Numerical Investigation of Errors in STFT Analysis Applied to PDV

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Purpose of Investigation

- Reduce ‘trial and error’ in PDV analysis.
  - Given signal frequency, sampling rate and noise should be deterministic problem.
- Inform frequency offsetting choices.
  - Can now ‘choose’ the frequency in an experiment.
  - Is a well sampled low frequency preferable to a poorly sampled high frequency?

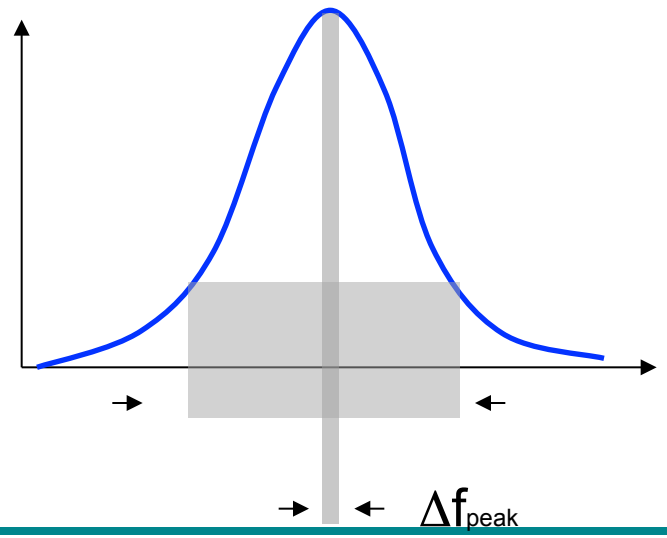
Plan of Attack

- In PDV want to extract frequency, variables are:
  - Signal frequency (can affect this with f offset).
  - Sampling rate (scope limited).
  - Analysis window length (time resolution reqs).
  - Noise.
- Use simulation to get *quantitative* error values.
- Calculate frequency error as a function of variables.
  - Would be nice to get a mathematical relation.

Previous investigation

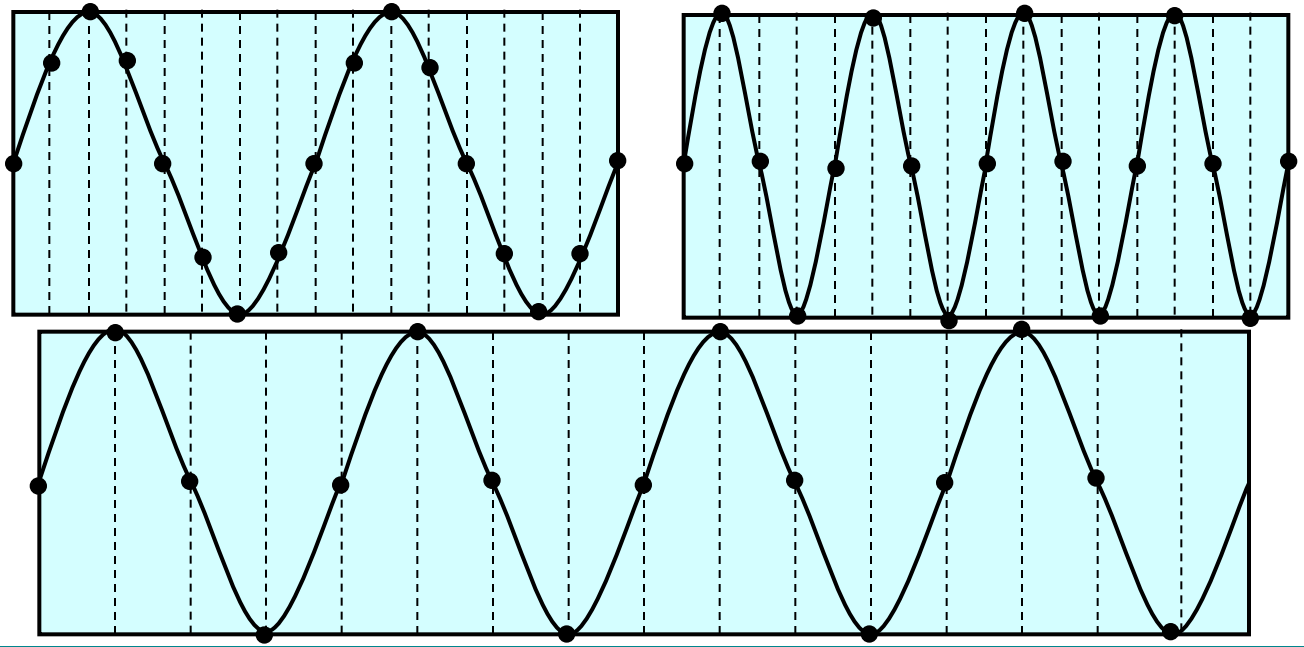
- The peak of the distribution can be located much more accurately than the width of the distribution.

- Dolan's paper reported a simulation, used:
  - 1 ns window
  - 25 samples per window
  - *Varied* frequency



Equivalence

- Doubling the signal frequency *while* keeping the same recording duration *and* sampling rate is equivalent to doubling the recording duration and halving the sample rate for a signal of the initial frequency.

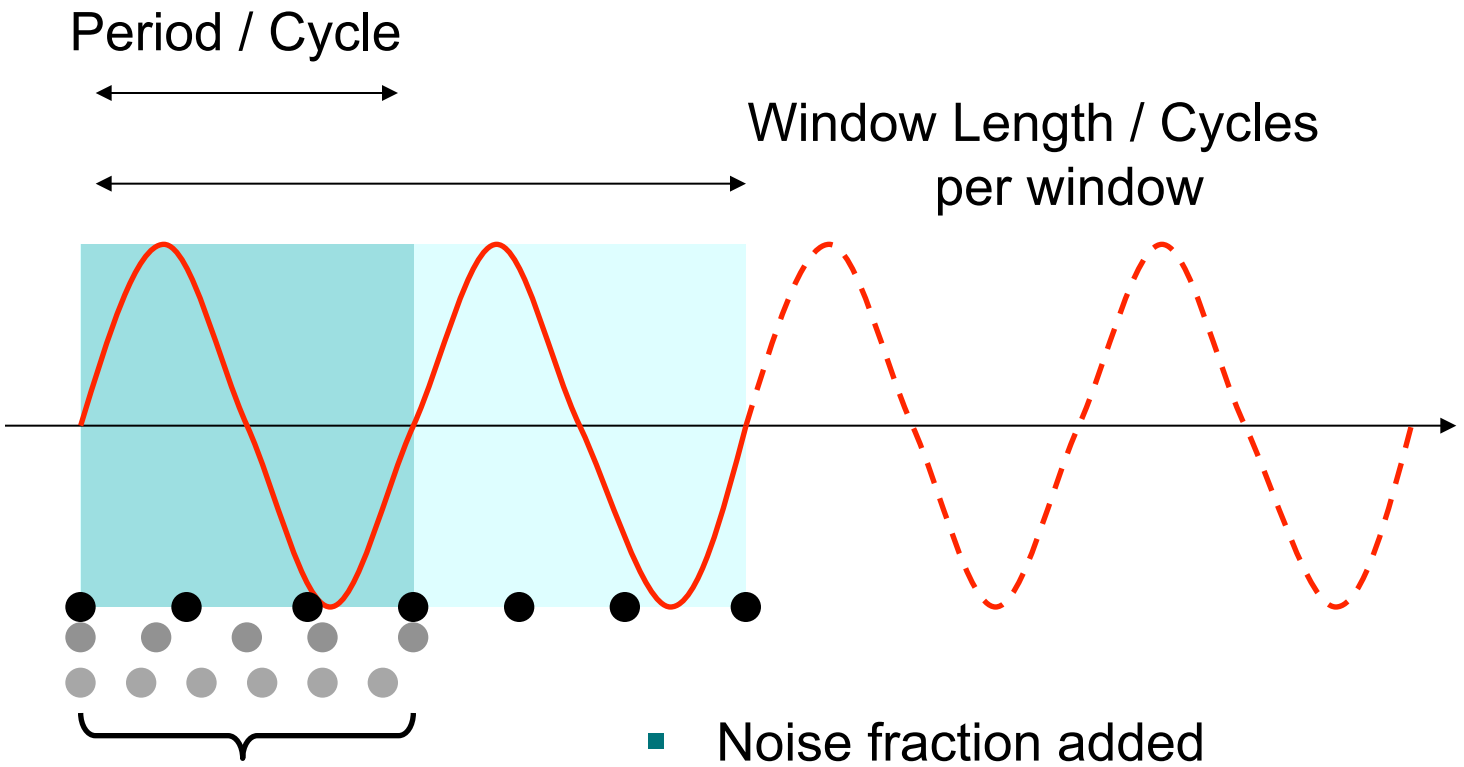


What are the variables?

- Signal frequency.
- Sampling rate.
- Analysis window length.
- ‘Noise’.

- One (1)
- ‘samples per cycle’ (ρ)
- ‘cycles per window’ (τ)
- ‘Noise’ (σ).

Simulation Details

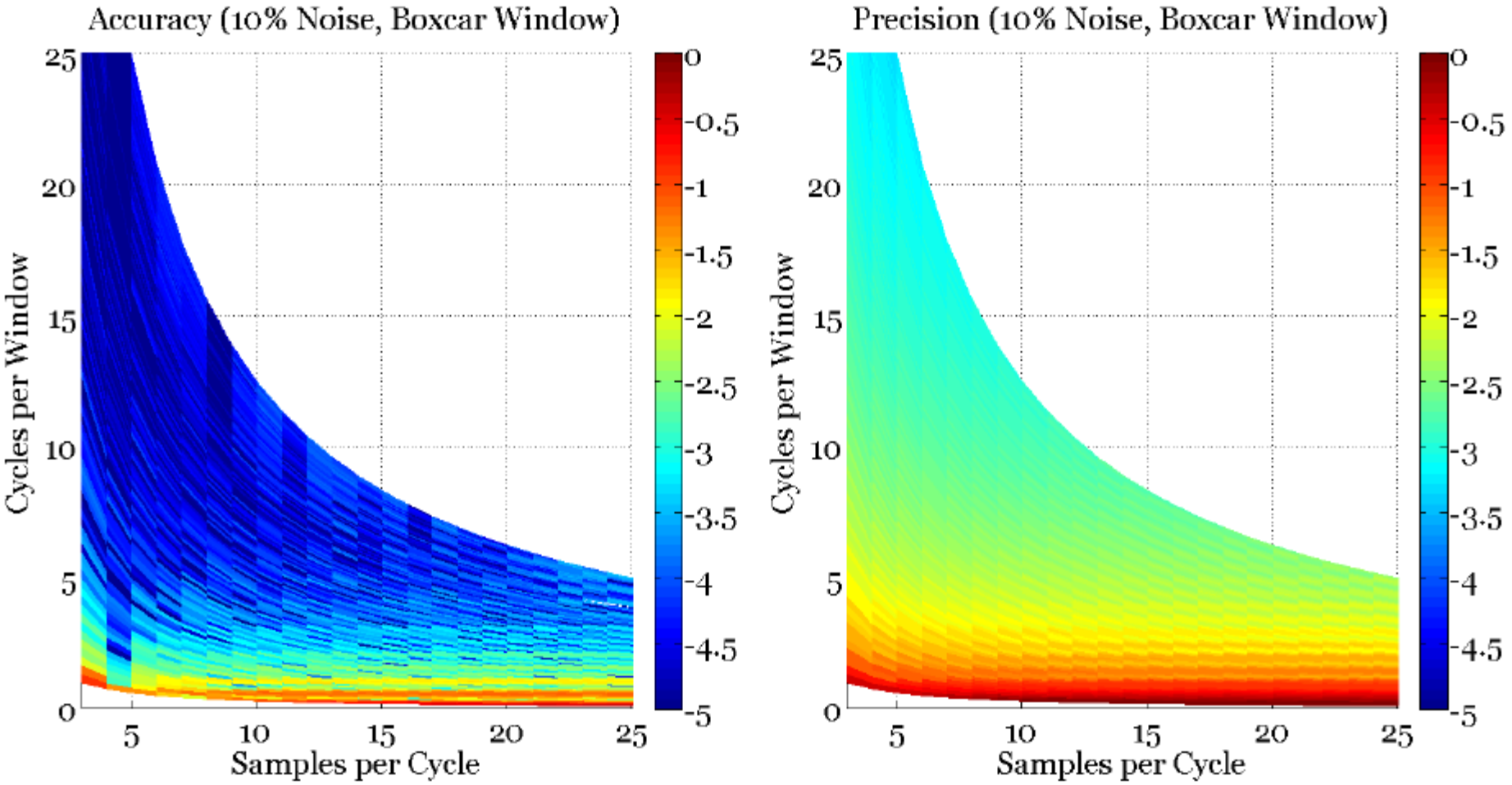


- Noise fraction added
  - 10%
  - 50%

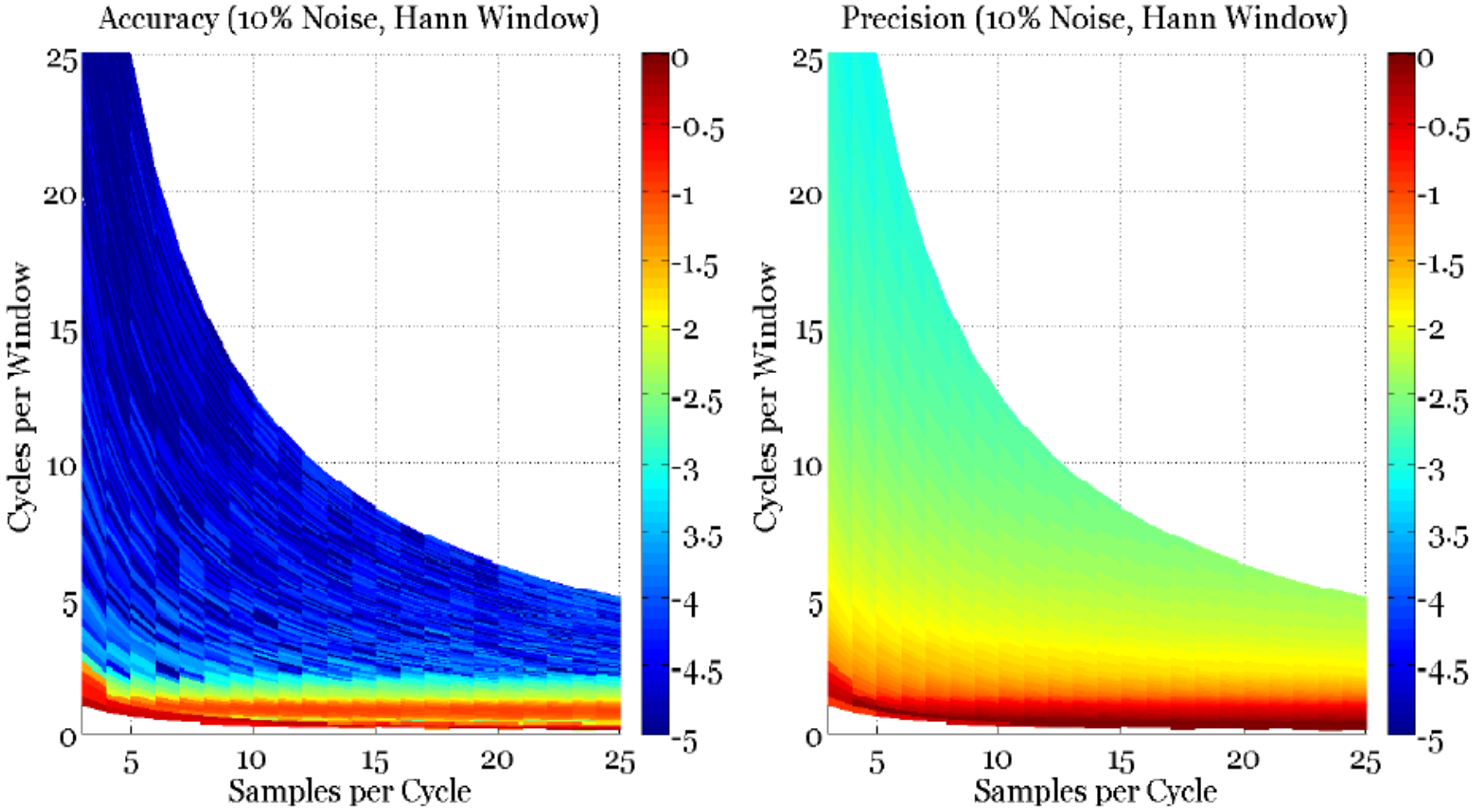
Simulation Details

- Generate random phase.
  - FTs are phase sensitive.
  - Phase effectively random in experiment.
- Generate signal with basis frequency 1 (the mathematical f value used).
- Add noise (10% or 50%)
- Boxcar and Hann windows.
- Need 10,000 runs for each point!

Results 10% Noise, Boxcar

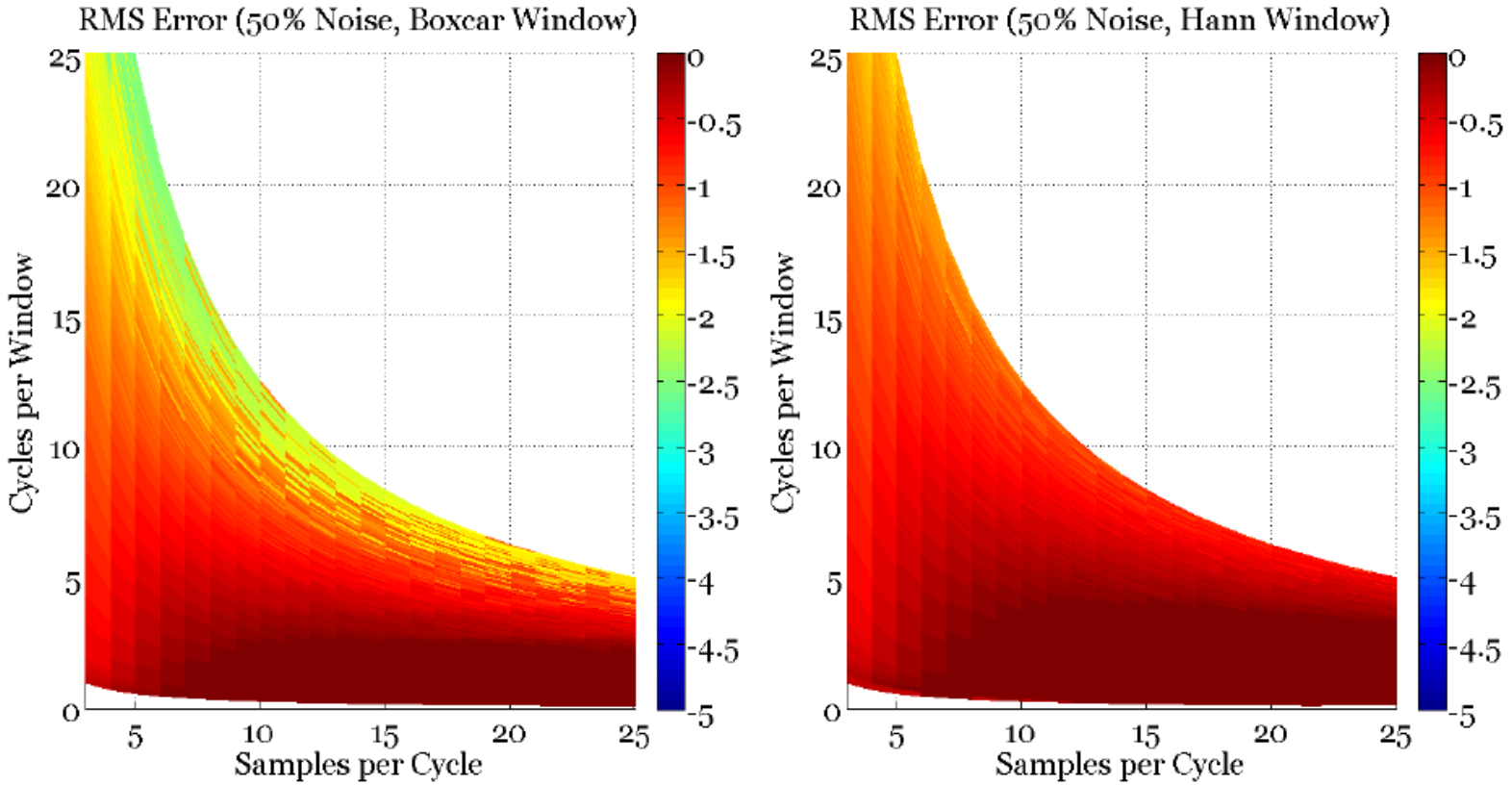


Results 10% Noise. Hann



Error 50% Noise

- Calculate error as quadrature sum of accuracy and precision.



Theoretical Expression

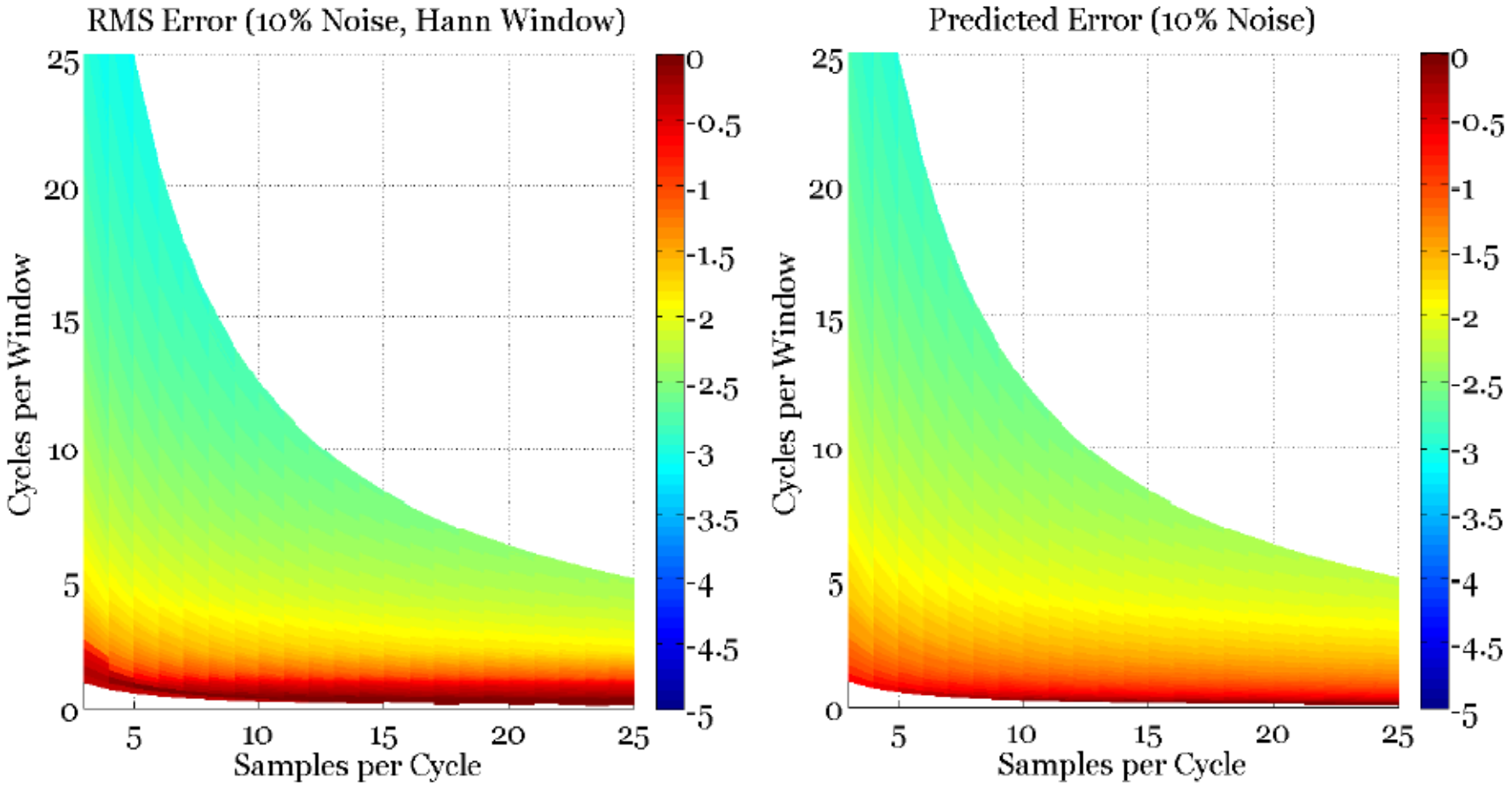
- Rife and Boorstyn suggest the variance of these results should be:

$$\mathrm{Var}(\omega) = \frac{12\sigma^2}{T^2 N(N^2 - 1)}$$

- The standard deviation can be simplified then to:

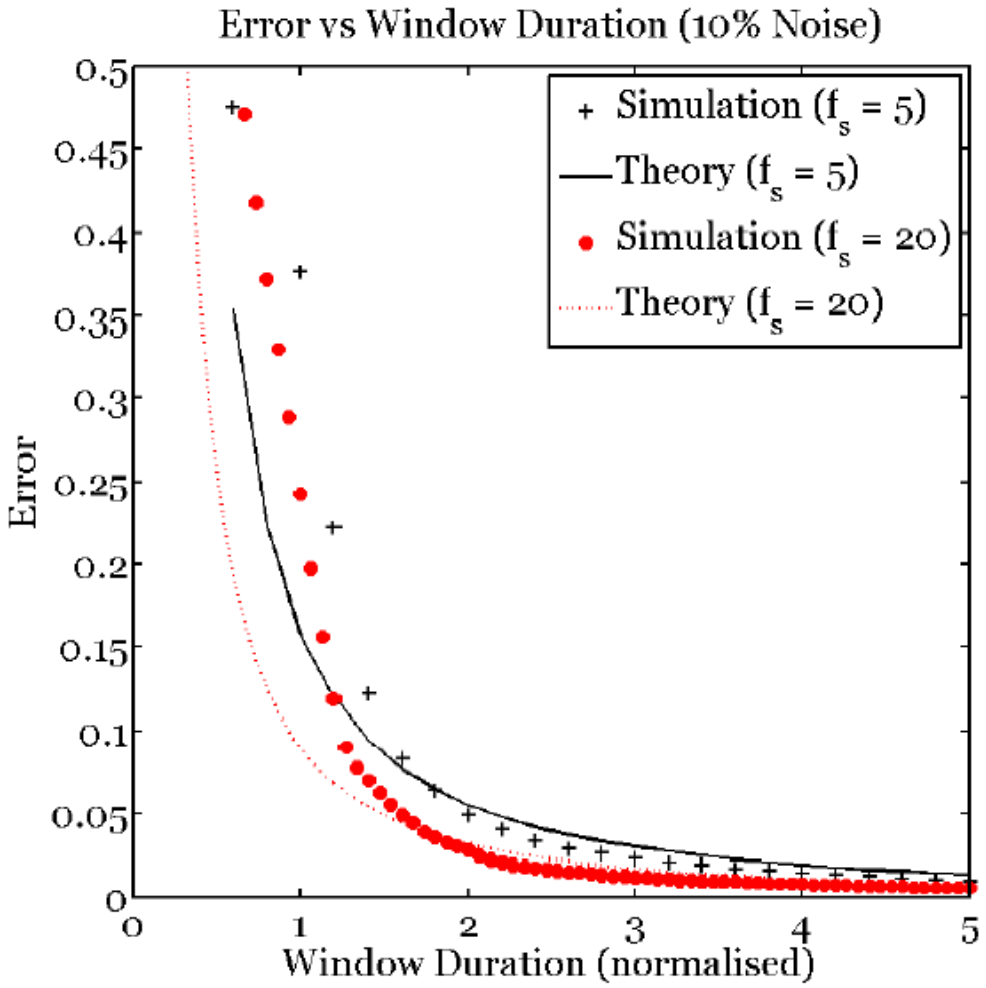
$$s(\omega) = \sqrt{\frac{12}{\rho}}\ \sigma\tau^{-3/2}$$

Theory vs. Simulation



Theory vs. Simulation

- Good match between theory and simulation
- Similar to previously reported expressions
- Over estimate due to error rather than precision?



Conclusions

- Modelling puts numbers to intuition.
- Modelling and theory match well.
- Diminishing returns for excessive window durations and sampling rates.
  - Increasing window duration generally better:

$$\frac{1}{\sqrt{\tau^3}} \qquad \frac{1}{\sqrt{\rho}}$$