Presentation Describing Photonic-Doppler Velocimetry Paraxial Scalar-Diffraction Theory and Simulation

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LAWRENCE LIVERMORE NATIONAL LABORATORY

Photonic-Doppler-Velocimetry, Paraxial-Scalar Diffraction Theory and Simulation

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September 10, 2015

This talk is a synopsis of a technical report I wrote last year. The goals of that work were:

- develop an approximate method of calculating the peak frequency in a spectral sideband at an instant of time based on an optical diffraction theory for a moving target,
- compare the 'measured' velocity to the 'input' velocity to **gain insights** into how and to what precision PDV measures the component of the mass velocity along the optical axis, and
- investigate the effects of small amounts of roughness on the measured velocity,
- using a full three dimensional picture including tilted target, tilted mass velocity, and small amounts of surface roughness.

Important experimental fact:
PDV appears to measure the component of the velocity along the optical axis, seemingly independent of surface orientation.

"WHAT DOES 'VELOCITY' INTERFEROMETRY REALLY MEASURE?"

Dan H. Dolan

in Shock Compression of Condensed Matter - 2009, edited by M. L. Elert, W. T. Buttler, M. D. Furnish, W. W. Anderson, and W. G. Proud

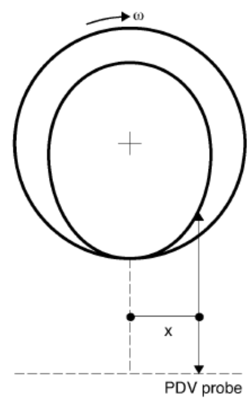


FIGURE 2. Retro-reflective PDV measurements of a rotating disk and cam.

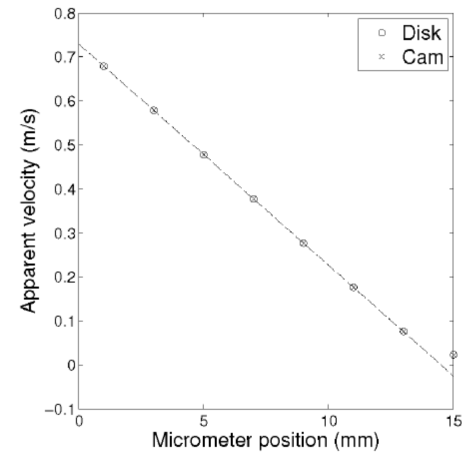


FIGURE 4. PDV measurements of a rotating disk and cam. The dashed curve is a linear fit.

"FUNDAMENTAL EXPERIMENTS IN VELOCIMETRY"

Matthew E. Briggs, Lawrence M. Hull, Michael A. Shinas

in Shock Compression of Condensed Matter - 2009, edited by M. L. Elert, W. T. Buttler, M. D. Furnish, W. W. Anderson, and W. G. Proud

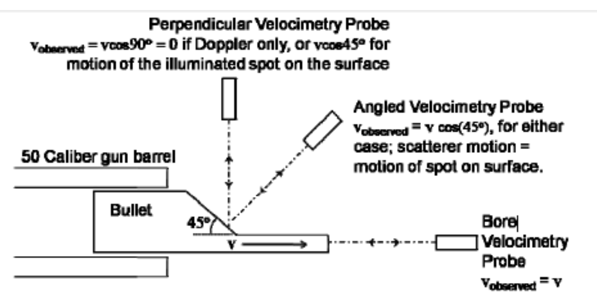
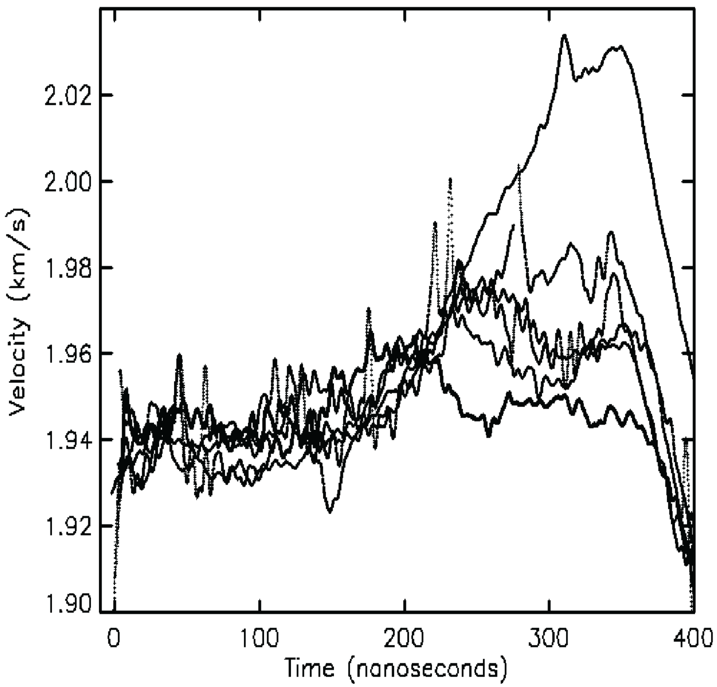
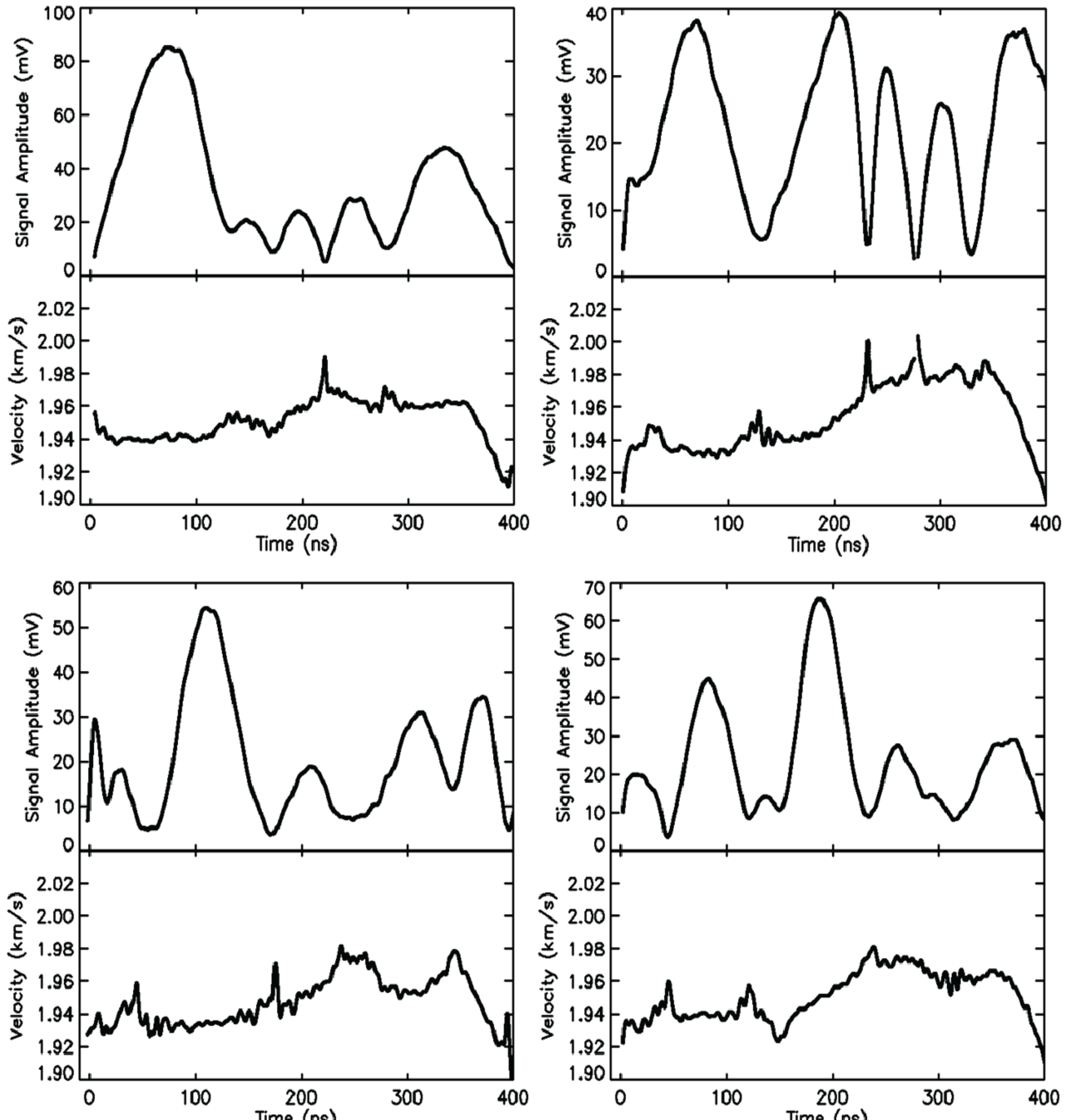


Figure 4. The three PDV probes shown track the bullet across the flat, up the ramp, and across the top at various angles as the bullet moves to the right at v = 532 m/s.

Common experimental observation:
At dips in the signal amplitude due to speckle, there are deviations in velocity of order of 1 % that are not “random” errors.

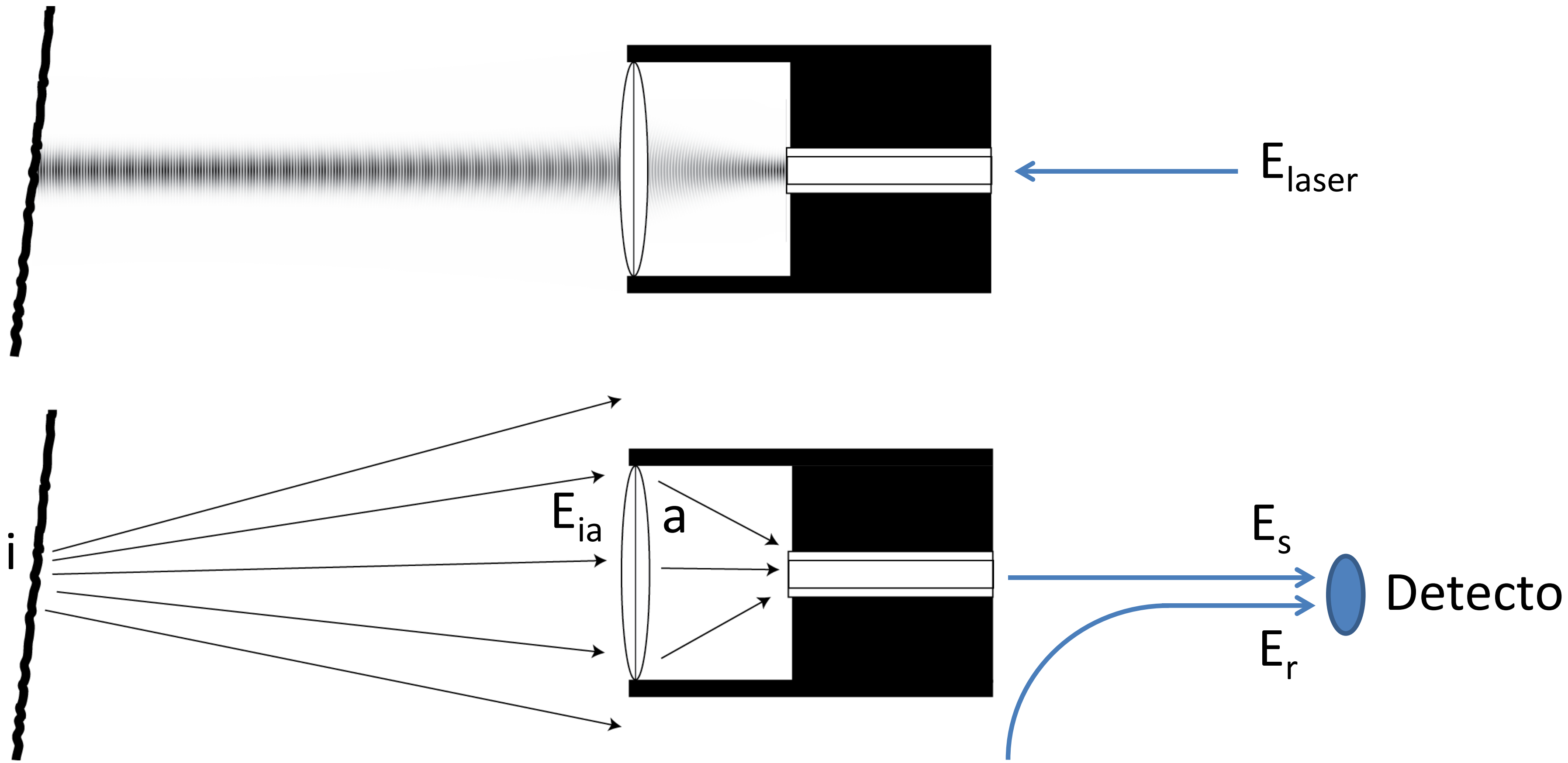


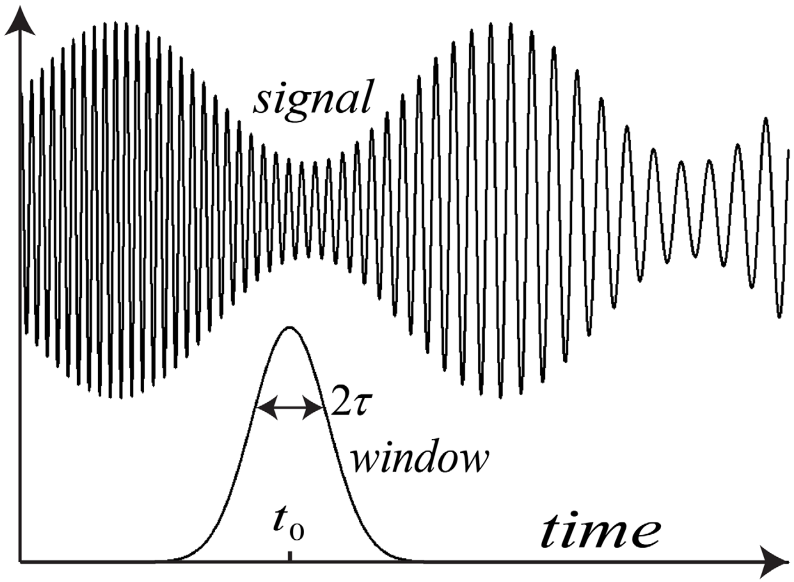


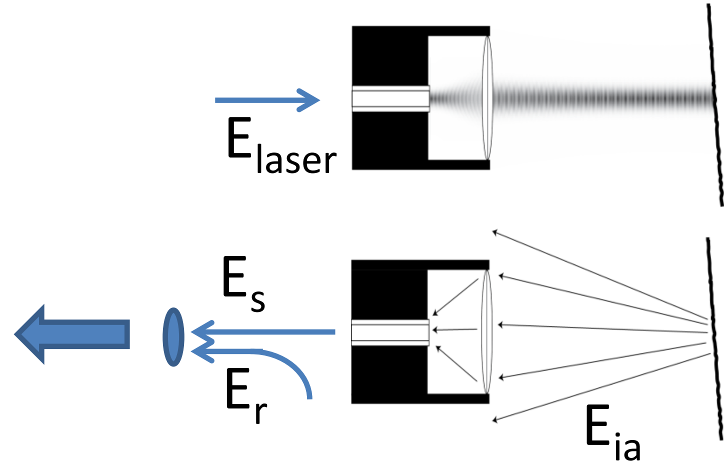
Essential elements of a theory needed to reproduce experimental facts

- PDV interference uses phase memory of events “in the past.”
- The optical probe can be treated using diffraction theory for a non-harmonic field (more convenient to work in the time domain).
- Light-target interactions needs a scattering theory recognizing:
  - fields propagate as waves
  - matter-field interactions occur at (scatter from) points
- Surface-roughness is not necessary to reproduce the idea that the measured velocity is nearly the component along the optic axis. This conclusion depends on the paraxial nature of most PDV experiments.
- Due to the three dimensional nature of “real” experiments, the scattered light field consists of a narrow spectrum of scattered fields.
- The theory allows adding surface roughness to the picture, which disrupts a delicate balancing act in a narrow spectrum of scattered fields.

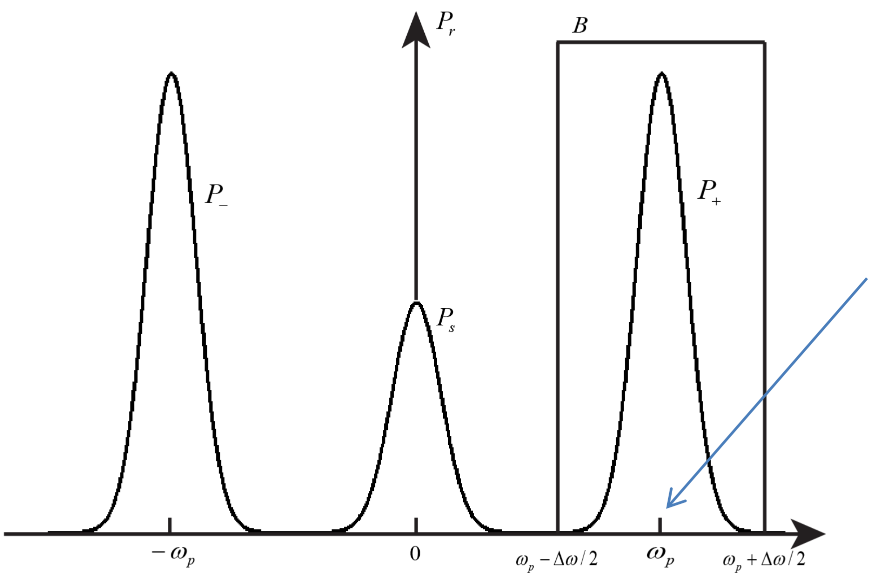
Basic elements of a PDV experiment





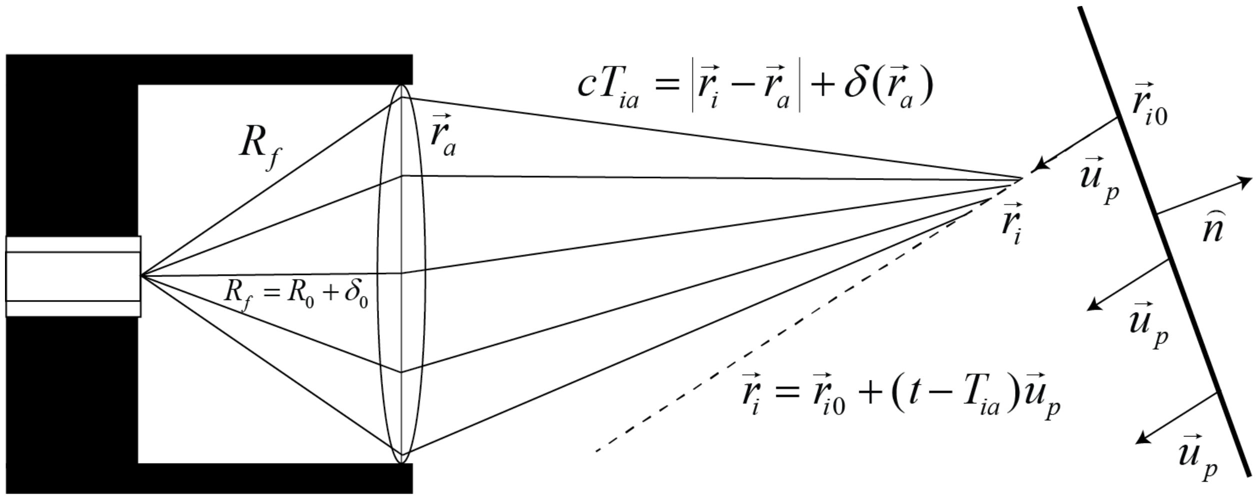


Frequency-time transform



"The" signal frequency is the rate of change of the *phase difference* between a reference and total of scattered light fields at the detector

$$\omega_p = -\frac{\partial}{\partial t}\Phi$$



$$
\begin{aligned}
\omega_p &= -\frac{\partial}{\partial t}\Phi \\
&= \mathrm{Re}\left[\frac{1}{(E_r^{\,*} E_s)}\; i\, \frac{\partial}{\partial t}\; (E_r^{\,*} E_s)\Big|_{t=t_0} \right] \\
&= \sum_{lens} \sum_{surface} \omega_{ia}\, \mathrm{Re}\left[\frac{E_{ia}}{\sum_{lens}\sum_{surface} E_{ia}} \right]
\end{aligned}
$$

The signal frequency is a weighted average of a narrow spectrum of frequencies

$$\omega_{ia}$$

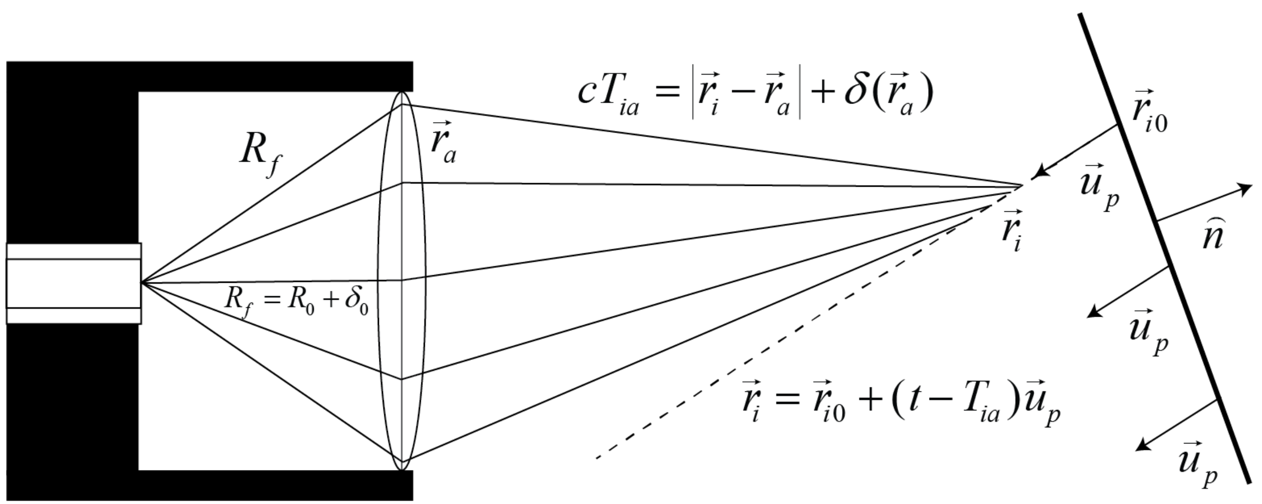
associated with individual pairs of scattering events (i) and lens aperture points (a) weighted by the laser field with appropriate delays (T_{ia}).

For laser wave fronts that are "not too curved," the dominant terms in the spectrum of scattered frequencies are:

$$\omega_{ia} \approx \frac{\partial}{\partial t}\left(\vec{k}\cdot\vec{r}_i + \omega_r T_{ia}\right)$$

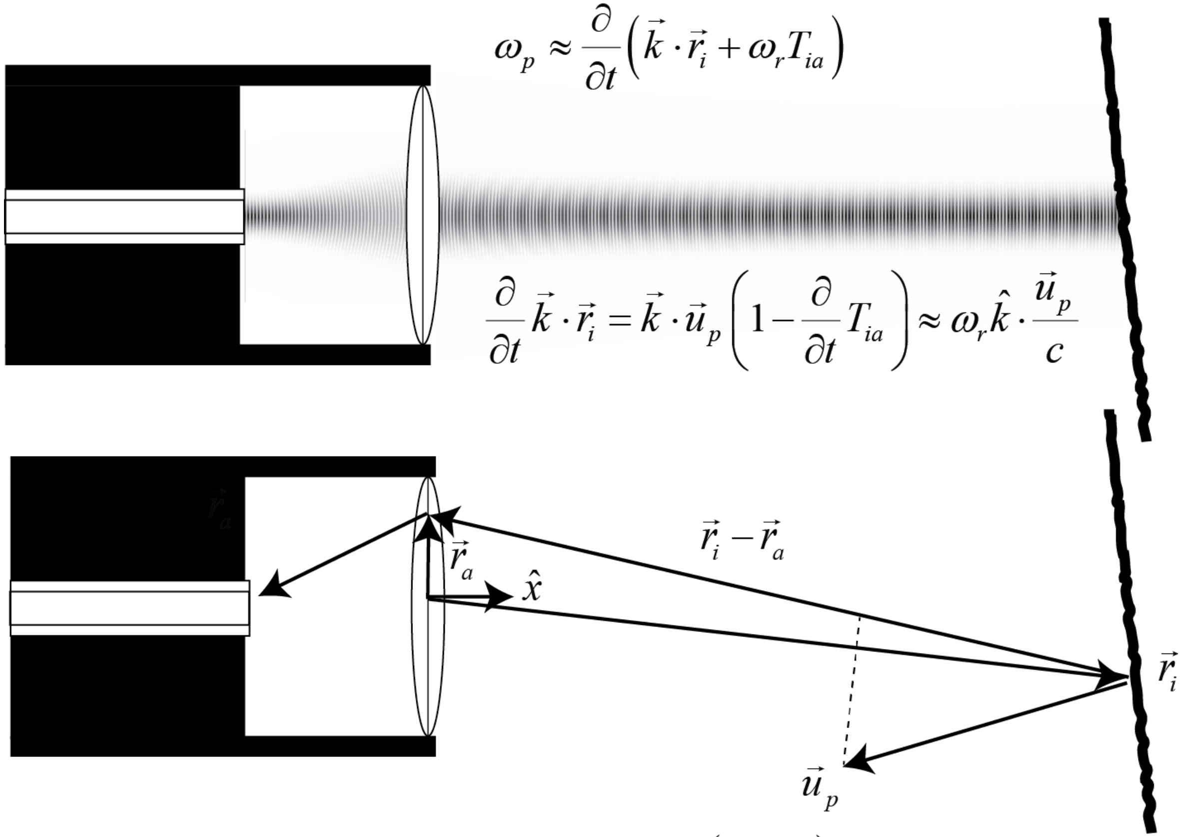
"Plane wave" like space terms in the laser at time dependent scattering positions

Time dependent delay from scattering events to lens aperture points



The theory is consistent with a measured velocity that is nearly the component along the optical axis:

$$\omega_p \approx \frac{\partial}{\partial t}\left(\vec{k}\cdot\vec{r}_i + \omega_r T_{ia}\right)$$



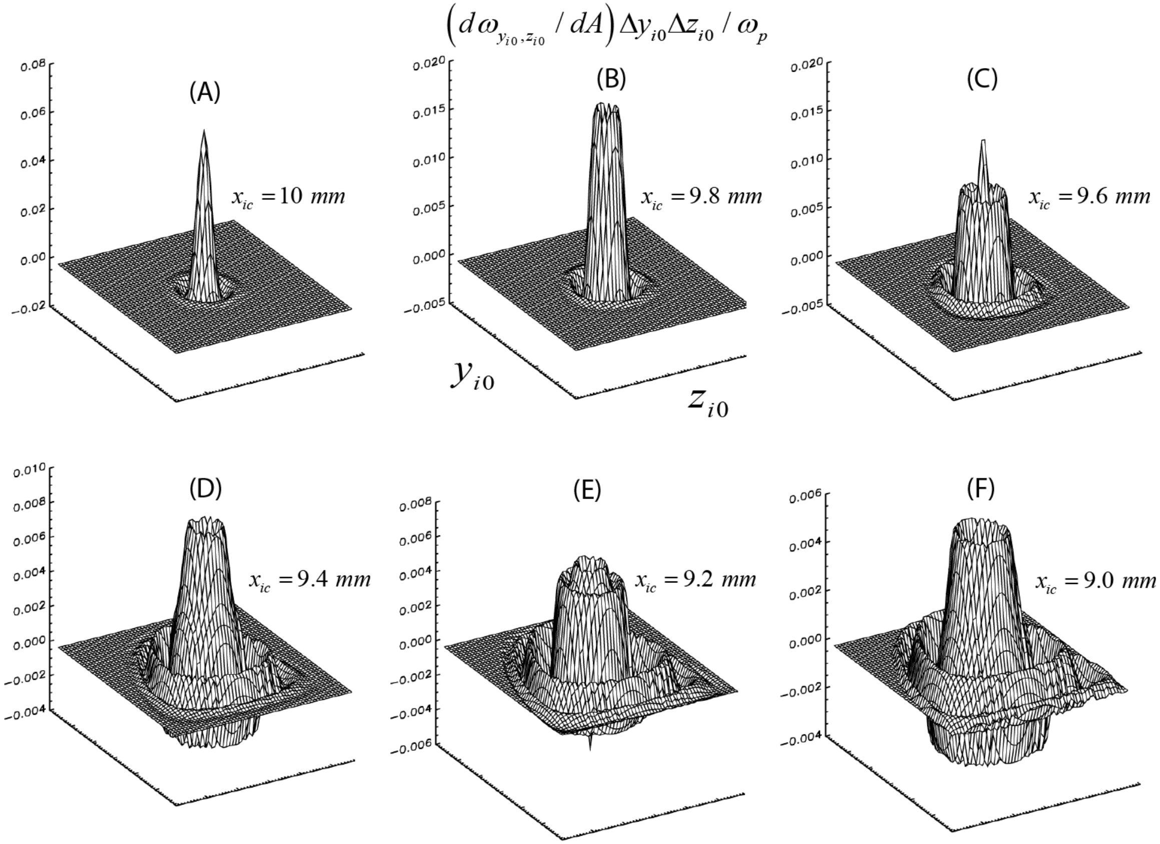
$$\frac{\partial}{\partial t}\vec{k}\cdot\vec{r}_i = \vec{k}\cdot\vec{u}_p\left(1 - \frac{\partial}{\partial t}T_{ia}\right) \approx \omega_r \hat{k}\cdot\frac{\vec{u}_p}{c}$$

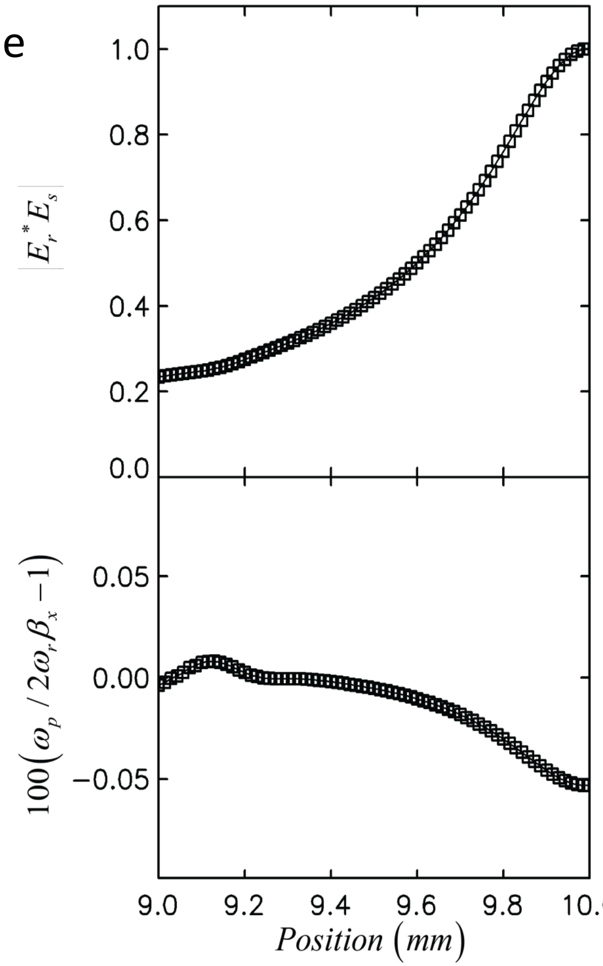
$$\omega_r \frac{\partial T_{ia}}{\partial t} = \omega_r \frac{\vec{u}_p}{c}\cdot\frac{\left(\vec{r}_i - \vec{r}_a\right)}{\left|\vec{r}_i - \vec{r}_a\right|} \approx \omega_r \frac{u_{px}}{c}$$

To the extent that we can ignore wave front curvature and that the scattered light paths are paraxial (parallel to the optical axis), the rate of change of space- and time-dependent terms in the phase difference are each nearly $\omega_r u_{px} / c$

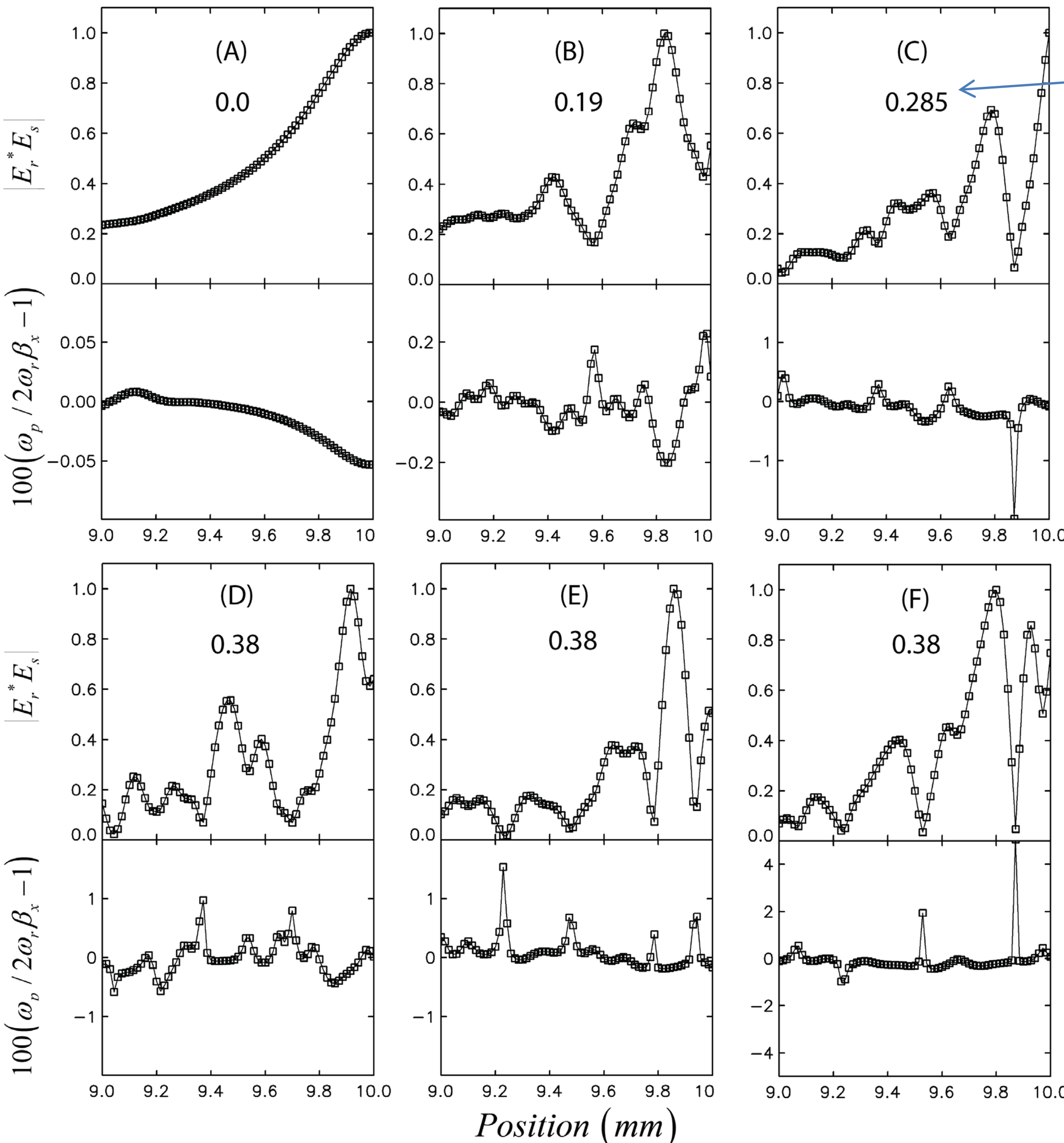
$$\sum_{lens} \omega_{ia} \, \mathrm{Re}\left[\frac{E_{ia}}{\sum_{lens} \sum_{surface} E_{ia}} \right]$$

After summing over the lens, field-weighted frequencies at different particle positions oscillate:





For a perfectly smooth surface, the sums add up to the component along the optical axis to within 0.05 %!



Average roughness in micrometers

When we add surface roughness, the theory reproduces the expected speckle modulation in the amplitude. The theory also predicts frequency deviations, which we infer are from destructive interference over the lens aperture leading to "unusual" averages of the field-weighted spectrum of scattered frequencies having a width of about 1 %.