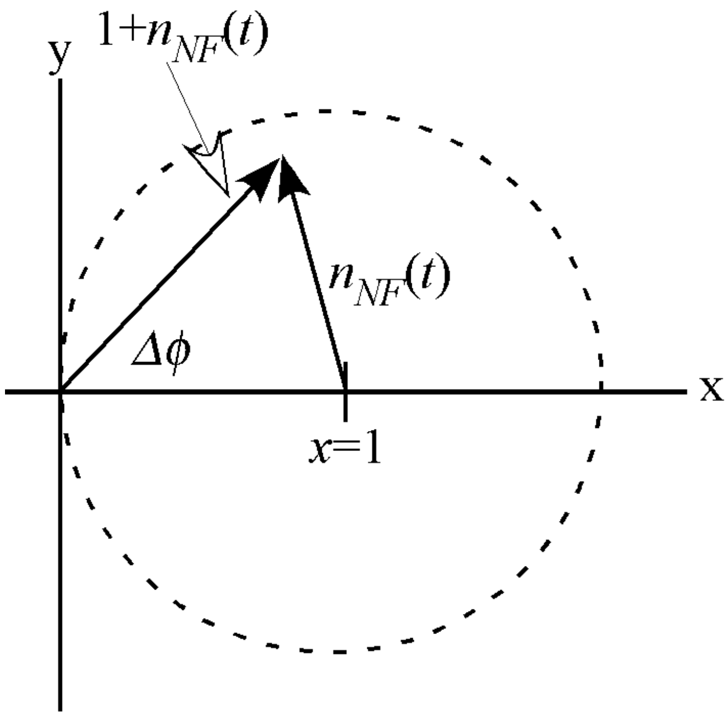
The Capture Effect

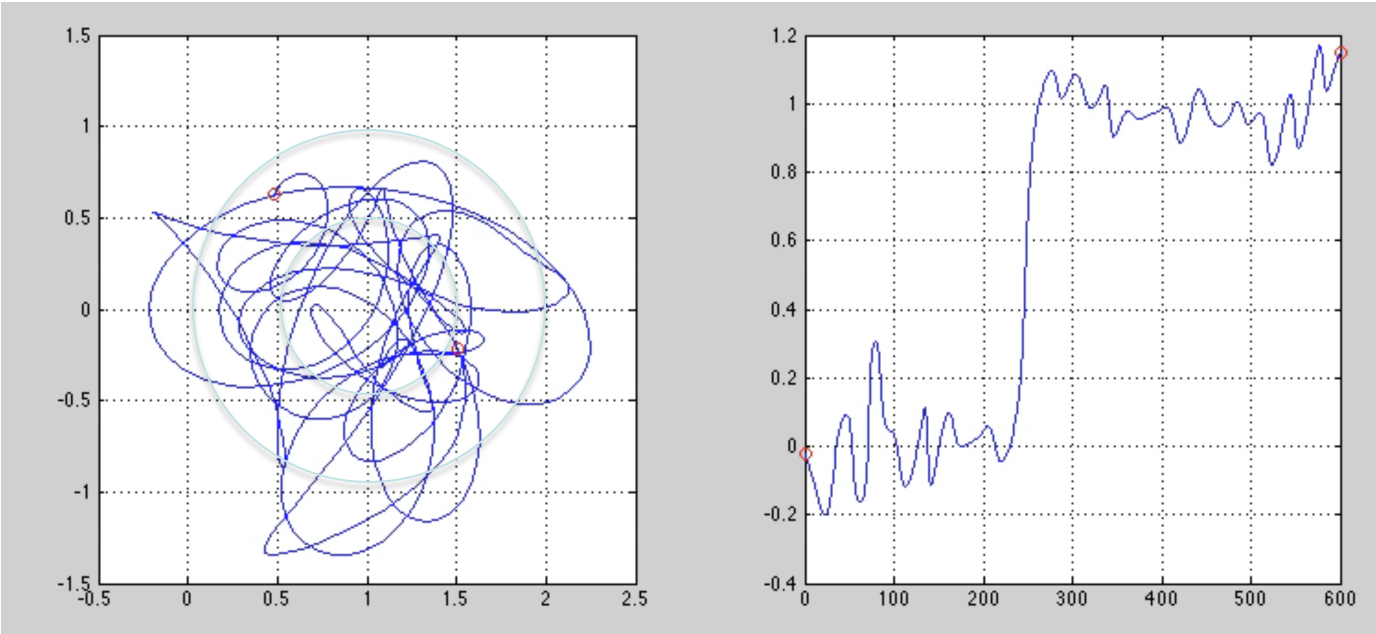


- If $|n_{NF}(t)|<1$ the phase error can never exceed 90°.
- So, the average phase error over many cycles is zero.
- Called the *capture effect*, because the largest signal captures the phase and frequency determination.

If the capture effect is operating (i.e. $|n_{NF}(t)|<1$), the phase error never exceeds 90°.

What happens when the noise is too large?

$1 + n_{NF}(t)$ $\Delta\varphi(t)/(2\pi)$

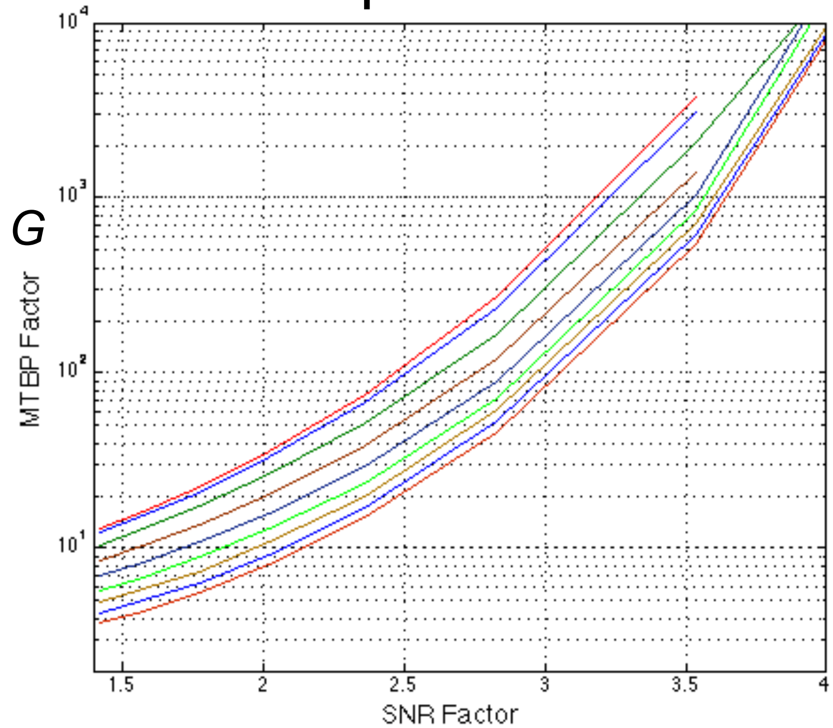


I call the rapid shift by 2π in the phase error a “pop” because that is what it sounds like on an FM radio.

Pop Distribution I

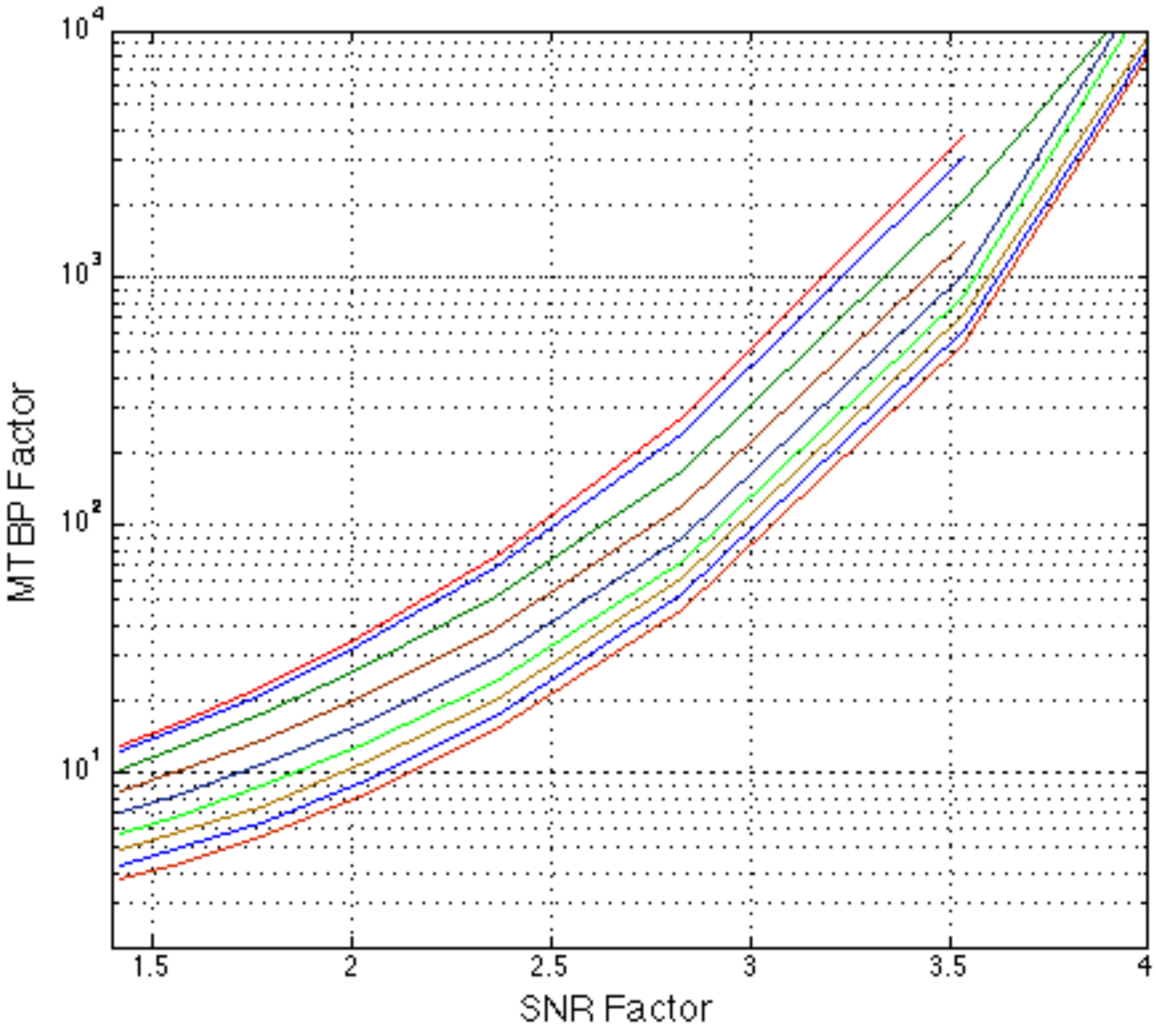
- A pop is a jump in phase and an impulse in frequency.
- Pops are distributed in time with a Poison distribution, described by Mean Time Between Pops.

$$T_{MTBP} = \frac{1}{B} G\left(SNR\sqrt{\frac{f_s}{B}}, \frac{|\Delta f|}{B/2} \right)$$



- B = BP filter bandwidth.
- f_s = sample rate.
- Δf = difference between filter center and signal frequency

Pop Distribution II



$$T_{MTBP} = \frac{1}{B} G\left(SNR\sqrt{\frac{f_s}{B}}, \frac{|\Delta f|}{B/2} \right)$$

$$E[N(T)] = T / T_{MTBP}$$

Curves are for 2nd arg = 0, 1/8, 2/8, …, 8/8 — starting at top.

Increasing SNR by a factor of 2 increases T_{MTBP} by a factor of 100 to 1000.

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Example

f_s = 20 GHz,
SNR = 0.5,
Δf = 0.25

$$T_{MTBP} = \frac{1}{B} G\left(SNR\sqrt{\frac{f_s}{B}}, \frac{|\Delta f|}{B/2} \right)$$

SNR Factor   Rel. Offset

B	1	.5	Comment
SNR Factor	2.23	3.16	From formula
Rel. Offset	0.25	0.5	From formula
G	30	200	From graph
MTBP	30 ns	400 ns	G/B
Pops/µsec	33	2.5	1/MTBP

Factor of 2 reduction in *B* gives factor of 13 improvement.
At low noise there is only a factor of 1.414 improvement.

Conclusions

- Errors in phase and frequency at low SNR are now quantitatively understood.
  - Same as at high SNR with added:
    - 2π Jumps in phase.
    - Delta-like functions in frequency.
- At low SNR choosing tight bounds on spectrogram is very important.
- Bounds should be chosen using multiple window lengths.