Decomposition of the Cylinder Test Velocity Vector

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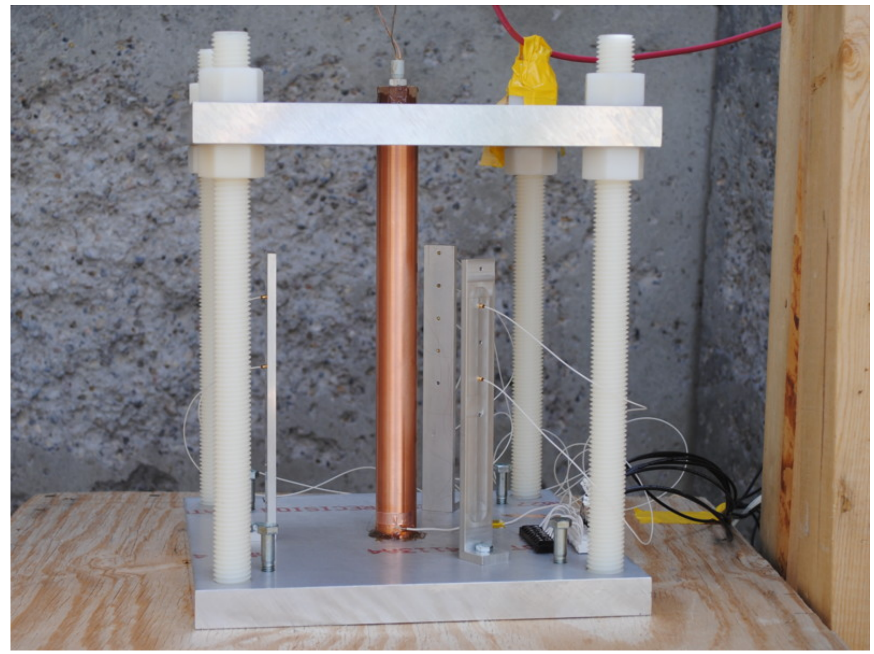
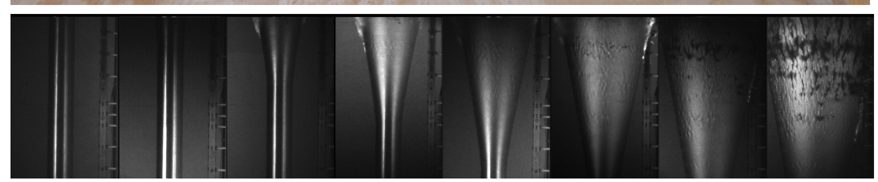
Focused Experiments, WX-3, LANL

What is a Cylinder Test?

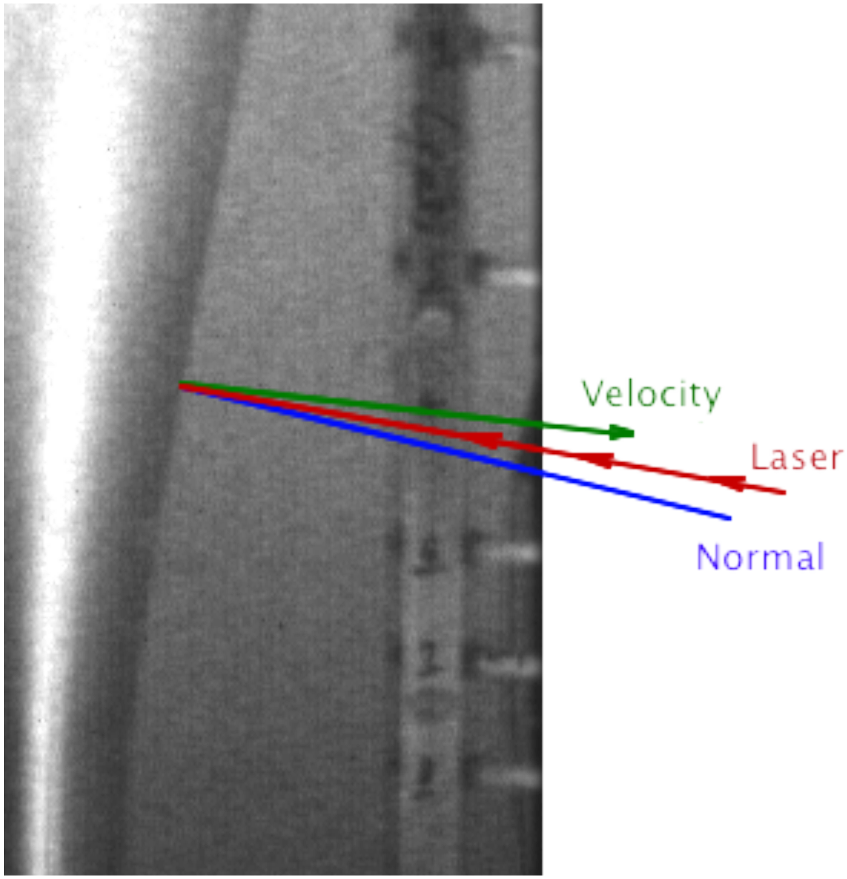
- **Copper cylinder filled with High Explosive (HE)**
  - Used for Equation of State (EOS) measurements
    (Pressure vs. product gas Specific Volume)
  - Instrumented with one to many PDV probes (8 is typical at LANL)
- **Assumptions of the physics:**
  - Detonation is *steady* (constant detonation speed)
  - Wall expansion is *self-similar* at all measurement locations (probes with identical angles produce identical results, independent of location on the wall)





Why decompose the PDV velocity vector?

- **PDV measures Material Velocity in the direction of laser travel**
  - Data is immune to phase velocity
    (This is excellent!)
  - Laser direction is not normal to the copper surface ($\theta_N(t) \neq 0$)
    (Not so excellent.)
  - Material travel is not parallel to the laser direction ($\theta_M(t) \neq 0$)
    (Not so excellent.)
  - Material travel is also not parallel to the surface normal ($\theta_M(t) \neq \theta_N(t)$)
    (Complicates the analysis!)



- **Wanted:**
  - Copper material velocity for a single, small portion of the surface (a speck of copper).

Decomposition Algorithm

- **Fit the extracted PDV velocity, including shock reflections**
- **Remove (ignore) copper reflections**
- **Assume acceleration is normal to the (inner) copper surface**
  - Consistent with acceleration applied solely by the interior gas pressure
  - Neglects transverse acceleration due to shock reflections in the copper
- **Bootstrap material position at each time**
  - Requires computation of the laser impact point at each time in the data
  - Requires assumptions of a self-similar expansion and a steady detonation speed
- **Rebuild the time base for a single material portion to yield a consistent material path line and velocity**

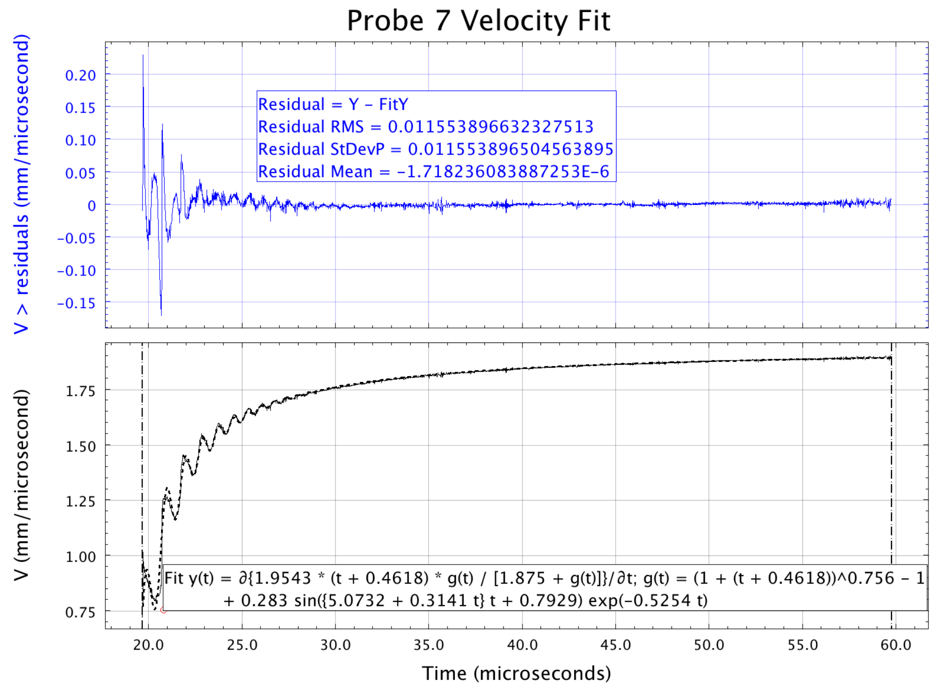
Velocity Fit

- **Root equation is taken from previous work on Cylinder Test analysis (Hill, et al.)**

$$R(t) = \frac{V_\infty g(t)(t - t_0)}{\frac{2\omega V_\infty}{acc_0} + g(t)}, \quad g(t) = \left[1 + (t - t_0)\right]^{\omega - 1} - 1$$

- **Additional harmonic term is added with a linearly "sliding" frequency (to match reflections)**

$$V(t)_{refl} = a \sin\left[(bt + c)t - \phi\right] \exp\left[-\lambda t\right]$$



Bootstrapping Surface Profile

- **Stepwise method is given to the right, with parameters:**
  - Distance "*z*" = $-D_0 * t$
    (D_0 is the detonation velocity)
  - Subscript "*p*" indicates value from probe fit
    R = position, V = velocity, A = acceleration
    Vr = radial velocity
    Vz = axial velocity
  - Angles are defined relative to initial normal (=0º)
    α = as-built laser (probe) angle
    ß = surface normal
    ø = velocity angle

- **Because the time cannot be adjusted when R_j, z_j are adjusted at step (10), the time base is rebuilt after bootstrapping according to:**

$$t_{j+1} = 2\frac{R_{j+1} - R_j}{Vr_{j+1} - Vr_j}$$

1) $z_{j+1} = z_j - \left(t_{j+1} - t_j\right)D_0$ (assumes steady detonation)

2) $V_j = \dfrac{V_{j,p}}{\cos\left[\alpha - \phi_j\right]}$ $\quad A_j = \dfrac{A_{j,p}}{\cos\left[\alpha - \beta_j\right]}$

3) $Vr_j = V_j \cos\left[\phi_j\right]$ $\quad Vz_j = V_j \sin\left[\phi_j\right]$

4) $Vr_{j+1} = Vr_j + Ar_{j-1}\left(t_{j+1} - t_j\right)$ $\quad Vz_{j+1} = Vz_j + Az_{j-1}\left(t_{j+1} - t_j\right)$

5) $R_{j+1} = R_j + 0.5\left(Vr_j + Vr_{j+1}\right)$ $\quad z_{j+1} = z_j + 0.5\left(Vz_j + Vz_{j+1}\right)$

6) $\dfrac{\partial R}{\partial z} = \dfrac{R_{j+1} - R_{j-1}}{z_{j+1} - z_{j-1}}$

7) $\beta_j = \arctan\left[-\dfrac{\partial R}{\partial z}\right]$

8) $Vr_{j+1} = Vr_j + 0.5\left(t_{j+1} - t_j\right)\left(Ar_{j-1} + A_j \cos\left[\beta_j\right]\right)$
$\quad Vz_{j+1} = Vz_j + 0.5\left(t_{j+1} - t_j\right)\left(Az_{j-1} + A_j \sin\left[\beta_j\right]\right)$

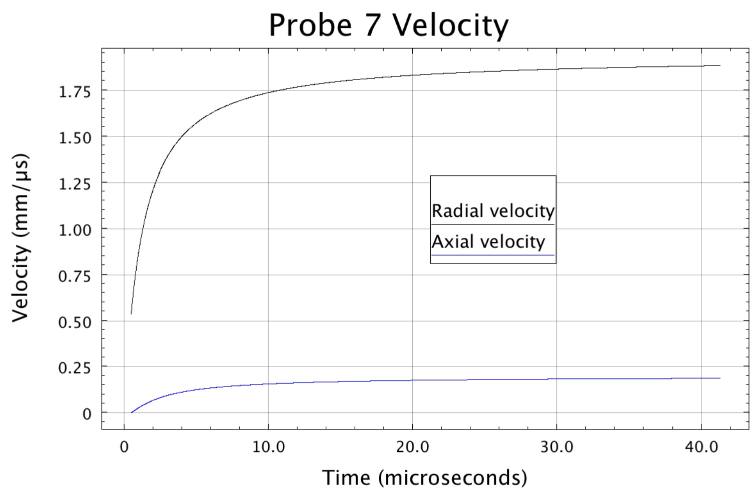
9) **Repeat steps (5)-(8) to convergence**

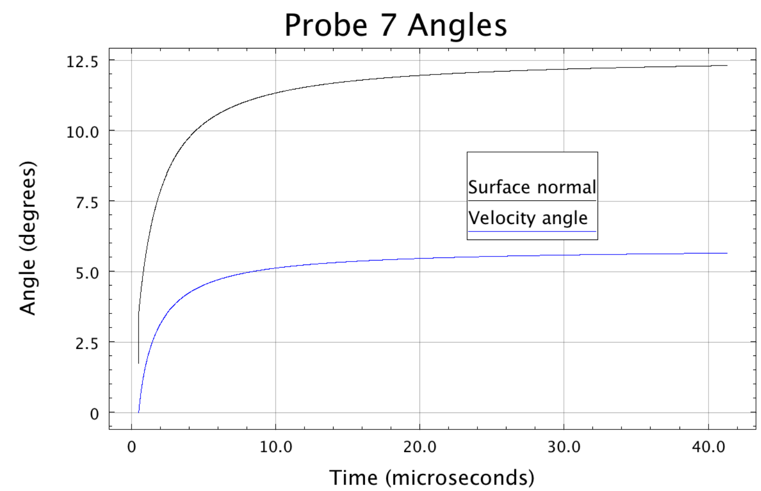
10) **Compute intersection of laser with quadratic fit to points [j-1] to [j+1], and adjust position of R_j, z_j to this intersection (assumes self-similarity)**

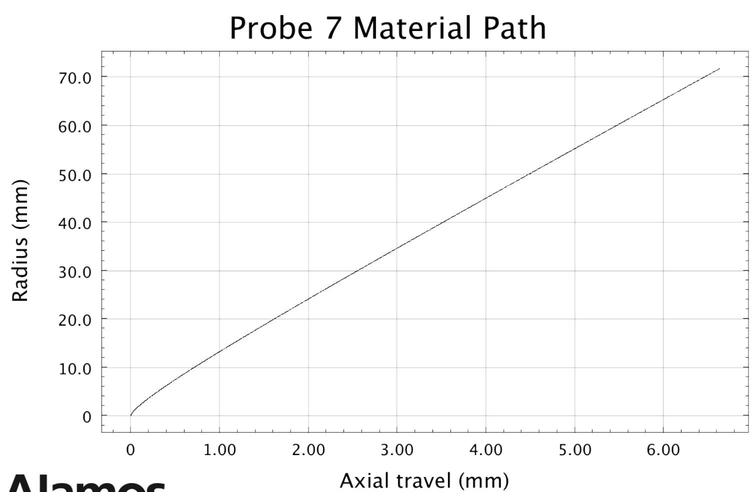
11) $\phi_{j+1} = \arctan\left[Vz_{j+1} / Vr_{j+1}\right]$ $\quad \beta_{j+1} = \arctan\left[-\dfrac{\partial R}{\partial z}\right]$

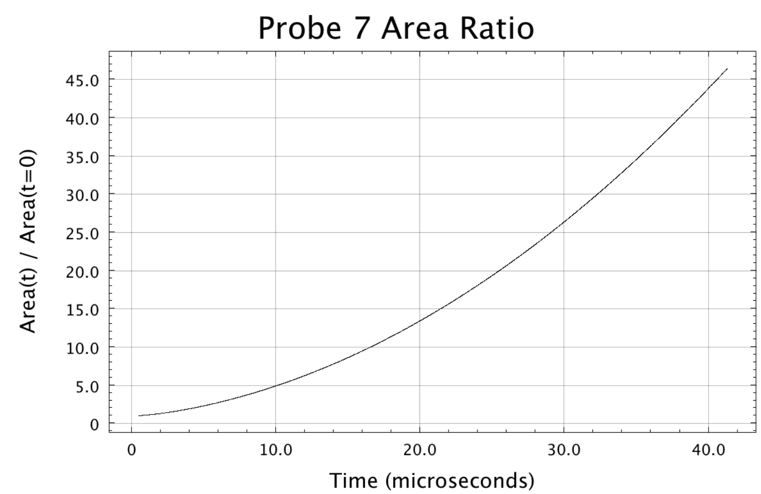
12) ***Increment counter "j" and return to step (1)***

Results of Decomposition









Future Advances

- **Include shock reflections in the copper in the final traces**
  - Likely will not include transverse acceleration of the wall due to the shockwaves
  - Should be able to include shock reflections in the normal acceleration at the surface, and follow through to their effects on the velocity
- **Propagate uncertainty through the analysis**
  - Uncertainty is already estimated for the extracted PDV velocities, so uncertainty propagation through the bootstrapping should be straightforward
- **Use 2D characteristics code to match data and estimate Pressure and Specific Volume within the cylinder (and hence the product gas EOS)**
  - This would further be fit with JWL, JWL++, and/or other analytic EOS forms
  - The pressure at the inside copper surface is not sufficient to estimate the EOS due to rarefactions within the gas – 2D characteristics analysis is necessary