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Mapping Rise Time Information with Down-Shift Analysis

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Introduction

Using new analysis techniques, this presentation demonstrates that the PDV diagnostic can capture sufficient information in short rise time shock events on a scale that is comparable to or better than VISAR. These new analysis techniques unfold the displacement profile from the digital down shift (DDS) at the mid point velocity of a shock and expose a minimum in the displacement profile at the mid point. The techniques then extract estimates for the mid point and rise time information from this minimum. These estimates are then input to a forward modeling analysis for refinement. The process runs very quickly which makes the process a good candidate for the ~ 1 million Monte Carlo simulations to map out errors as function of baseline velocity, velocity jump, rise time, and signal-to-noise ratios. More sophisticated analysis techniques exist which require more CPU time.

Digital down shift (DDS), displacement profile, mid velocity, minimum, estimates, forward modeling analysis, ~ 1 million Monte Carlo simulations

Thank you's to: Dan Dolan (SNL), Rick Gustavsen, and Rob Hixson, David Holtkamp, (LANL)
Also to gallery of NSTec folks for constructive feedback regarding this presentation.

SignalB

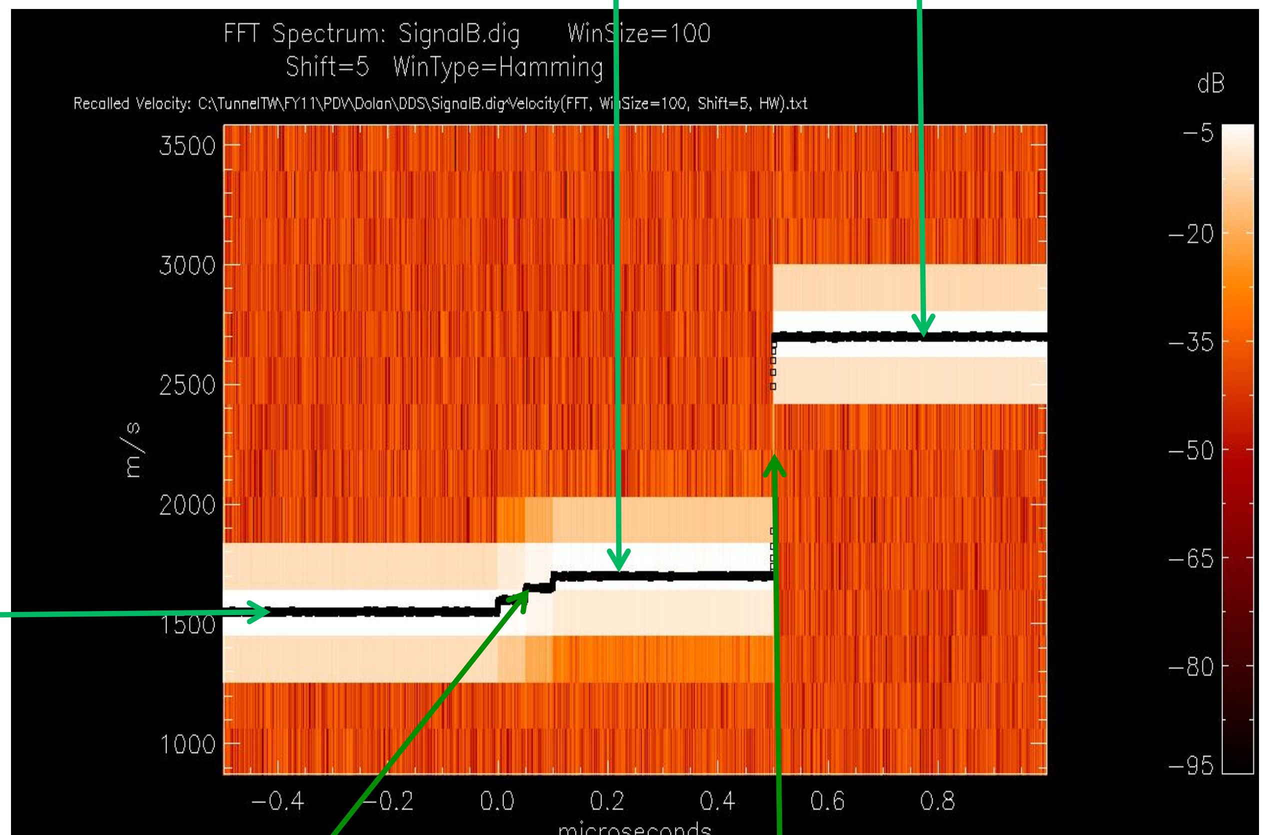
Consider SignalB from Dan Dolan's FY11 PDV analysis challenge. This signal consisted of 3 small velocity jumps (50 m/s with 2 ns rise times) early in the data followed by a delta function jump at 1 km/s late in the data set. All signals rode on top of a 1550 m/s (2 GHz) baseline velocity.

Spectrogram

1,700 m/s

2,700 m/s

1,550 m/s Baseline



3 Small jumps

1 km/s Delta Function Jump

Digitial Down Shift (DDS) and displacement

Details of procedure are in Appendix

The DDS procedure essentially subtracts a constant velocity from all spectral components in a signal.

Signal $A(t) *\cos[\Phi(t)]$ converts to $\Longrightarrow$ $A(t)*\cos[\Phi(t) - \Phi_v(t)]$

$\Longrightarrow$ $-A(t)*\sin[\Phi(t) - \Phi_v(t)]$

where $\Phi_v(t) = (2\pi\ V/(\lambda/2))\ (t-t[0])$ is the frequency (i.e., velocity) and phase shift due the DDS procedure.

From the continuous unfold of the arctangent, you can extract displacement as

$\Longrightarrow$ $X_V(t) = (\lambda/2) * (\Phi(t) - \Phi_v(t))/(2\pi)$

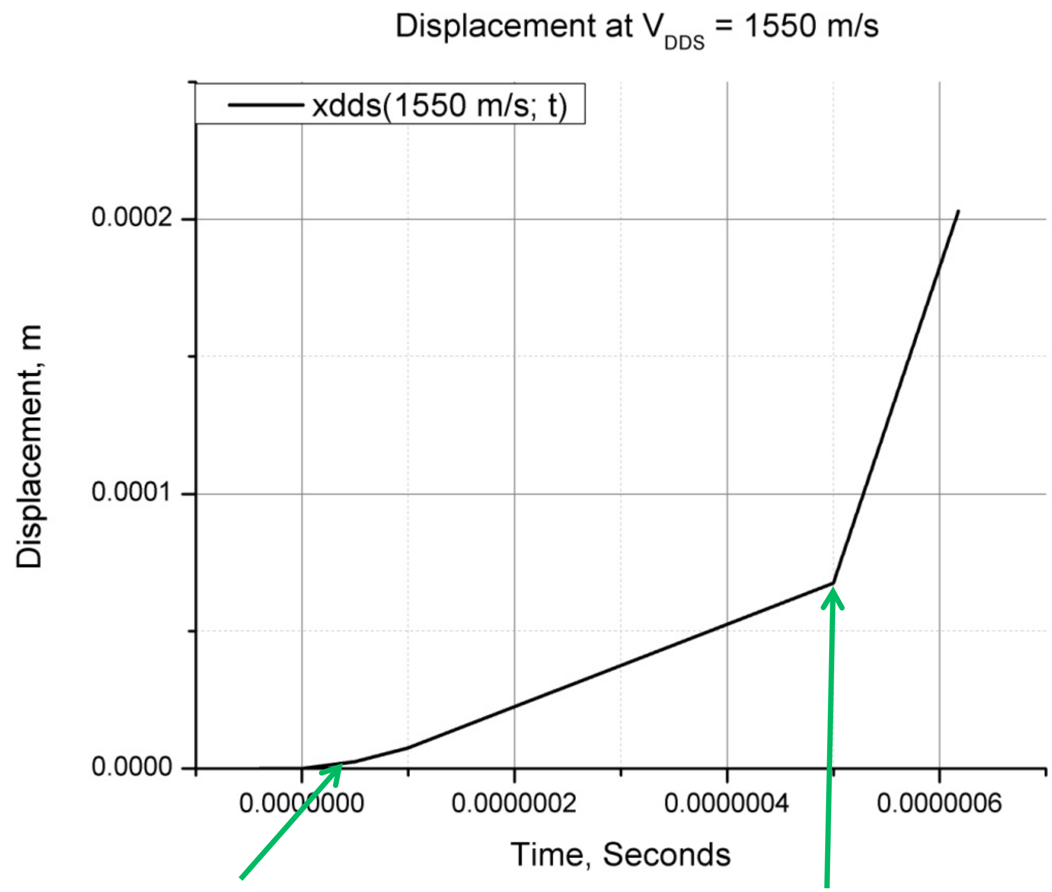
Displacement from DDS at baseline velocity

This viewgraph plots the displacement for SignalB after DDS at the baseline velocity of 1,550.

The rms error in position before time = 0.0 is 14.3 nm. With a noise-to-signal ratio of 0.10, we would expect an rms error

$$\epsilon = \frac{\left(\frac{\sigma}{A}\right)}{2\pi}\left(\frac{\lambda}{2}\right)$$

or 12.3 nm.



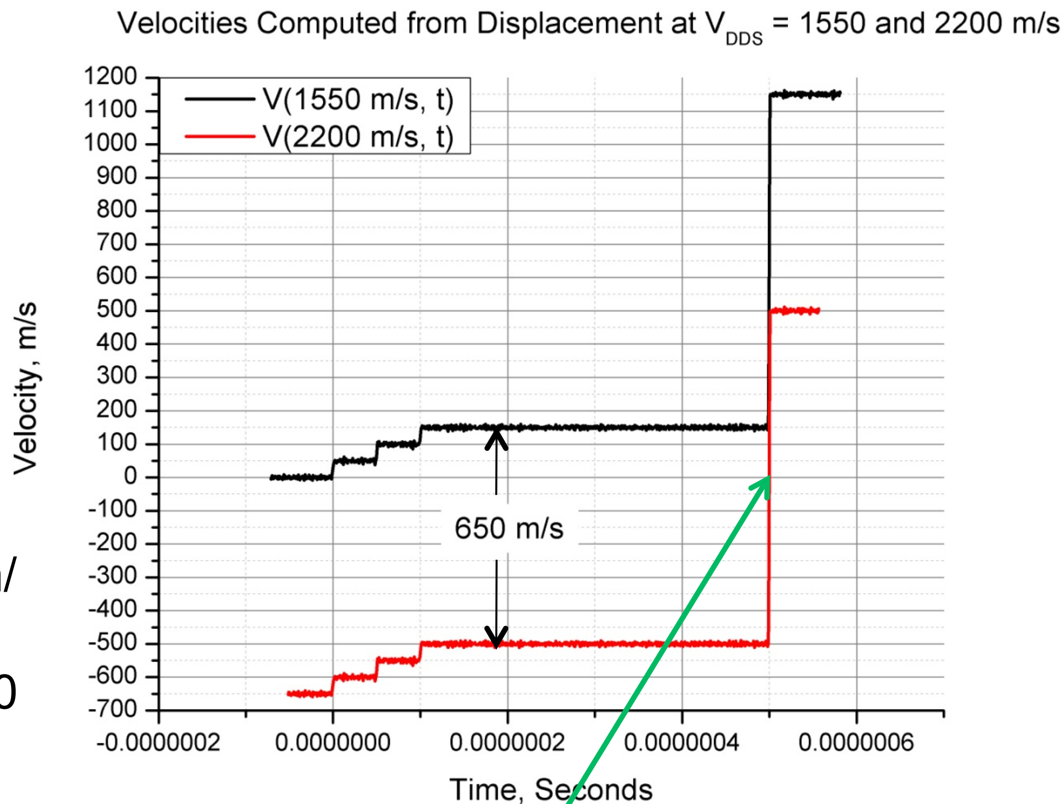
The 3 small jumps are hard to detect.

The delta function jump is obvious.

Velocity computed from linear fit to displacement for slow events

Black curve = velocity profile computed from linear fit to displacement profile at baseline velocity of 1,550 m/s

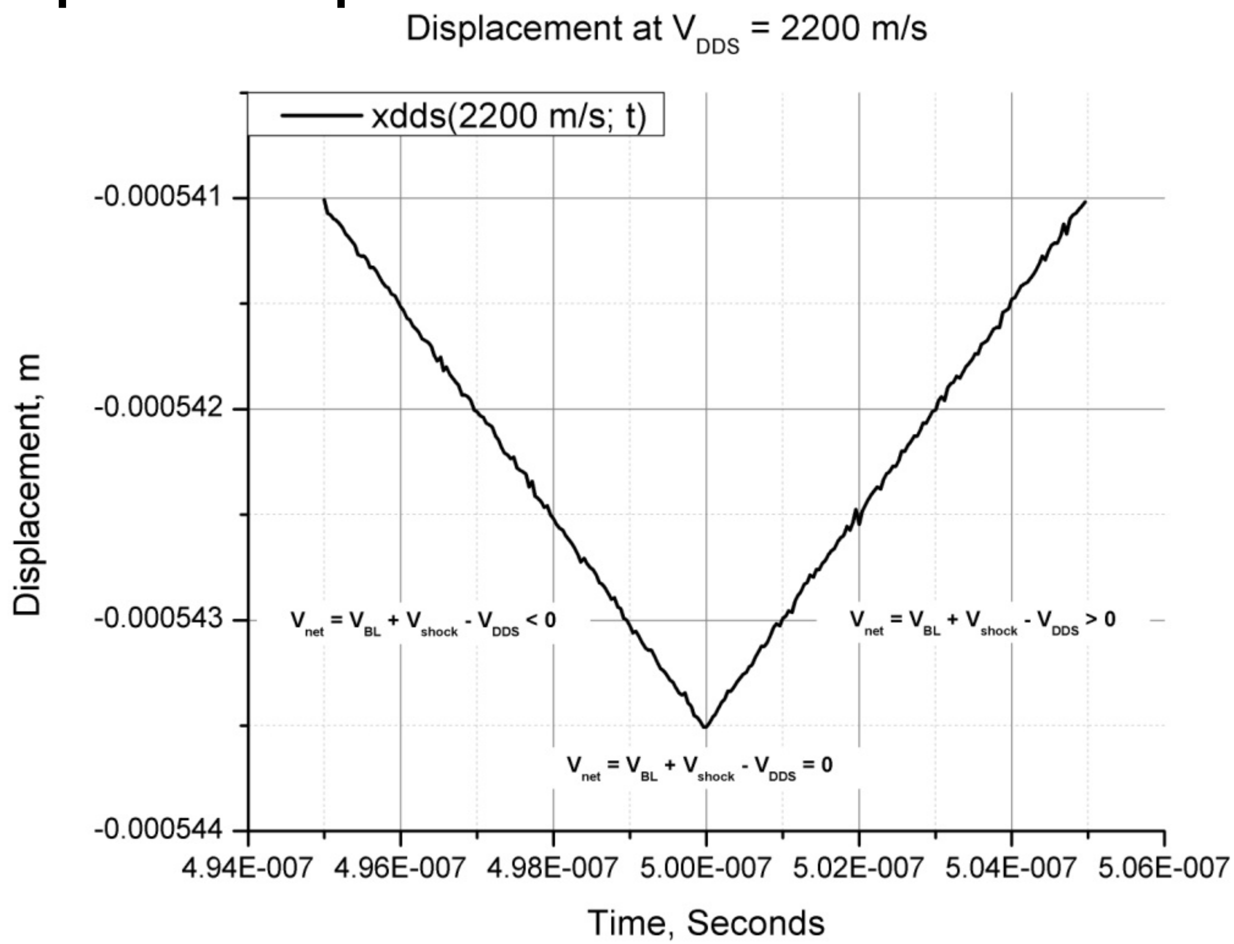
Red curve = velocity profile computed from linear fit to displacement profile at mid point velocity of 2,200 m/s. This 2,200 m/s is 650 m/s above the 1,550 m/s baseline. Velocities will then be 650 m/s lower.



The velocity values predict that going into the delta function jump, the displacement profile for DDS at 2,200 m/s will have negative slope and positive slope afterwards. The displacement profile will have a minimum as it passes from negative slope to positive slope.

DDS: Minimum in displacement profiles

The plot on the right shows the zoomed view of the displacement profile at the delta function after a DDS at the mid point velocity of 2,200 m/s. The minimum is obvious. The profile has negative slope ($V_{net} < 0$) as it approaches the minimum and positive slope ($V_{net} > 0$) as it leaves the minimum.



This plot was an accident. But Rick Gustavsen noted that the technique could be used to extract mid points. In the waning weeks of FY11, we realized that we could also extract rise time information without calculating the velocity profile.

Rise time information in the displacement profile

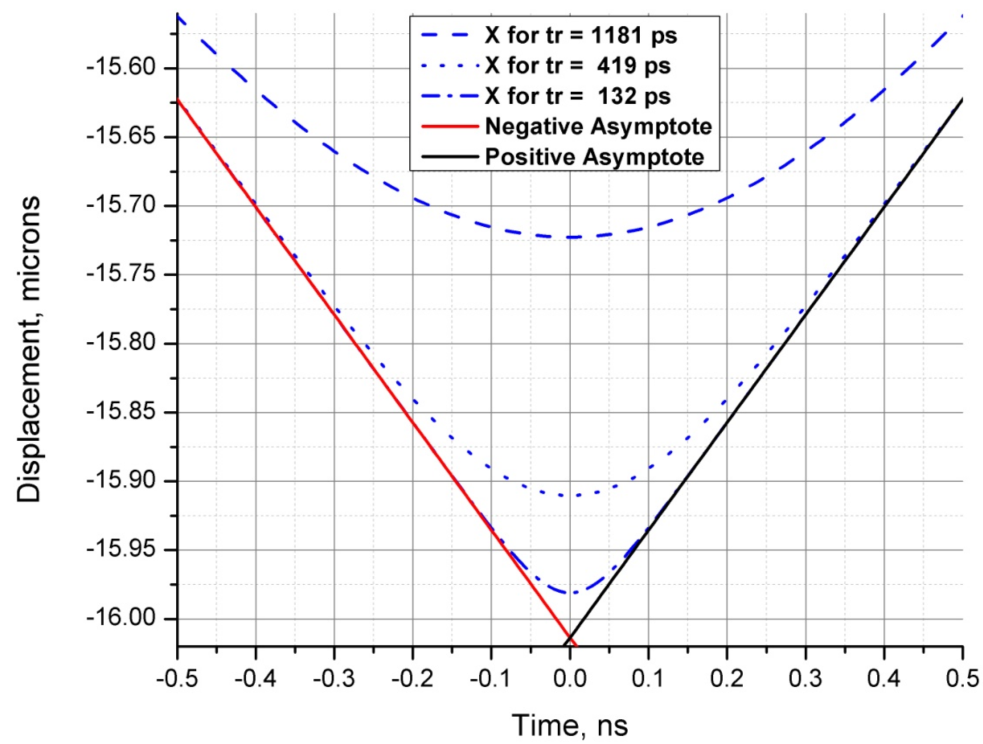
Consider shock (modeled as a sigmoid) with finite rise time:

$$v(t) = v_{BL} + \frac{\Delta v}{1 + e^{-\Gamma(t-T)}}$$

T is mid point (0). The rate Γ is related to 10-90% rise time, tr, as Γ = 4.4/tr. Plot on right shows model (no noise) displacement profile for 3 curves:

Baseline velocity, Vbl = 973 m/s
Velocity jump , ΔV = 1,564 m/s,
DDS of 1,755 m/s (mid point)

Comparison of displacement profiles for signals with different rise times



The rise times are:
tr = 1181 ps
tr = 419 ps
tr = 132 ps.

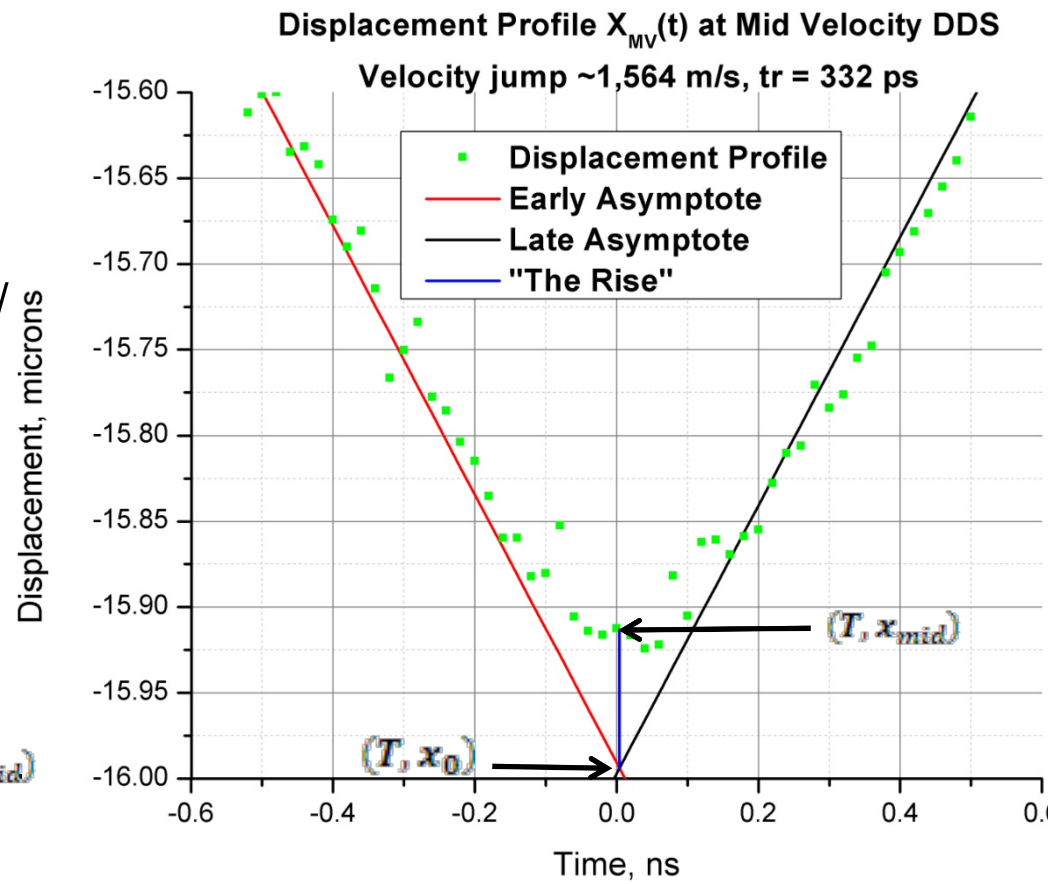
The early and late time displacement values describe lines that are plotted as the negative asymptote and positive asymptote. The mid point is at the intersection. The quick survey of the plot above shows that the displacement profiles penetrate deeper toward the intersection with decreasing rise time. Rise time information resides in the displacement profile.

With noise…

Now consider rise time of 333 ps, mid point of 7.2 ps, and signal-to-noise ratio of 7 (S/N=7). DDS is same at 1,755 m/s. Plot is at right.

The asymptotes intersect at (T, x_0) = (-4 ps, -15.9377 μ).

The blue line rises vertically upward until the line intersects the displacement data at (T, x_{mid}). We will call the length of the blue line the rise.



The rise in this case is 0.0813 microns which is calculated from the data. For a sigmoid, the rise is related to the rise time as Rise = ΔV log(2) tr / 4.4. A rise time of 330 ps is estimated. The estimated values for T and tr (4 ps and 330 ps) compare well with actual values of 7.2 ps and 333 ps. It's that simple.

Mapping with using Monte Carlo simulations

Approximately 1,000,000 Monte Carlo simulations were performed to map out how well the PDV diagnostic captures rise time information (mid point and rise time) in short rise time (< 1 ns) shock events. The shock was modeled as a sigmoid. This included 12 different cases of velocity jumps (described by table at right), 20 values of rise times between 132 – 1181 ps, at S/N ratios from 4 – 7. One thousand simulations were performed for each case, rise time, and S/N ratio combination. Each simulated data set was generated with 6,144 points at 6.667 ps sample interval.

Case	Baseline Velocity (m/s)	Velocity Jump (m/s)	Final Velocity (m/s)
1	487	512	999
2	487	1564	2050
3	487	3722	4209
4	973	512	1485
5	973	1564	2537
6	973	3722	4696
7	1460	512	1972
8	1460	1564	3024
9	1460	3722	5182
10	1946	512	2459
11	1946	1564	3510
12	1946	3722	5669

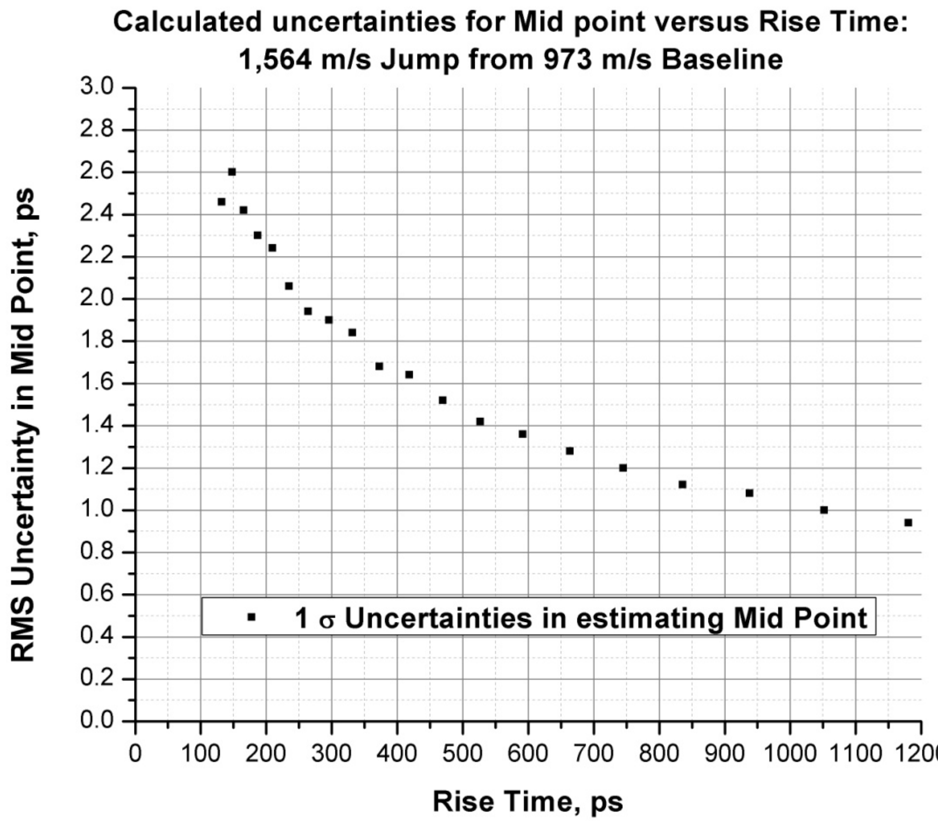
The mid point varied randomly between +/- 10 ps near the middle of the data set. The generated data were re sampled down to 20 ps and white noise was added. The data were processed in the manner just described to estimate mid point and rise time. Improved estimates were then obtained with a forward modeling analysis. RMS uncertainties (in ps) in extracting mid point were calculated as were average and RMS errors (%) in calculated rise times.

Mid point forward modeling results for Case=5 (1,564 m/s velocity jump from 973 m/s base line, S/N = 7)

The figure on the right plots the 1 σ uncertainties in extracting the mid point of the velocity jump for Case 5 and S/N = 7.The success rate for convergence with the forward modeling analysis was 99.9% for all rise times 180 ps and longer.

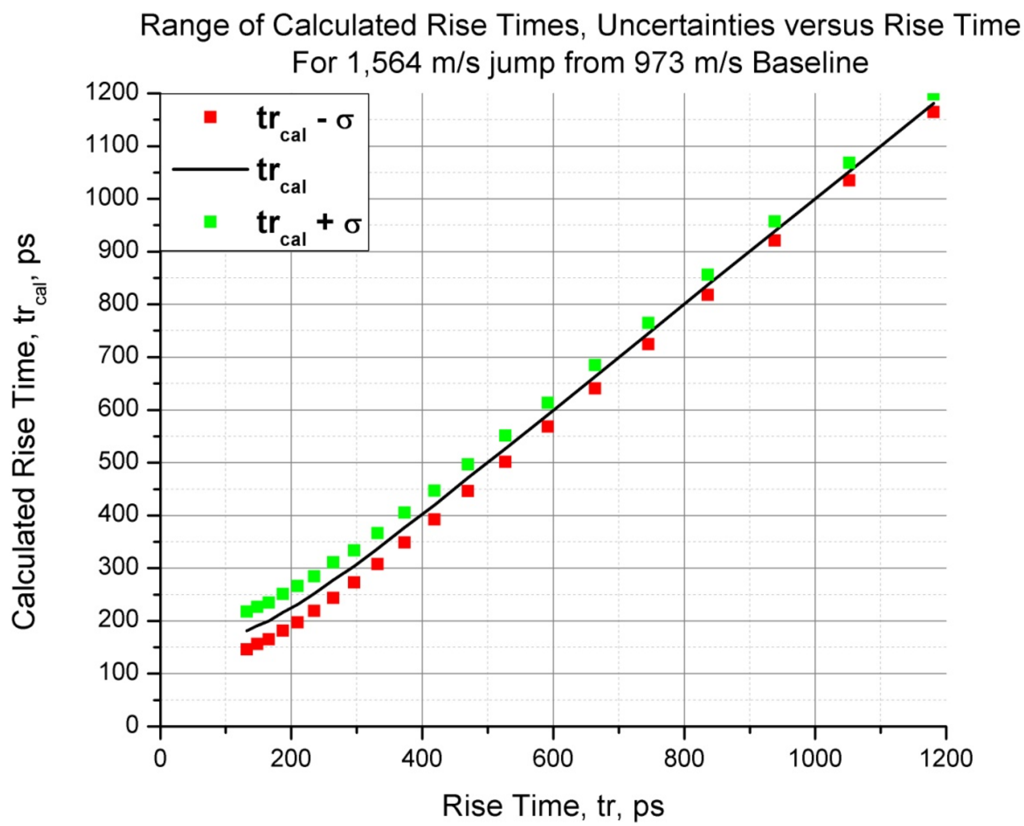
Success rate decreased to 99.2% for shortest case of 132 ps.

These uncertainties range from a high of ~2.5 ps for the shortest rise times (~130 ps) and steadily decrease down to 1 ps and less for 1 ns rise times.



Rise time forward modeling results for Case=5 (1,564 m/s velocity jump from 973 m/s base line, S/N = 7)

The figure on the right plots the average calculated rise time (with 1 σ uncertainties) versus rise time for Case 5 and S/N = 7.The success rate for convergence with the forward modeling analysis was 99.9% for all rise times 180 ps and longer. Success rate decreased to 99.2% for shortest case of 132 ps. The 1 σ uncertainties at 333 ps is about 4 % while the average error is 1% too long.



Errors decrease at longer rise times and increase at shorter rise times. Average error increases to 3 % too long for a 210 ps rise time and 1 σ uncertainties increase to 10%.

Abbreviated Table for S/N = 7

An abbreviated tabulation of errors for S/N = 7 is presented on the right. The errors tabulated are rms error, δt (ps), in calculating the mid point, average error, ε (%), in calculating the rise time, and rms error, σ (%), in calculating the rise time. Errors are tabulated as function baseline velocity, Vb, velocity jump, ∆V, and rise time, tr. In general, the error, ε, decreases with increasing baseline velocity. The errors, δt and σ, are less sensitive to increases in base line velocity. All errors tend to decrease with increasing velocity jump and rise time.

		Vb = 486 m/s			Vb = 973 m/s			Vb = 1,460 m/s			Vb = 1,946m/s		
ΔV (m/s)	tr (ps)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)
512	133	7	179	61	7	85	66	7	57	69	8	46	66
512	187	6	103	44	8	40	49	7	24	49	8	21	48
512	264	6	52	28	6	15	33	7	6	35	7	4	35
512	374	5	23	18	6	3	23	6	0	23	6	-1	24
512	528	5	7	12	5	-1	14	5	-1	14	5	-1	14
512	745	4	2	7	4	-1	8	4	0	8	4	-1	8
512	1053	3	0	4	3	-1	4	3	-1	5	3	-1	5
1,564	133	2	83	23	3	38	28	3	19	30	3	11	31
1,564	187	2	42	15	2	15	19	2	4	20	3	2	21
1,564	264	2	17	10	2	4	12	2	1	13	2	0	13
1,564	374	2	6	6	2	0	7	2	-1	7	2	0	8
1,564	528	1	1	4	1	0	4	2	-1	4	1	0	4
1,564	745	1	0	2	1	-1	2	1	0	2	1	0	2
1,564	1053	1	0	1	1	0	1	1	0	1	1	0	1
3,722	133	1	30	12	1	14	13	1	7	13	1	2	14
3,722	187	1	13	8	1	5	8	1	2	8	1	0	9
3,722	264	1	5	4	1	1	5	1	0	5	1	0	5
3,722	374	1	1	2	1	0	3	1	0	3	1	0	3
3,722	528	1	0	1	1	0	1	1	0	1	1	0	1
3,722	745	1	0	1	0	0	1	1	-1	1	1	0	1
3,722	1053	0	0	0	0	0	0	0	0	0	0	0	0

Errors for S/N = 7. RMS error, δt (ps) in mid point, average error, ε (%), and rms error, σ (%) in calculated rise time as function of baseline velocity, Vb (m/s), Velocity Jump, ΔV (m/s), and rise time, tr (ps)

Abbreviated Table for S/N = 7

An abbreviated tabulation of errors for S/N = 7 is presented on the right. The errors tabulated are rms error, δt (ps), in calculating the mid point, average error, ε (%), in calculating the rise time, and rms error, σ (%), in calculating the rise time. Errors are tabulated as function baseline velocity, Vb, velocity jump, ∆V, and rise time, tr. In general, the error, ε, decreases with increasing baseline velocity. The errors, δt and σ, are less sensitive to increases in base line velocity. All errors tend to decrease with increasing velocity jump and rise time.

		Vb = 973 m/s			Vb = 1,460 m/s			Vb = 1,946 m/s		
ΔV (m/s)	tr (ps)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)
512	133	7	85	66	7	57	69	8	46	66
512	264	6	15	33	7	6	35	7	4	35
512	528	5	-1	14	5	-1	14	5	-1	14
512	1053	3	-1	4	3	-1	5	3	-1	5
1,564	133	3	38	28	3	19	30	3	11	31
1,564	264	2	4	12	2	1	13	2	0	13
1,564	528	1	0	4	2	-1	4	1	0	4
1,564	1053	1	0	1	1	0	1	1	0	1
3,722	133	1	14	13	1	7	13	1	2	14
3,722	264	1	1	5	1	0	5	1	0	5
3,722	528	1	0	1	1	0	1	1	0	1
3,722	1053	0	0	0	0	0	0	0	0	0

Errors for S/N = 7. RMS error, δt (ps) in mid point, average error, ε (%), and rms error, σ (%) in calculated rise time as function of baseline velocity, Vb (m/s), Velocity Jump, ΔV (m/s), and rise time, tr (ps)

Summary

1. This presentation has demonstrated that short rise time information can be extracted from PDV data (S/N >5) on a scale that is comparable to or better than VISAR as noted by Dan Dolan. The results are based on a particular model for shock and results with other models (especially those with asymmetric shape) may have greater uncertainty.

2. The tabulations in the appendix map out this capability.

3. The analysis tool that was used for this mapping is simply one of the fastest. Uses no filtering. Preliminary results indicate that filtering improves calculations for S/N =4,5 ratios. More sophisticated tools exist.

4. Also, this presentation reminds us that, despite the name, PDV is a displacement interferometer and perhaps we have not exploited PDV to its fullest extent.

Appendix: DDS procedure

1. The first step determines the Nyquist frequency from the sample interval as: $f_{Nyquist} = \frac{1}{2\,\Delta t}$

2. The second step generates a mixing function at Nyquist as $\cos\left(2\pi\, f_{Nyquist}\, n\,\Delta t\right)$ which is an alternating sequence of +/- 1's.

3. In the third step, the DDS technique multiplies the digital data set by the mixing function as

$$P(n) = D(n)\cos\left(2\pi\, f_{Nyquist}\; n\,\Delta t\right)$$

   This multiplication step mixes all signals. This mixing maps all frequencies f into $f \rightarrow f_{Nyquist} \pm f$. In the digital frequency domain, the sum $(f_{Nyquist} + f)$ maps directly on top of the difference $(f_{Nyquist} - f)$.

4. Using FFT properties(sinc interpolation), the fourth step re-samples the product *P(n)* by a factor of 2. The re sampled product is *R(n)*. The FFT power spectrum is unchanged and frequencies above $f_{Nyquist}$ have no power.

5. The fifth step generates new mixing functions as $C(n) = \cos\left(2\pi\left(f_{Nyquist} - f_{DDS}\right)n\Delta t\right)$ and $S(n) = \sin\left(2\pi\left(f_{Nyquist} - f_{DDS}\right)n\Delta t\right)$

6. Continuing, the procedure multiplies the re sampled product *R(n)* by these new mixing functions in the sixth step $PC(n) = C(n) * P(n)$ and $PS(n) = P(n) * S(n)$. This step splits the frequency $(f_{Nyquist} - f)$ into a sum frequency $(2f_{Nyquist} - f - f_{DDS})$ which is above Nyquist and a difference $-(f - f_{DDS})$, which is below Nyquist. The separation occurs for both the *PC(n)* and *PS(n)*. The difference $-(f - f_{DDS})$ signal behaves identically to a signal with frequency $(f - f_{DDS})$ except for a sign difference for the sine series $PS(n)$, which can be corrected. The FFT spectrogram would be identical.

7. The seventh step reuses the sinc interpolation techniques to re sample the *C(n)* and *S(n)* products down by a factor of 2. The downward resampling eliminates the frequencies above Nyquist. The re sampled products are multiplied by 2 for proper normalization.

8. In rare circumstances, the frequency domain above ½ Nyquist will contain content of interest. In such cases, the data will first need to be re sampled up by a factor of 2. In addition, a final resampling down by 4 rather than by 2 will be required.

Summary Table for S/N = 6

An abbreviated tabulation of errors for S/N = 6 is presented on the right. The errors tabulated are rms error, δt (ps), in calculating the mid point, average error, ε (%), in calculating the rise time, and rms error, σ (%), in calculating the rise time. Errors are tabulated as function baseline velocity, Vb, velocity jump, ∆V, and rise time, tr. In general, the error, ε, decreases with increasing baseline velocity. The errors, δt and σ, are less sensitive to increases in base line velocity. All errors tend to decrease with increasing velocity jump and rise time.

		Vb = 486 m/s			Vb = 973 m/s			Vb = 1,460 m/s			Vb = 1,946m/s		
ΔV (m/s)	tr (ps)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)
512	133	8	184	67	9	93	76	9	66	76	8	53	76
512	187	7	101	48	8	43	52	8	24	54	8	23	56
512	264	7	55	35	8	14	37	8	10	39	8	9	40
512	374	7	23	21	7	1	27	7	1	27	7	0	27
512	528	6	7	15	6	0	16	6	0	17	6	-1	17
512	745	4	1	9	5	-1	10	5	-1	10	5	-1	10
512	1053	4	0	5	4	-1	5	4	-1	5	4	-1	5
1,564	133	3	83	26	3	38	31	3	20	34	4	8	35
1,564	187	2	42	18	3	15	21	3	6	22	3	1	25
1,564	264	2	18	12	2	4	14	3	0	15	2	-1	15
1,564	374	2	6	8	2	0	8	2	0	9	2	-1	9
1,564	528	2	1	5	2	-1	5	2	0	5	2	0	5
1,564	745	1	0	3	1	0	3	1	-1	3	1	0	3
1,564	1053	1	0	1	1	0	1	1	0	1	1	0	1
3,722	133	1	30	14	1	14	15	1	7	16	1	3	17
3,722	187	1	13	9	1	5	10	1	2	10	1	0	10
3,722	264	1	5	6	1	1	6	1	0	6	1	0	6
3,722	374	1	1	3	1	0	3	1	0	3	1	0	3
3,722	528	1	0	2	1	0	2	1	0	2	1	0	2
3,722	745	1	0	1	1	0	1	1	0	1	1	0	1
3,722	1053	1	0	0	0	0	0	0	0	0	1	0	0

Errors for S/N = 6. RMS error, δt (ps) in mid point, average error, ε (%), and rms error, σ (%) in calculated rise time as function of baseline velocity, Vb (m/s), Velocity Jump, ΔV (m/s), and rise time, tr (ps)

Summary Table for S/N = 5

An abbreviated tabulation of errors for S/N = 5 is presented on the right. The errors tabulated are rms error, δt (ps), in calculating the mid point, average error, ε (%), in calculating the rise time, and rms error, σ (%), in calculating the rise time. Errors are tabulated as function baseline velocity, Vb, velocity jump, ∆V, and rise time, tr. In general, the error, ε, decreases with increasing baseline velocity. The errors, δt and σ, are less sensitive to increases in base line velocity. All errors tend to decrease with increasing velocity jump and rise time. **Preliminary results indicate that filtering improves results.**

		Vb = 486 m/s			Vb = 973 m/s			Vb = 1,460 m/s			Vb = 1,946m/s		
ΔV (m/s)	tr (ps)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)
512	133	10	187	81	10	105	89	9	77	82	9	70	81
512	187	9	110	55	10	51	60	10	33	64	57	34	65
512	264	8	54	40	9	20	45	10	15	45	9	11	45
512	374	54	24	26	9	6	32	9	0	31	9	2	32
512	528	57	8	18	8	0	20	9	0	21	8	-2	20
512	745	6	2	11	34	0	31	6	-1	12	6	-1	12
512	1053	4	0	6	5	-1	7	5	-1	7	5	-1	7
1,564	133	3	83	34	4	42	37	4	21	39	4	15	39
1,564	187	3	43	22	4	15	26	3	6	28	4	2	30
1,564	264	3	18	15	3	4	17	3	0	18	3	-1	17
1,564	374	2	6	9	2	0	10	2	-1	10	3	-1	11
1,564	528	2	1	6	2	-1	6	2	0	6	2	-1	6
1,564	745	2	0	3	2	0	4	2	0	3	2	-1	3
1,564	1053	1	-1	2	1	-1	2	1	0	2	1	-1	2
3,722	133	2	28	16	2	13	18	2	6	19	2	3	20
3,722	187	1	13	11	1	4	12	1	1	12	1	1	12
3,722	264	1	5	7	1	1	7	1	0	7	6	0	18
3,722	374	1	1	4	1	0	4	1	-1	4	1	0	4
3,722	528	1	0	2	1	0	2	1	0	2	1	0	2
3,722	745	1	0	1	1	0	1	1	0	1	1	0	1
3,722	1053	1	0	0	1	0	0	1	0	0	5	0	3

Errors for S/N = 5. RMS error, δt (ps) in mid point, average error, ε (%), and rms error, σ (%) in calculated rise time as function of baseline velocity, Vb (m/s), Velocity Jump, ΔV (m/s), and rise time, tr (ps)

Summary Table for S/N = 4

An abbreviated tabulation of errors for S/N = 4 is presented on the right. The errors tabulated are rms error, δt (ps), in calculating the mid point, average error, ε (%), in calculating the rise time, and rms error, σ (%), in calculating the rise time. Errors are tabulated as function baseline velocity, Vb, velocity jump, ∆V, and rise time, tr. In general, the error, ε, decreases with increasing baseline velocity. The errors, δt and σ, are less sensitive to increases in base line velocity. All errors tend to decrease with increasing velocity jump and rise time. **Preliminary results indicate that filtering improves results.**

		Vb = 486 m/s			Vb = 973 m/s			Vb = 1,460 m/s			Vb = 1,946m/s		
ΔV (m/s)	tr (ps)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)	δt (ps)	ε (%)	σ (%)
512	133	123	204	314	110	143	484	130	128	459	192	132	461
512	187	95	121	214	107	71	212	118	60	219	177	73	362
512	264	61	65	140	113	44	240	84	21	56	163	34	214
512	374	131	28	93	114	18	154	149	20	158	173	19	186
512	528	88	7	24	111	7	87	112	3	95	162	6	91
512	745	9	1	14	44	-1	15	62	1	46	170	4	79
512	1053	82	0	18	81	0	34	127	1	44	170	5	70
1,564	133	19	84	86	19	46	118	20	33	100	22	24	153
1,564	187	4	43	28	25	19	75	7	8	39	28	14	132
1,564	264	4	18	18	20	3	22	29	4	84	35	4	84
1,564	374	20	8	37	27	2	45	20	0	41	34	1	43
1,564	528	20	2	27	20	0	28	27	0	31	31	2	41
1,564	745	33	1	18	20	0	9	29	1	24	32	0	18
1,564	1053	12	-1	3	35	1	17	33	0	13	38	1	21
3,722	133	7	30	42	5	14	37	7	8	37	2	3	25
3,722	187	2	12	14	8	7	35	11	3	40	9	1	33
3,722	264	11	6	26	4	1	13	8	0	17	11	1	29
3,722	374	1	2	5	9	1	19	12	1	23	8	0	12
3,722	528	11	0	9	9	0	9	7	0	9	6	0	8
3,722	745	1	0	2	6	0	5	9	0	7	5	-1	2
3,722	1053	10	0	5	9	0	5	13	0	6	16	0	7

Errors for S/N = 4. RMS error, δt (ps) in mid point, average error, ε (%), and rms error, σ (%) in calculated rise time as function of baseline velocity, Vb (m/s), Velocity Jump, ΔV (m/s), and rise time, tr (ps)