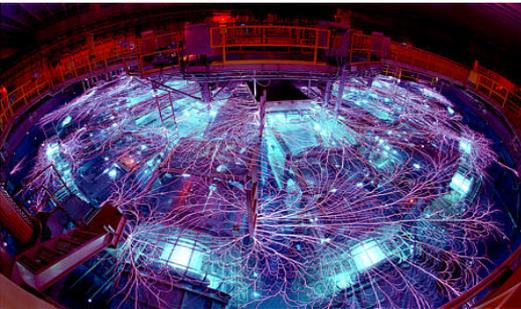


Complexities, complications and other gory details of Photonic Doppler Velocimetry



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Photonic Doppler Velocimetry Workshop
Albuquerque, NM, October 22-23, 2012



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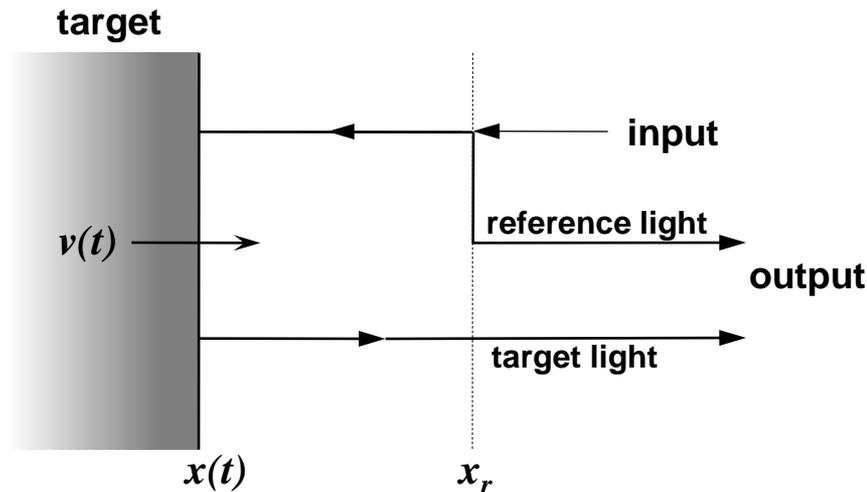
Overview

- PDV analysis
 - Displacement interferometry
 - Fourier Transform
- Accuracy and precision
 - Velocity resolution
 - Time resolution
- Signal interferences
 - Numerical
 - Electrical
 - Optical

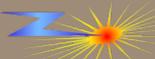


Heterodyne velocimetry

- Measured signal: $s(t) = aI_R(t) + bI_T(t) + 2\sqrt{abI_R(t)I_C(t)} \cos\left[\Phi(t_i) + 4\pi \frac{x(t) - x(t_i)}{\lambda_0}\right]$



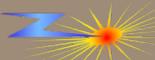
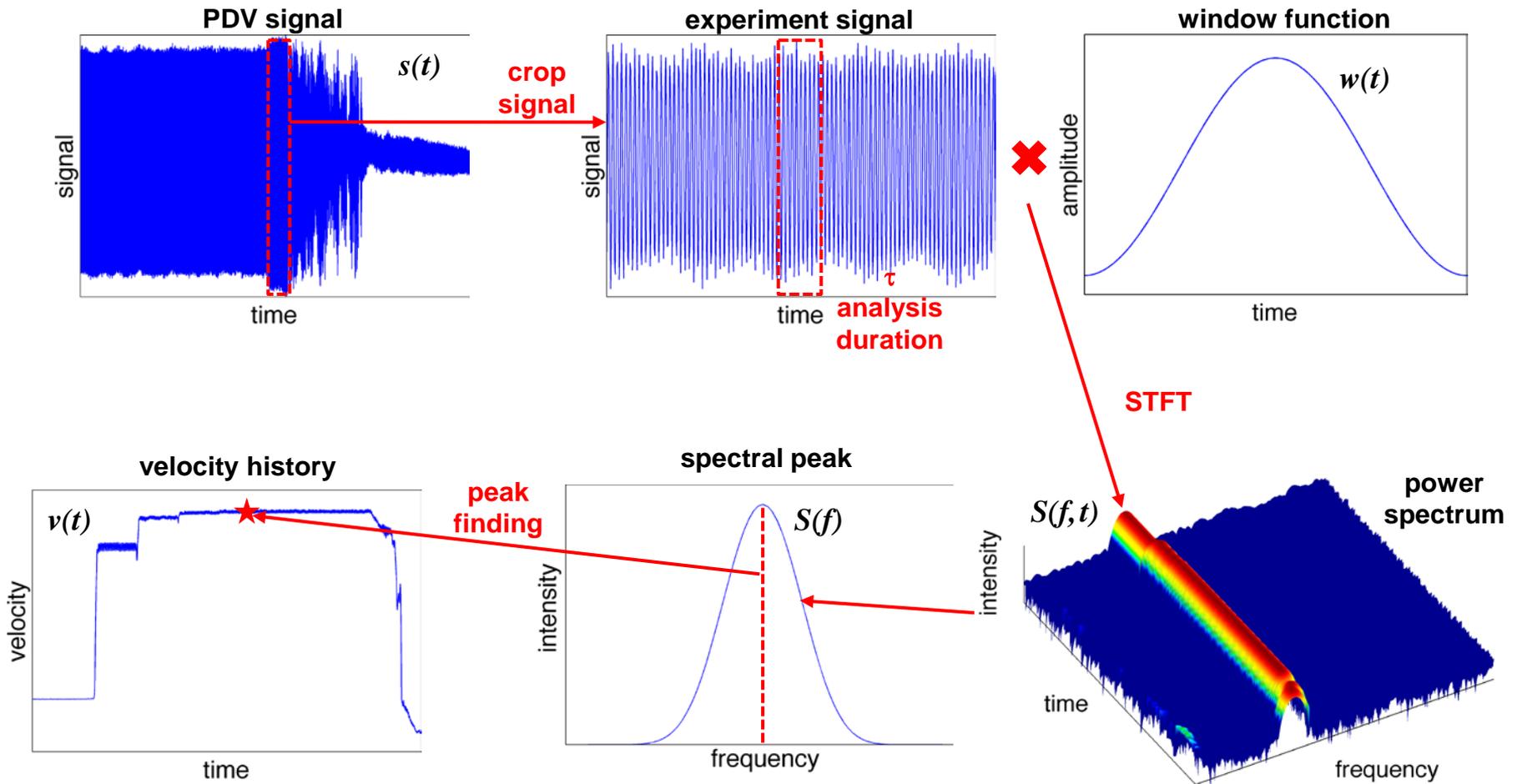
- Velocity mode analysis
 - Target velocity changes relatively slowly over analysis duration τ
 - Detector signal (PDV approximation): $s(t) = A \cos(\bar{\Phi} + 2\pi \bar{f} [t - \bar{t}])$
 - Velocity extracted from spectral peak of beat frequency: $\bar{v} = \frac{\lambda_0}{2} \bar{f}$



Time-frequency analysis

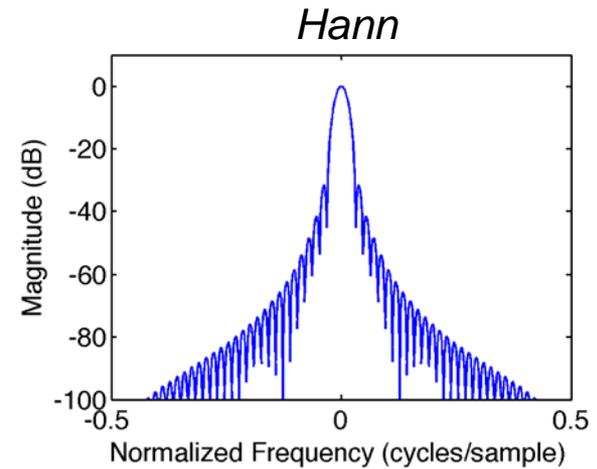
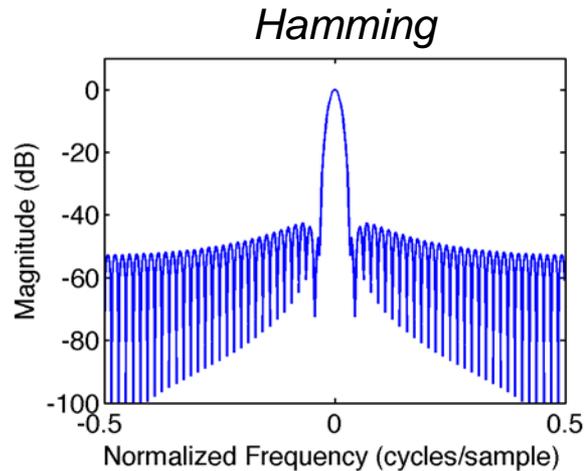
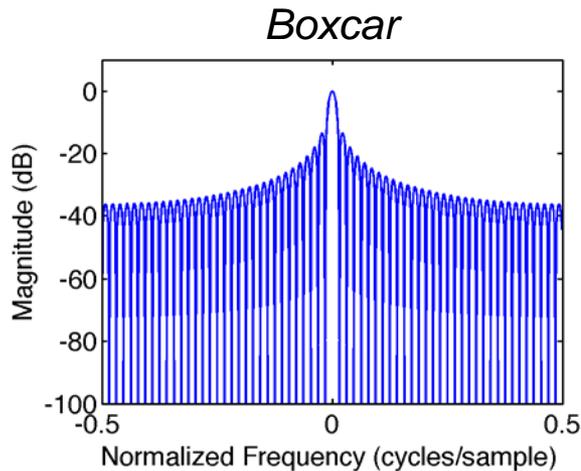
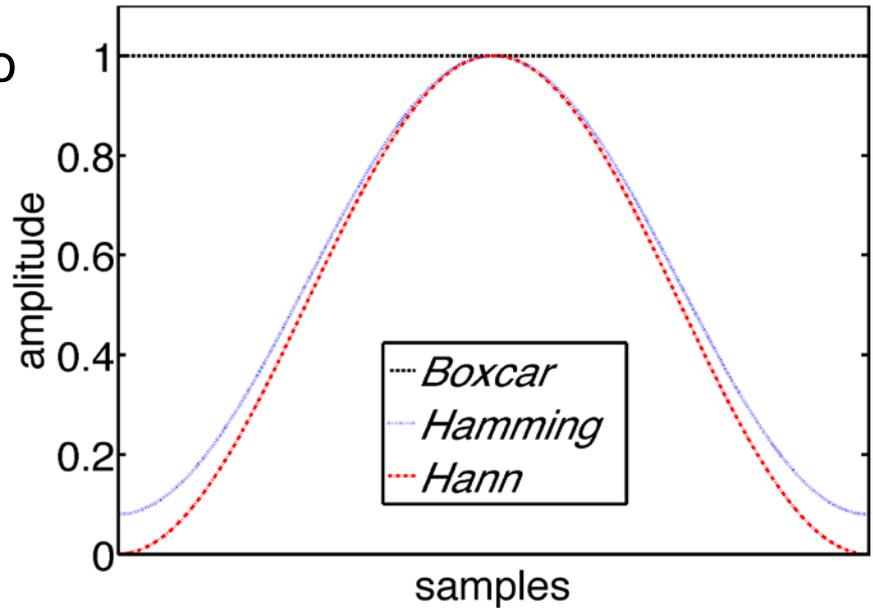
- Short-time Fourier transform (STFT) of signal to power spectrum:

$$S(f, t) = 2\pi \int_{-\infty}^{\infty} s(t)w(t)e^{-i2\pi ft} dt$$

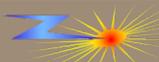


Window weighting functions

- Data multiplied by window function to reduce spectral leakage associated with finite observation intervals
- Spectral response goals
 - Narrow main-lobe
 - Low first-side-lobe
 - Rapid fall-off of side-lobes



F. Harris, Proceedings of the IEEE **66**, 51 (1978)



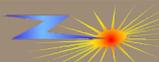
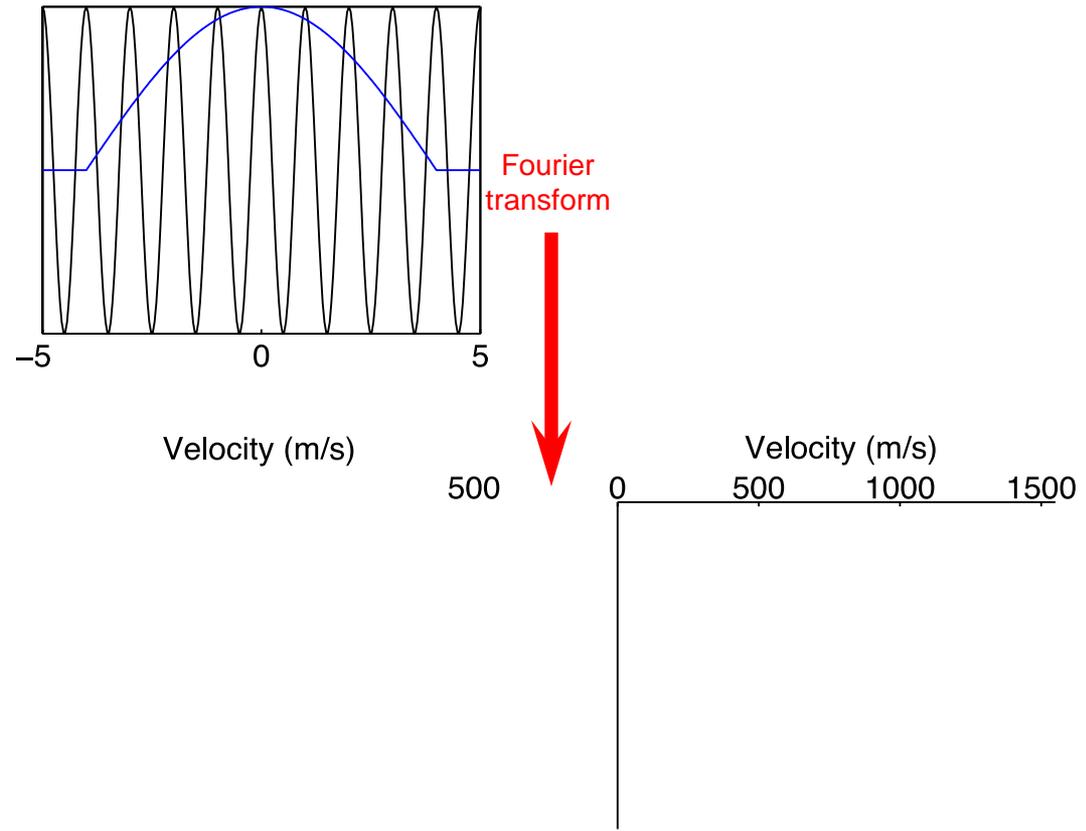
Uncertainty principle

- Time-velocity uncertainty product greater than a constant

$$(\delta f)(\delta t) \geq \frac{1}{4\pi} \quad \longrightarrow \quad (\delta v)(\tau) \geq \frac{\lambda_0}{8\pi}$$

- Velocity resolution $\delta v \sim 1$ m/s requires analysis duration $\tau > 62$ ns !
(@ 1550 nm)

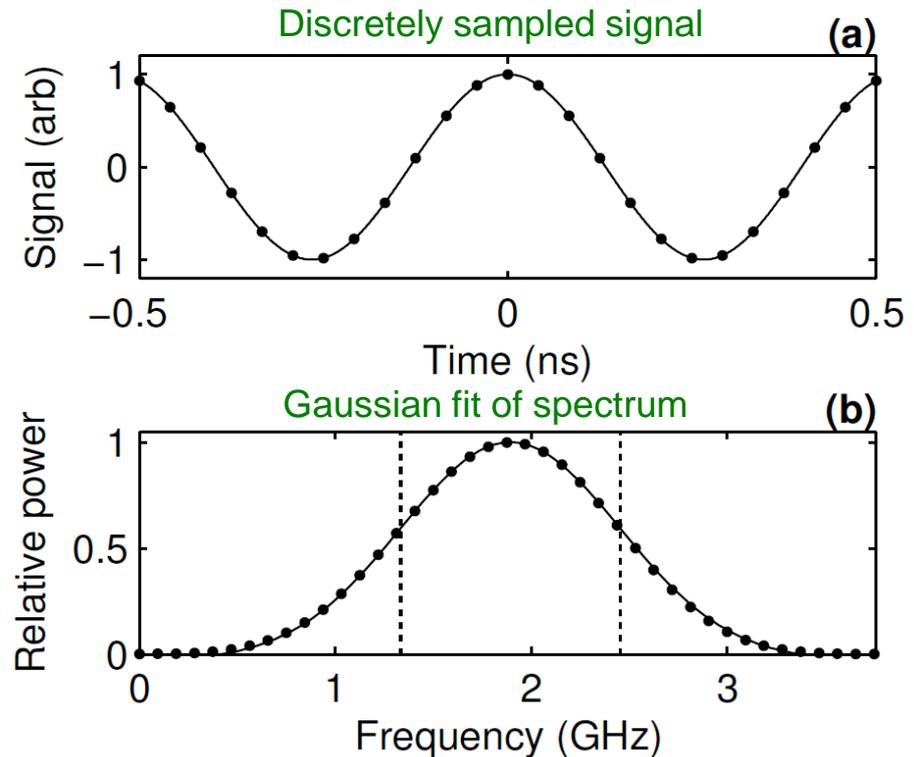
- Really a separability criterion, not location uncertainty
- No reference to noise or sampling rate



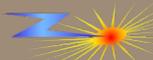
Peak fitting of PDV power spectrum

Example

- Sinusoid with frequency 1.880 GHz sampled 25 times over 1 ns time interval
- PDV measurements on typical digitizers
 - 25 Gsamples/s, 6 GHz bandwidth
- Gaussian fit
 - Peak location: 1.894 GHz
 - Peak width: 1 GHz
 - Fit uncertainty: 0.003 GHz (2 m/s)
 - Error: 0.014 GHz
- Peak location can be determined more narrowly than uncertainty bound
 - However, the results may be systematically wrong!
 - Error depends on many factors



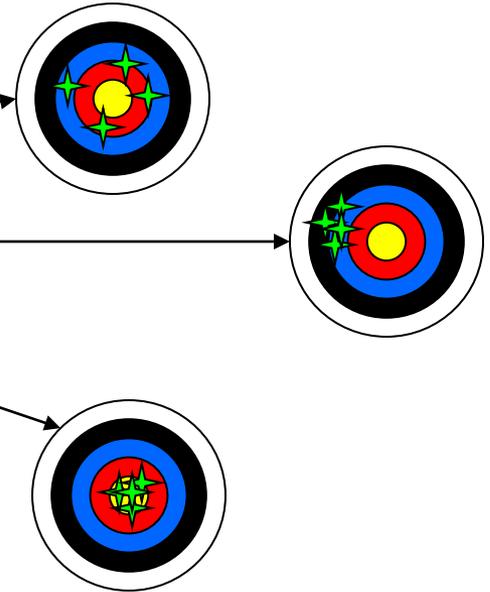
D.H. Dolan, Rev. Sci. Instrum. 81, 053905 (2010)



Accuracy and precision of PDV

- Quantitatively, how good can PDV be?

- Accuracy: being right (on average)
- Precision: variability about the average
- Small numbers mean “high performance”



- Things that are neglected

- Window corrections (0.2-1%)
- Probe effects/cosine corrections (<0.1%-?)
- Absolute wavelength errors (<10-100 ppm)
- Digitizer clock errors (<10 ppm)

Looking for an equivalent to the “1-2% of a fringe rule (VISAR)”

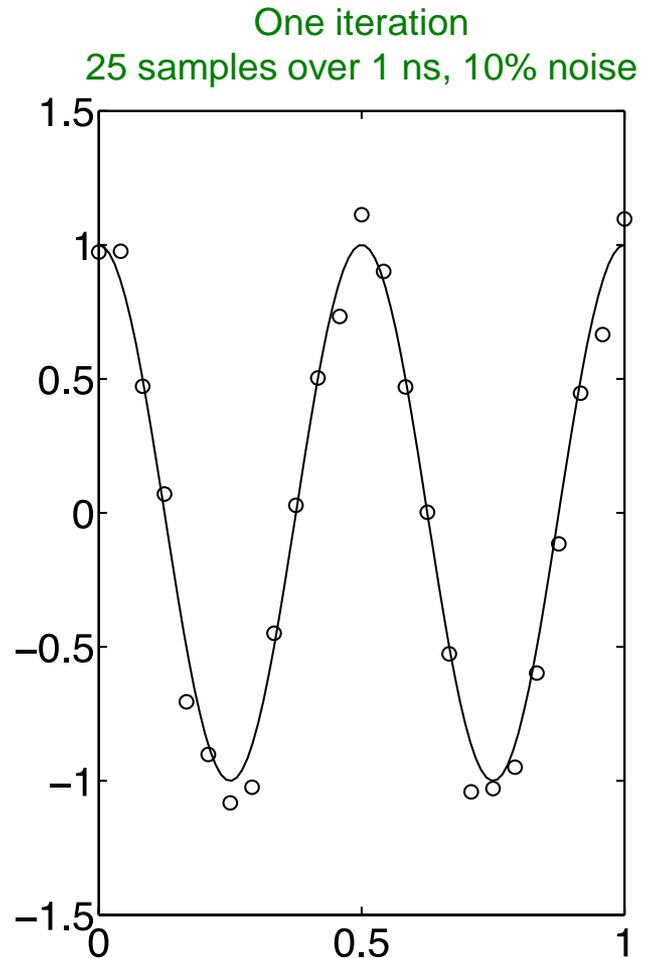


Monte Carlo simulations

- Generate discretely sampled signal

$$s_n = \cos(2\pi f_0 T n + \delta) + \sigma R_n, \quad (n = -M \dots M)$$

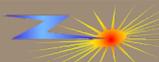
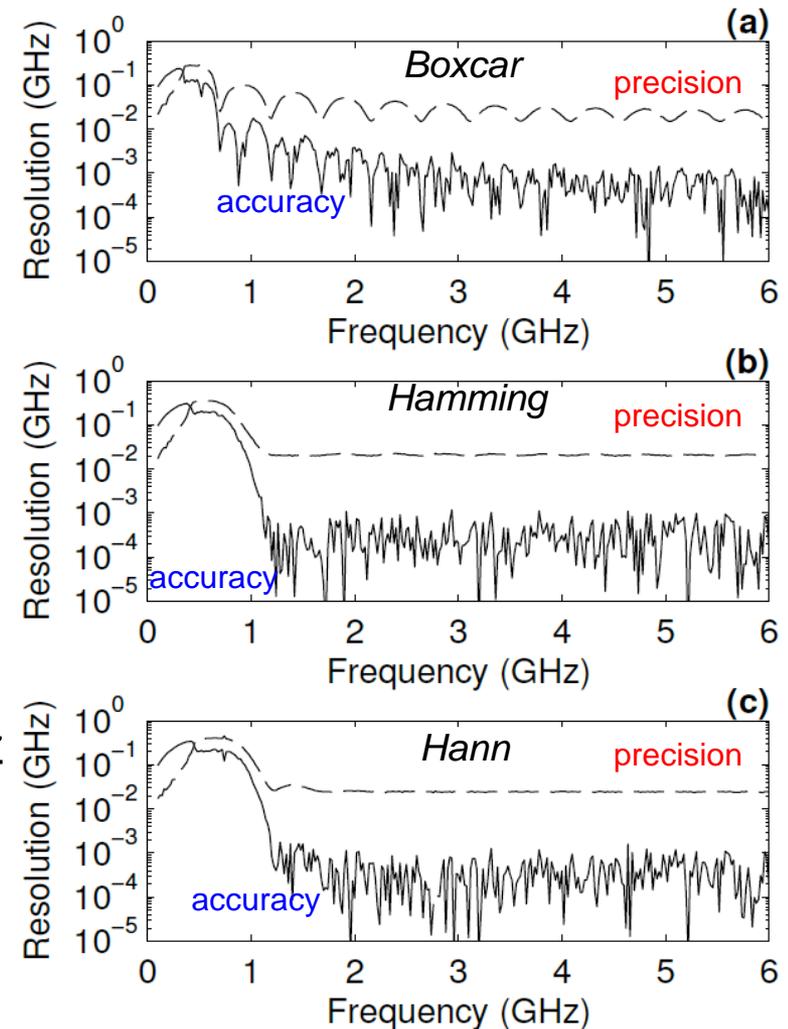
- Specified input frequency f_0
 - $k = -M \dots M$ ($2M+1$ points)
 - Sampling interval T
 - Noise fraction σ
 - Random phase δ
 - Random noise array R
- Extract frequency from signal
 - FFT analysis, etc.
 - Compare the result to input frequency
 - Repeat MANY times with different phases and noise arrays



Velocity resolution: accuracy and precision

- Inputs
 - 1 ns duration, 25 GS/s
 - 10% noise fraction
 - 0.1-6 GHz (0.02 GHz steps)
 - ~ 590 million iterations total
- Analyze many random signals
 - Ensemble of frequency results from a set of inputs
 - **Accuracy**: difference between ensemble mean and input (solid lines)
 - **Precision**: ensemble standard deviation (dashed lines)
- Poor accuracy and precision below 1 GHz (partial fringe)
- Measurements are precision-limited above 1 GHz

Monte Carlo simulations: 10 % noise

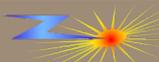
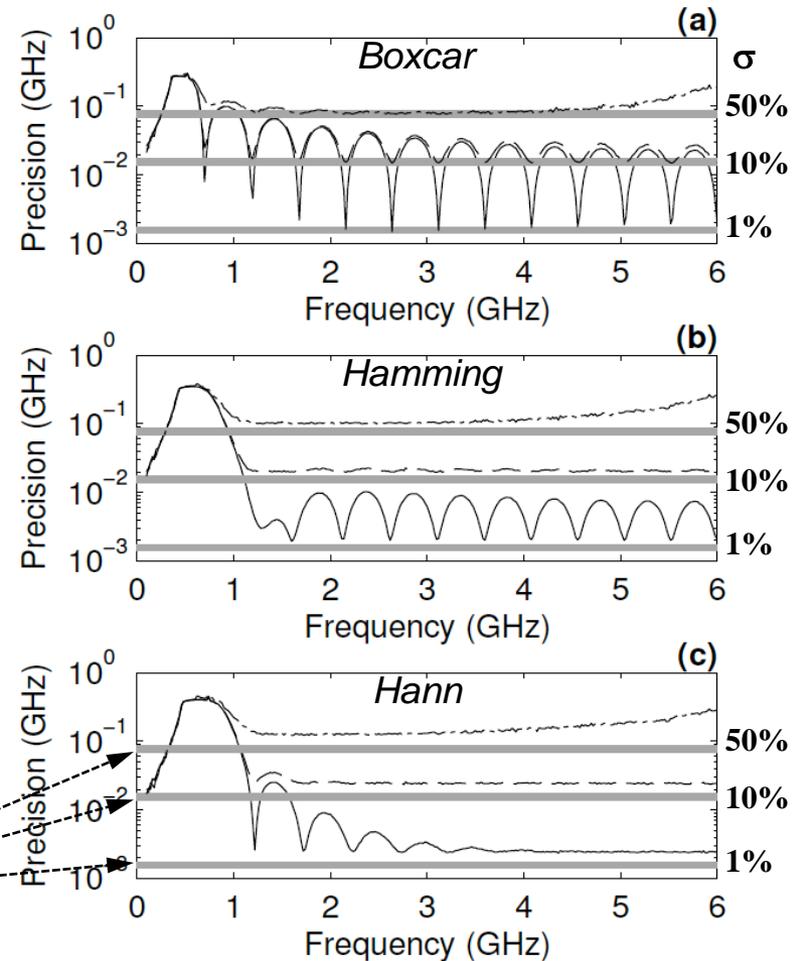


Velocity precision versus noise

- Simulations performed at various noise fractions
 - Solid lines: 1%
 - Dashed lines: 10%
 - Dot-dash lines: 50%
- Window performance
 - Hann window best at low noise fractions
 - Boxcar best at high noise fractions
- Precision benefits are largely constant above low frequency “shoulder”
 - Precision limit of PDV measurement scales with signal noise fraction σ , inversely with sampling rate f_s , and to $-3/2$ power with analysis duration τ

$$\delta f = \sqrt{\frac{6}{N} \frac{\sigma}{\pi \tau}} = \left(\sqrt{\frac{6}{f_s} \frac{\sigma}{\pi}} \right) \tau^{-3/2}$$

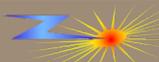
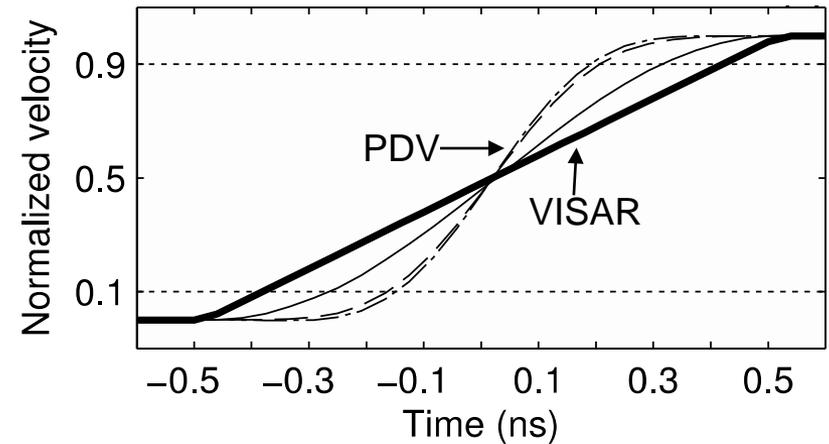
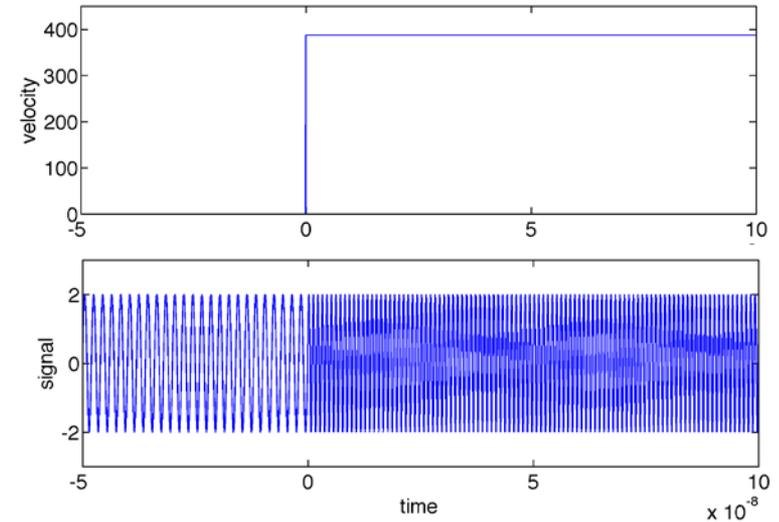
Monte Carlo simulations: 1, 10, 50% noise



Time resolution of PDV and VISAR

- Velocity step
 - Convert velocity profile to PDV and VISAR signals
 - Analyze PDV and VISAR signals to get velocity profile
 - Measure 10-90% risetime
- VISAR signal
 - Time delay of 1 ns (heavy solid line)
 - Linear risetime of 0.8 ns
- PDV signal
 - Analysis duration $\tau = 1$ ns

Digital window	Risetime (ns)
Boxcar (solid)	0.58
Hamming (dash)	0.37
Hann (dot-dash)	0.34



Comparison of PDV versus VISAR

- Resolution ratio ($\rho = \text{PDV}/\text{VISAR}$)

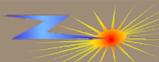
- PDV noise fraction $\sigma \sim 10\%$
- VISAR fringe resolution $\varepsilon \sim 2\%$
- Common delay/analysis time

$$\rho = \frac{1550}{532} \sqrt{\frac{6}{f_s \tau}} \frac{\sigma}{\pi \varepsilon}$$

- For 1 ns analysis, ratio is 2.3: VISAR better (maybe)
- For 10 ns analysis, ratio is 0.72: PDV better
- PDV time scale is adjustable, VISAR fixed by hardware

- Mitigating factors

- PDV rise faster than VISAR for common delay/analysis time (improvements depend on digital window)
- PDV uses one signal, VISAR uses 4-8 signals (2-2.8x improvement)
- VISAR requires good characterization (5-10% fringe resolution not unusual)

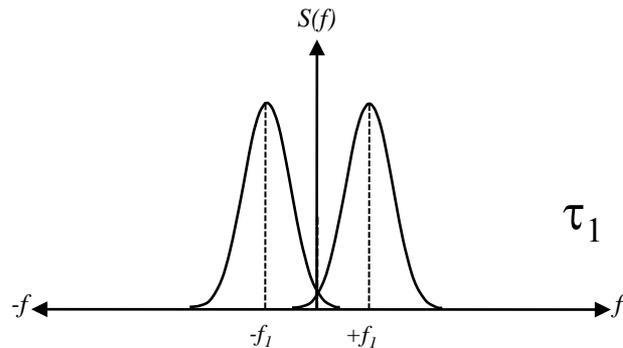


Numerical signal interference

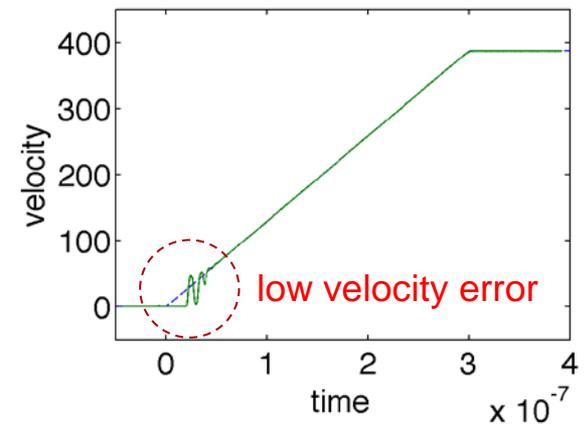
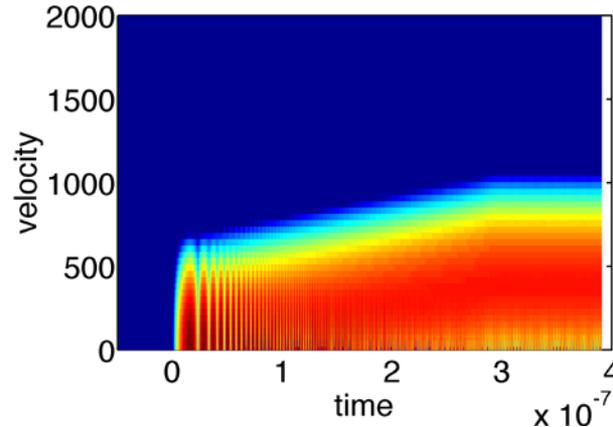
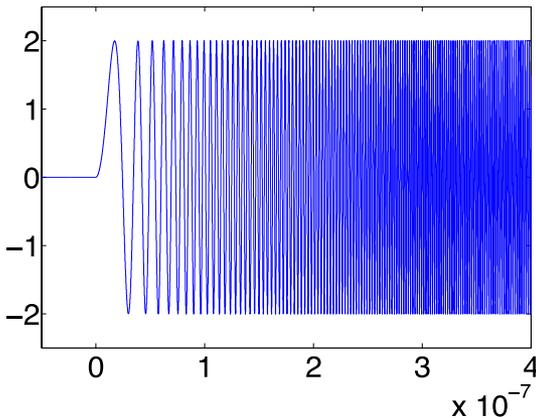
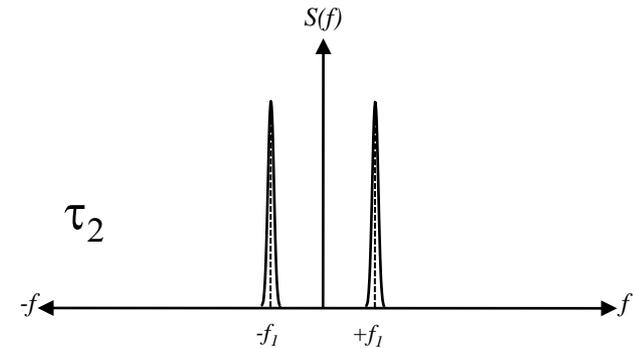
- PDV signal consists of negative and positive frequency components:
- Conventional PDV
 - Interference from negative frequency components

$$s(t) \approx \cos(2\pi f_k t) = \frac{e^{i2\pi f_k t} + e^{-i2\pi f_k t}}{2}$$

- Increasing analysis duration τ narrows frequency widths

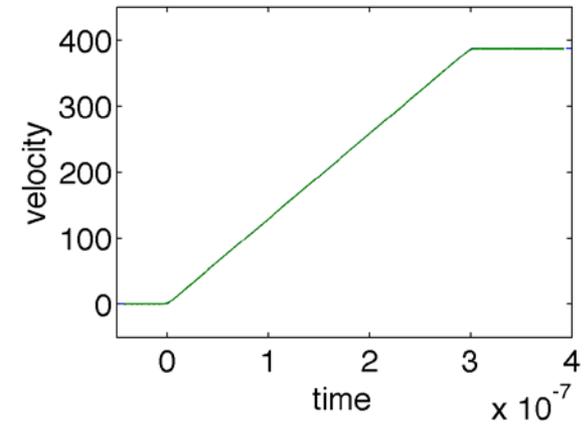
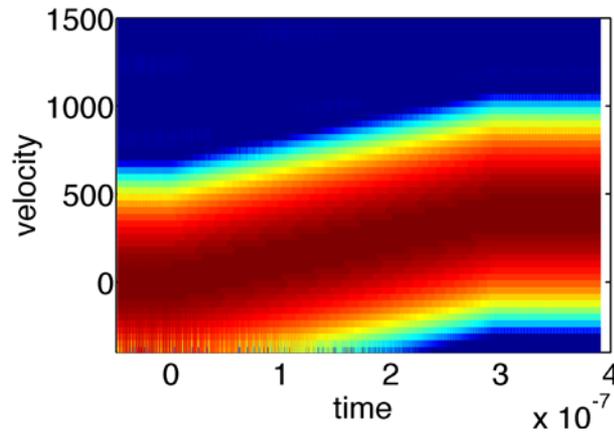
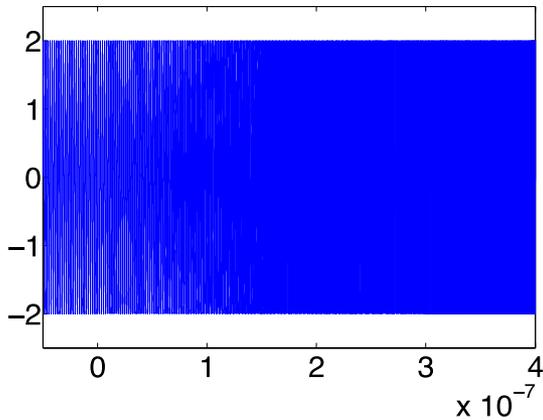
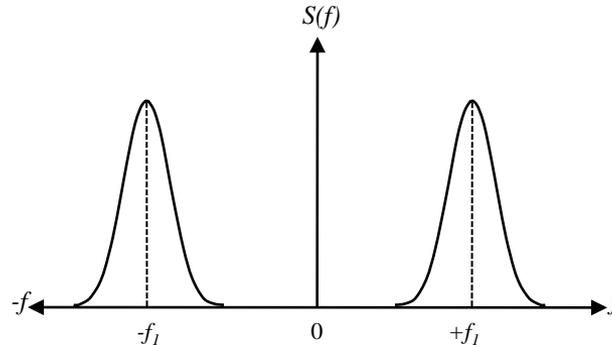


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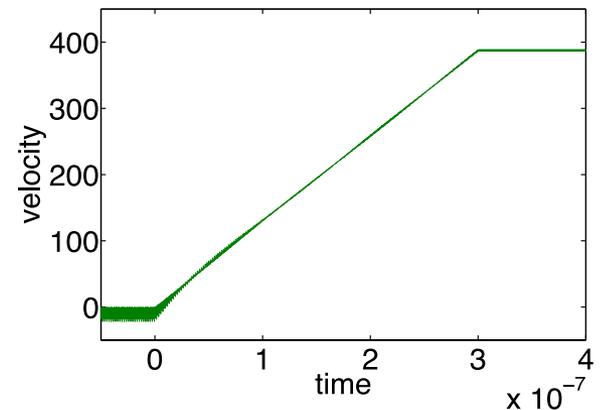
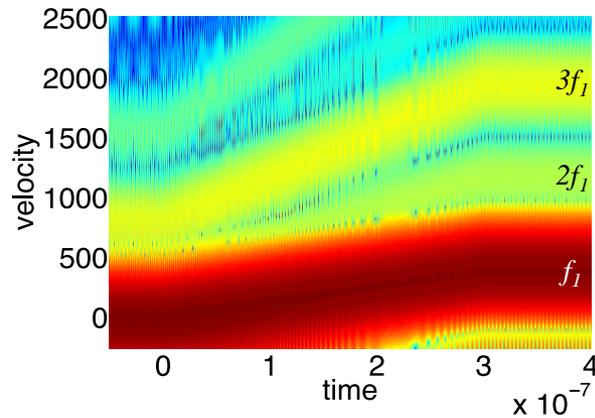
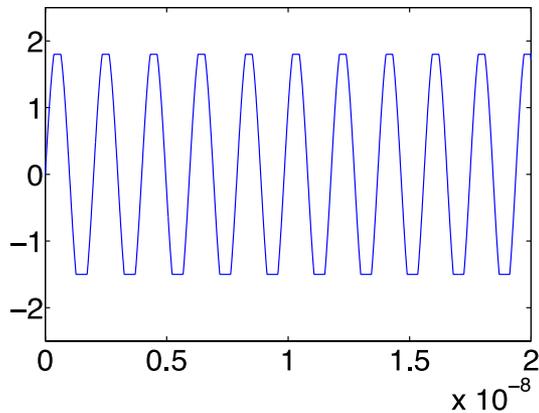
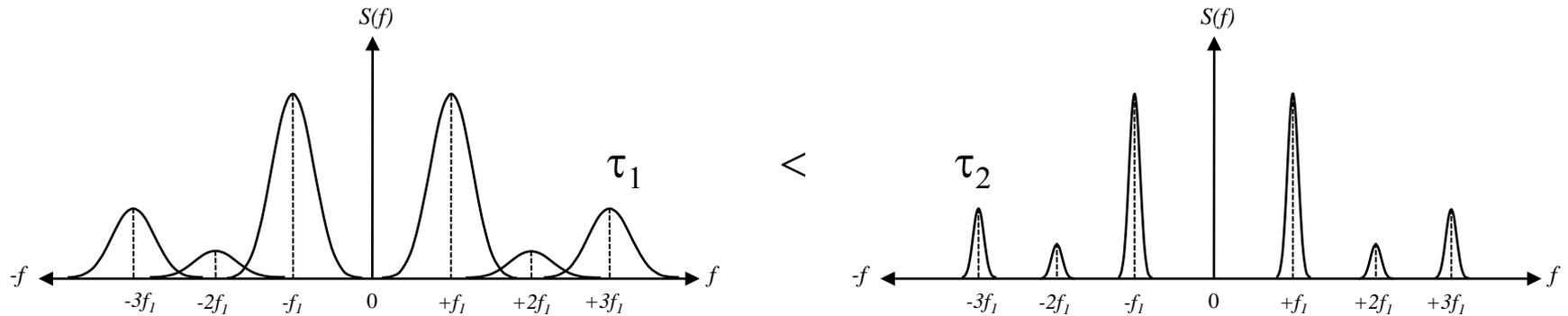
Better solution to numerical signal interference

- Frequency-conversion PDV
 - Separates positive and negative frequency components
 - Enables short analysis duration τ at low velocities



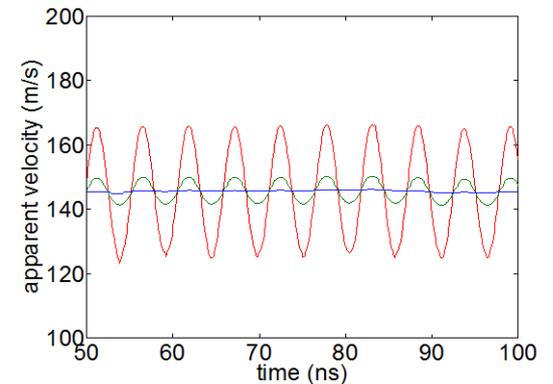
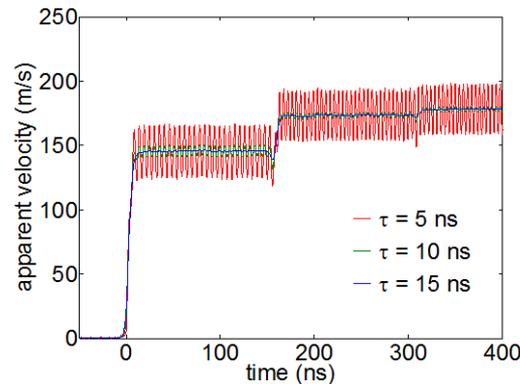
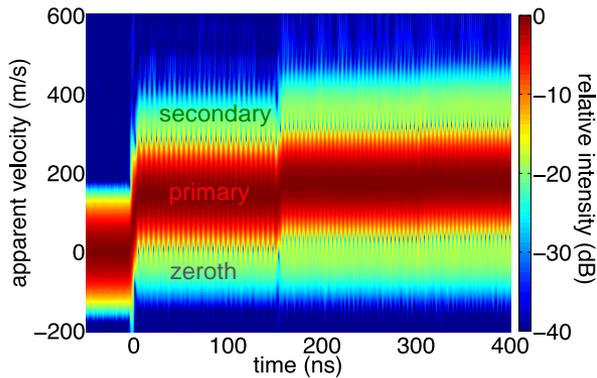
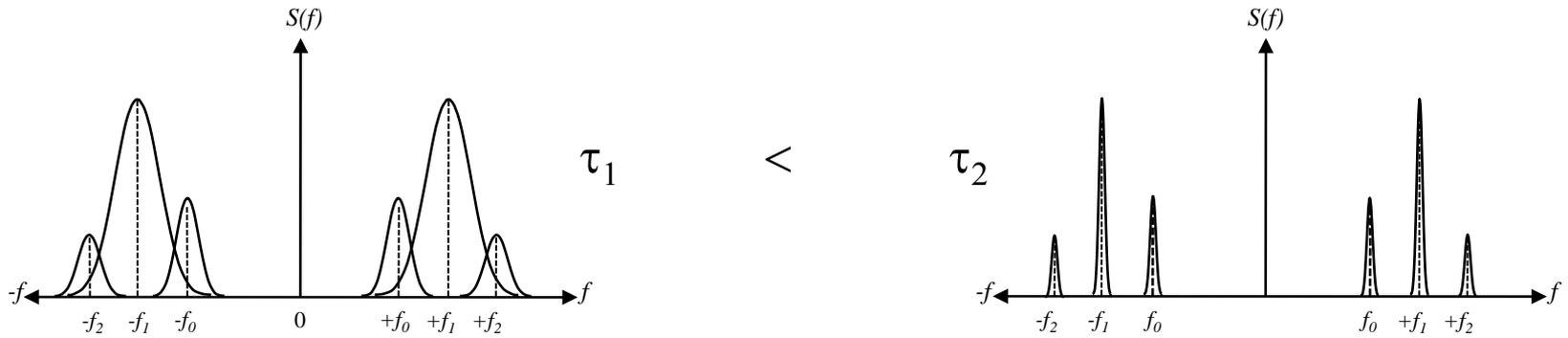
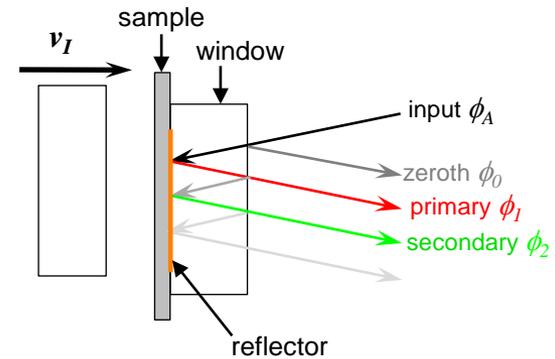
Electrical signal interference

- Clipping of signal and detector nonlinearity
 - Introduces higher harmonics



Optical signal interference

- Signal leakage, back reflections (probe, fiber connections)
- Window reflections
 - Oscillations within velocity profile due to other reflections perturbing the primary

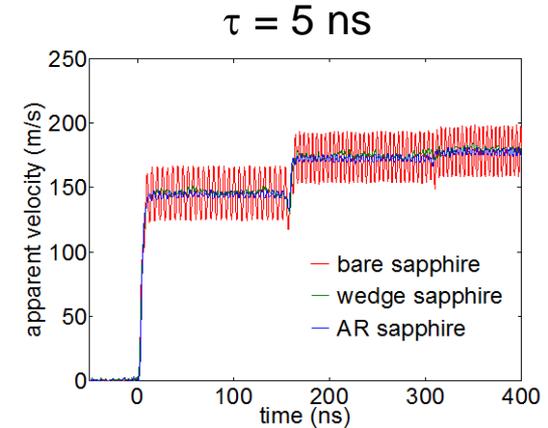
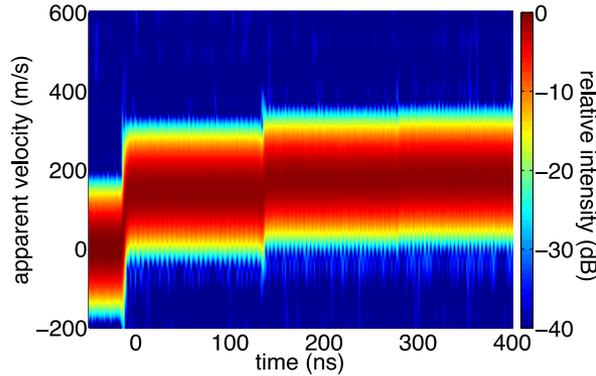
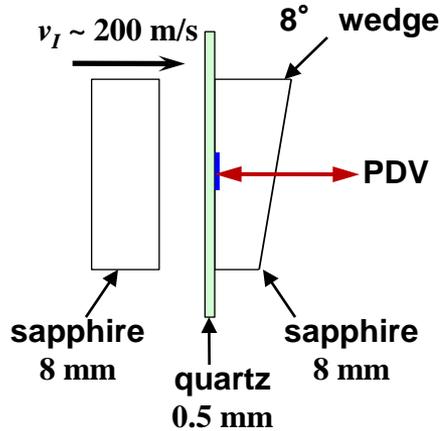


T. Ao and D.H. Dolan, Rev. Sci. Instrum. **82**, 023907 (2011)

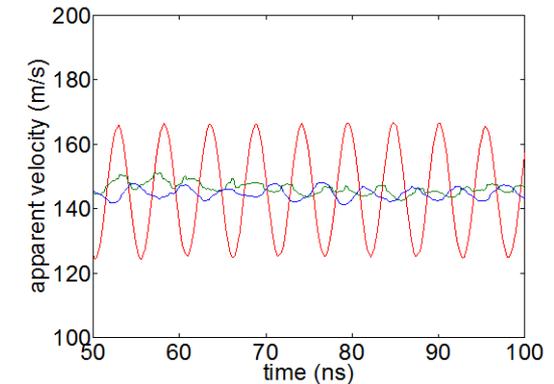
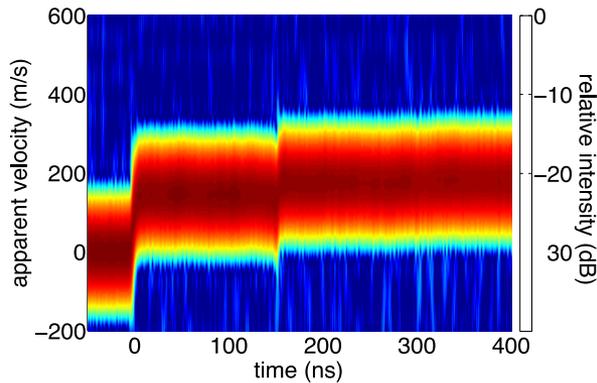
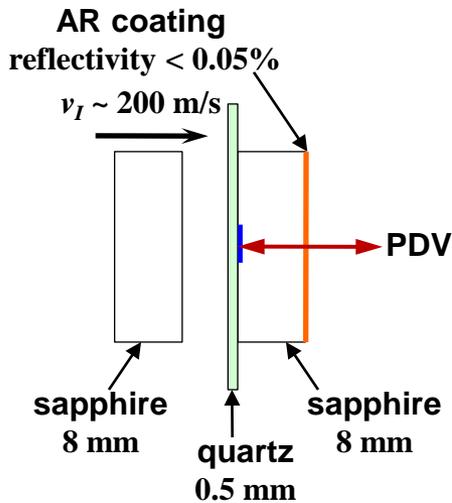


Better solution to optical interference from window reflections

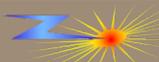
- Wedge windows deflect reflections from free surface



- Anti-reflective coating attenuates reflections from free surface



T. Ao and D.H. Dolan, Rev. Sci. Instrum. **82**, 023907 (2011)



Summary

- Monte Carlo simulations of PDV
 - Inaccurate and imprecise at low frequencies
 - Accurate and (potentially) precise otherwise
 - Limiting performance can be tied to sampling rate, noise fraction, and analysis time duration
- PDV versus VISAR
 - Competitive time and velocity precision with VISAR, despite wavelength difference
 - PDV requires fewer signals than VISAR
- Frequency-conversion is a good thing
 - Underlying beat frequency resolves low velocity features while maintaining good time resolution
 - Separates negative and positive frequency spectral functions
 - Windows should either be wedged or have sufficient anti-reflective coating to mitigate secondary reflections

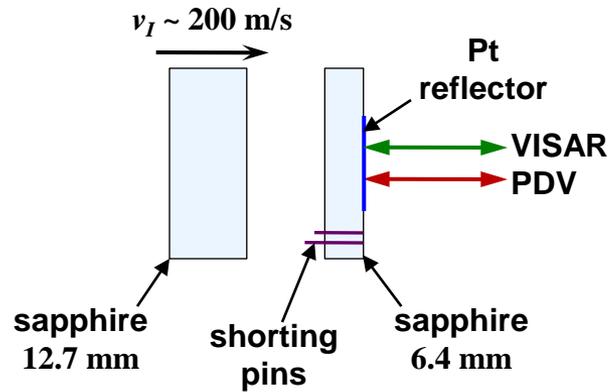


Extra slides

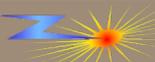
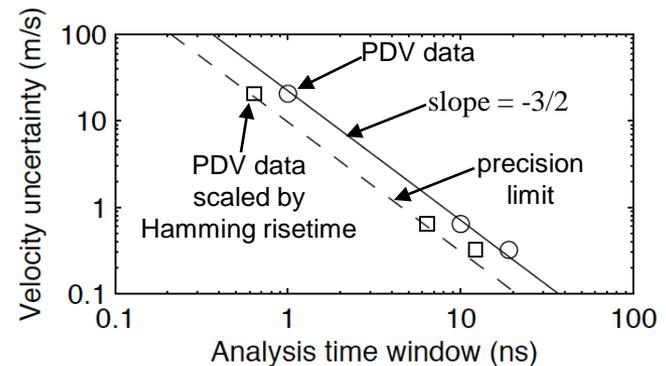
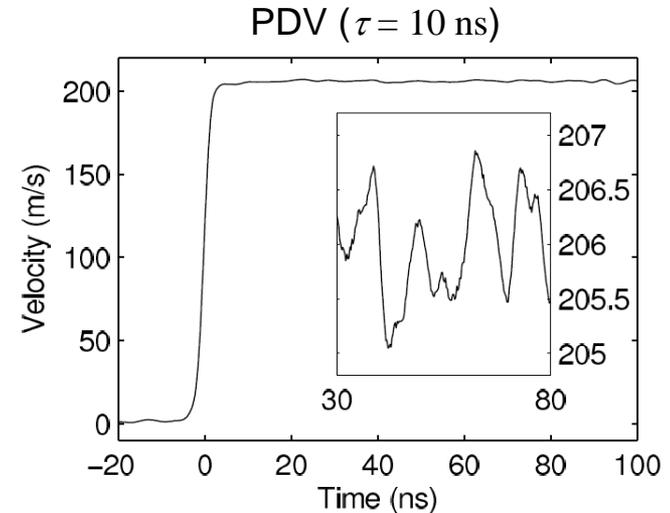


PDV validation

- Free surface velocity of shocked sapphire
 - Measured with shorting pins, air-delay VISAR, frequency-conversion PDV



Diagnostic	Time scale (ns)	Velocity change (m/s)
Pins	>1000	206.6 ± 1.1
VISAR	19	206.11 ± 0.28
PDV	19	205.95 ± 0.32
PDV	10	205.93 ± 0.64
PDV	1	206 ± 21



Limiting performance

- There is a limit to how well frequency can be determined from a discretely sample signal

$$\delta f = \sqrt{\frac{6}{N}} \frac{\sigma}{\pi\tau} = \left(\sqrt{\frac{6}{f_s}} \frac{\sigma}{\pi} \right) \tau^{-3/2}$$

- Fixed parameters:
 - Sampling rate
 - Noise fraction (digital filtering does NOT help)
- Adjustable parameters:
 - Analysis time duration
 - Improvement faster than uncertainty principle predicts
- FFT windows alters the effective duration
 - This expression assumes all data is treated equally (boxcar window)

