FROM RESEARCH TO INDUSTRY

PDV: NOISE ANALYSIS USING SHORT-TIME FOURIER TRANSFORM (STFT)

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PDV User Workshop

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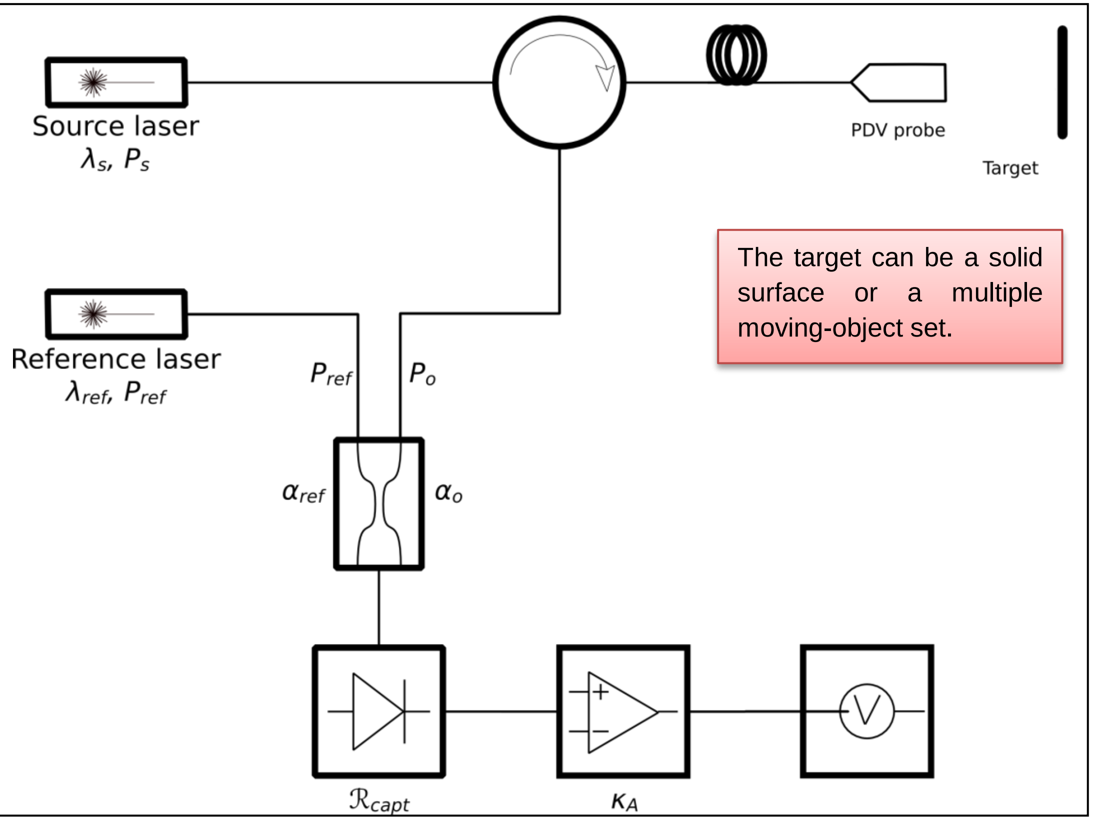
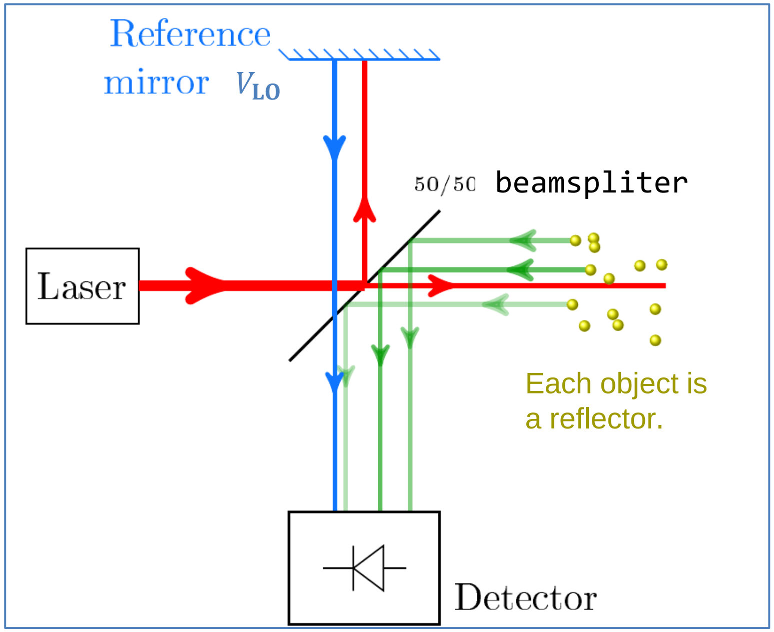
INTRODUCTION

WE CONSIDER THE FOLLOWING PDV DESIGN:

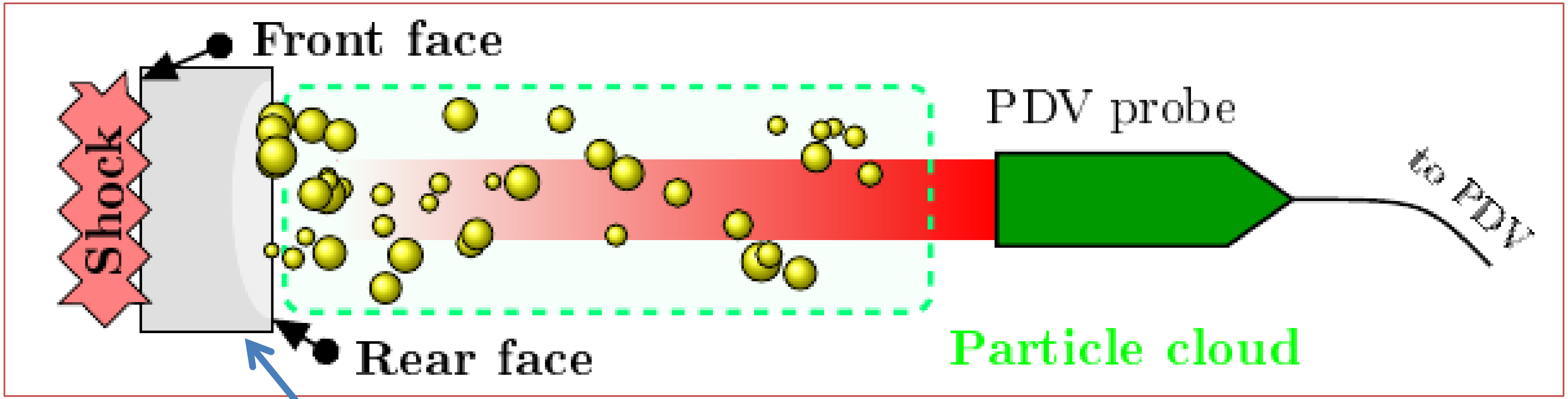


- Working @ 1.55 µm,
- Two laser configuration: source and local oscillator.

The design is **equivalent to a Michelson interferometer.**
(It is equivalent to have a reference mirror moving with a constant velocity $V_{\mathbf{LO}}$ (equal to $\lambda_s/2\,\Delta f$))

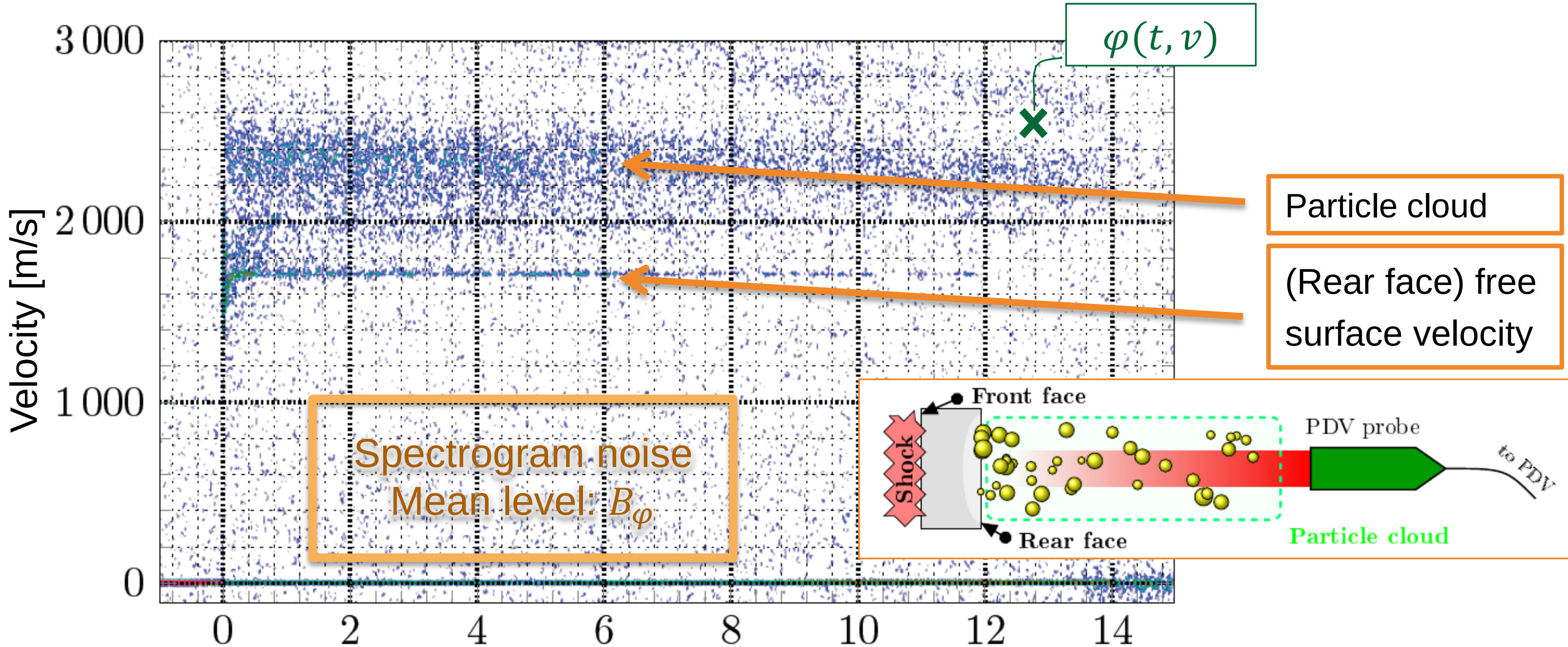


HIGH-VELOCITY PARTICLE CLOUDS IN SHOCK PHYSICS



- A metallic (tin) plate undergoes a shockwave (produced by a laser shock, high-explosives or impact)
- Many mechanisms (spalling, microspalling, microjetting, melting, …) generate a **cloud of micron-sized particles accelerating up to several km/s**.

THE NEED: PARTICLE CLOUDS



Tin micro-jetting first PDV measurement at the CEA (Mercier *et al.*, 2006)

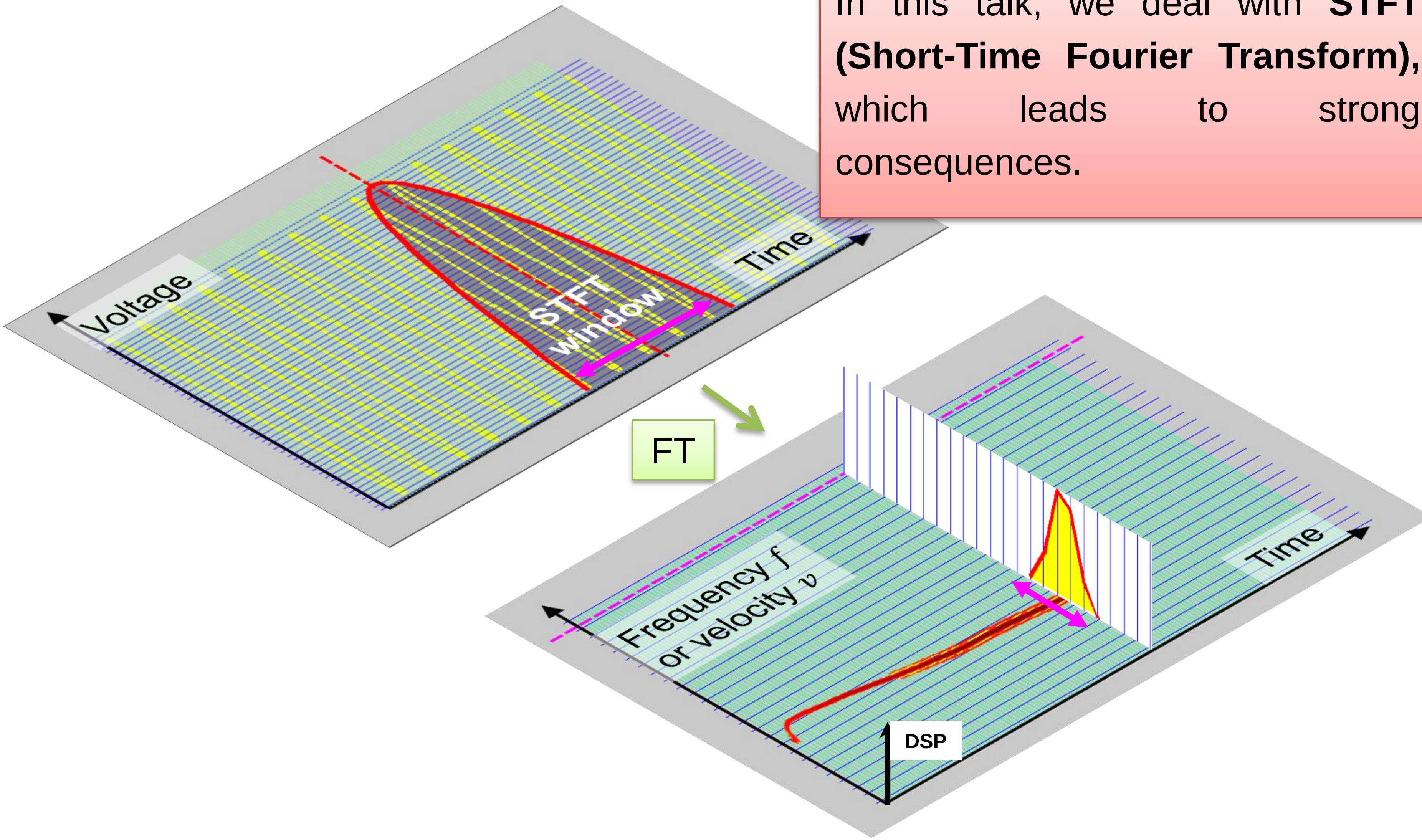
Are the fastest particles detected?

How deep is the cloud analyzed?

- ➢ Noise on spectrogram may limit the visibility of the cloud by PDV.
- ➢ Noise is linked to the PDV design.

SHORT-TIME FOURIER TRANSFORM

In this talk, we deal with **STFT (Short-Time Fourier Transform),** which leads to strong consequences.



WHAT ARE THE NOISE SOURCES OF THE STANDARD PDV?

(STANDARD = NOT MPDV)

HOW DO WE SEPARATE THE NOISE SOURCES?

(Ph): shot noise

- Quantum optics : the photons are inseparable amounts of power.

(D): Dark noise

Includes background IR (BLIR) noise, electronic noise, thermal noise, …

(Q): digitizer noise

- Generally expressed as a number of “equivalent bit”.

(S): spontaneous-emitted photons – ASE noise

- Due to the lasers and amplifiers of the PDV line.

- **We will express all noise sources as an optical “NEP” (Noise Equivalent Power”, unit : $W/\sqrt{Hz}$).**
  - **This choice will be explained in the 2nd part.**
  - **Relation between optical power P_{det} and electrical u_e : $u_e = \mathcal{R}\, P_{det}$.**
- **First, we derive the power P_{det} on the PDV detector in order to estimate noise levels.**

BACK TO BASICS: THE OPTICAL POWER COLLECTED BY THE DETECTOR

- For a single free surface: $P_{det}(t) = \alpha_{LO}P_{LO} + \sqrt{\alpha_{LO}P_{LO}\alpha_o P_p}\,\kappa_{LO}^p \cos[\Omega_p t + \phi_{LO}^p] + a_o P_p$
- **For N objects (i.e, a particle cloud), in order of importance:**

$$P_{det}(t) = \underbrace{\alpha_{LO}P_{LO}}_{(1)} + \underbrace{\sum_p \sqrt{\alpha_{LO}P_{LO}\alpha_o P_p}\,\kappa_{LO}^p \cos[\Omega_p t + \phi_{LO}^p]}_{(2)} + \underbrace{\alpha_o \sum_{p,q} \sqrt{P_p P_q}\,\kappa_p^q \cos[(\Omega_p - \Omega_q)t + \phi_p^q]}_{(3)}$$

- p (or q) : points to a reflector (i.e. a free surface or a particle),
- α_{LO}, α_o: coupling efficiencies,
- κ_p^q: polarization dot product (between p and q),
- $\Omega_p = \frac{4\pi}{\lambda_{PDV}}(v_p - V_{LO})$: relative angular frequency.

- (1) **constant term**, strong but useless.
  - $\approx$ few 100 µw?
- (2) **beating term** (LO coupling), weak but that's we are looking for!
  - $\approx \sqrt{(\text{few } 100\ \mu\text{w})(?\text{ to } 1\ \mu\text{W?})} \lesssim 10\ \mu W$ for each p.
  - $N \times (\text{few } 10\ \mu W)$ spread over the velocity spectrum.
- (3) **cross-beating term**, very weak, neglected in general!
  - $N^2 \times (\lesssim 1\ \mu W)$: weak when N is not too large.
  - the sum could be separated in two parts:

    - (3c) $p = q$, $\alpha_0 \sum_p P_p$: **c**onstant part.

    - (3n) $p \neq q$, $\alpha_0 \sum_{(p\neq q)} X_p^q \cos[(\Omega_p - \Omega_q)t + \phi_p^q]$: (nearly) random freq. -> **n**oise ?!

WHICH TERMS ARE IMPORTANT FOR THE NOISE? 1/4

Shot noise (or photonic noise)

Because a lot of beats is recorded inside the FT window $\left(T \gg \frac{2\pi}{\Omega_p}\right)$, we consider the constant part of the signal:

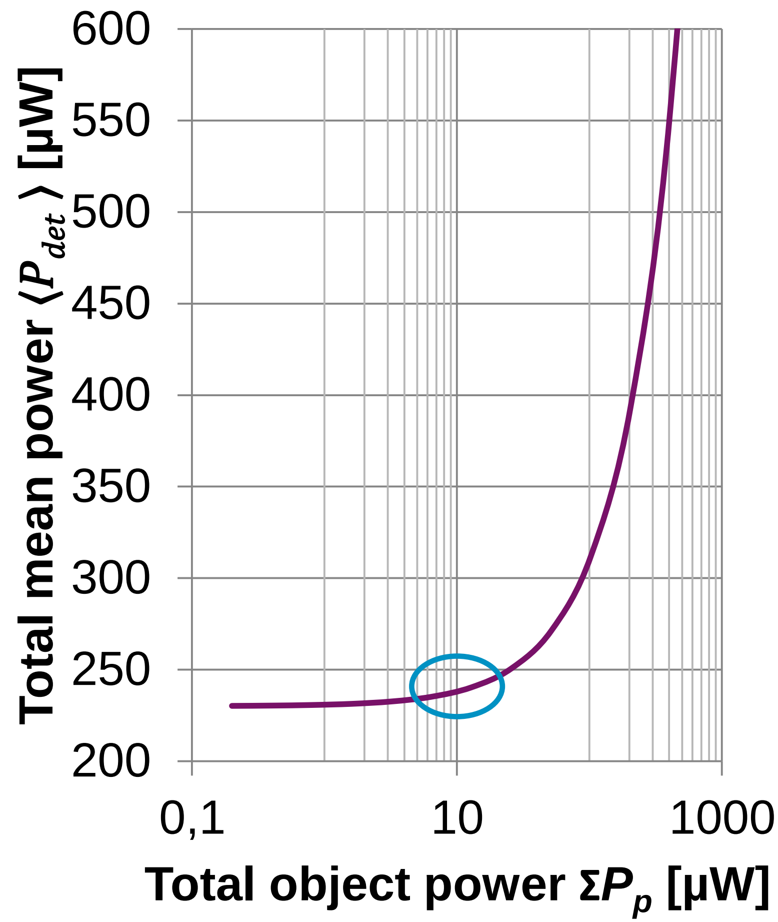
$$\langle P_{det}\rangle = \underbrace{\alpha_{LO} P_{LO}}_{(1)} + \underbrace{\alpha_o \sum_p P_p}_{(3c)}$$

- **If $\boldsymbol{\alpha_o \sum_p P_p \leq 10\ \mu W}$, we suppose that $\boldsymbol{P_{det}}$ is constant and equal to (1).**
- **The shot-noise spectrum density is:**

$$< P_{ph} > = K_{ph}\sqrt{f_{BW}} = \sqrt{\frac{2\ hc_0\ f_{BW}\ \alpha_{LO} P_{LO}}{\eta\ \lambda_{PDV}}}$$

$$K_{ph} = \sqrt{\frac{2\ hc_0\ \alpha_{LO} P_{LO}}{\eta\ \lambda_{PDV}}}$$

Total intensity on detector (CEA PDV System)



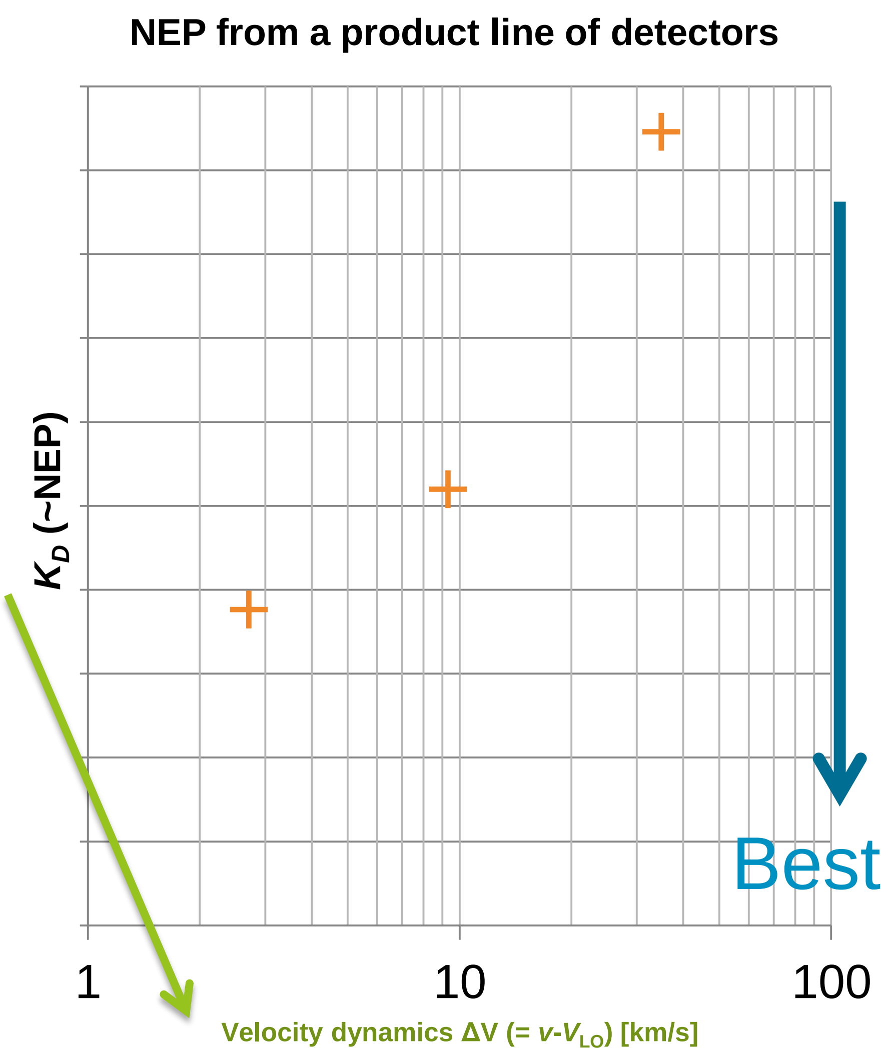
WHICH TERMS ARE IMPORTANT FOR THE NOISE? 2/4

Dark noise

- Values taken from datasheet (NEP),
- Or measured.

$$< P_D > = K_D(f_{BW})\sqrt{f_{BW}}$$

- $K_D(f_{BW})$ is the NEP of the detector.
- It depends on the detector technology.
- In practice, it depends on the **detector bandwidth**.



WHICH TERMS ARE IMPORTANT FOR THE NOISE? 3/4

Digitizer noise

- **The constant term (1) may be canceled with an offset**
- **The beating term (2) will set the dynamics of the digitizer, and its noise level.**

- **Mainly used characteristic to estimate digitizer noise : ENOB (effective number of bits), composed of:**
  - noise floor level,
  - dynamic noise level,
  - distortions (total harmonic – TH).

$$K_Q = \frac{\Delta U_{P-P}}{2\sqrt{3}\,(2^{ENOB} - 1)\, f_{BW}}$$

- **NB : distortions create harmonics, it is not so disadvantageous for free surface velocities**

WHICH TERMS ARE IMPORTANT FOR THE NOISE? 4/4

ASE noise

- **Generated by laser source, local oscillator and optical amplifier (EDFA).**
- **We do not use EDFA.**
- **We use very high-coherency laser (frequency width < 100 kHz), the spontaneous emission is really weak!**
- **For our PDV systems,**

$$K_S = 0$$

TOTAL « NEP » K_T OF THE PDV SYSTEM IS:

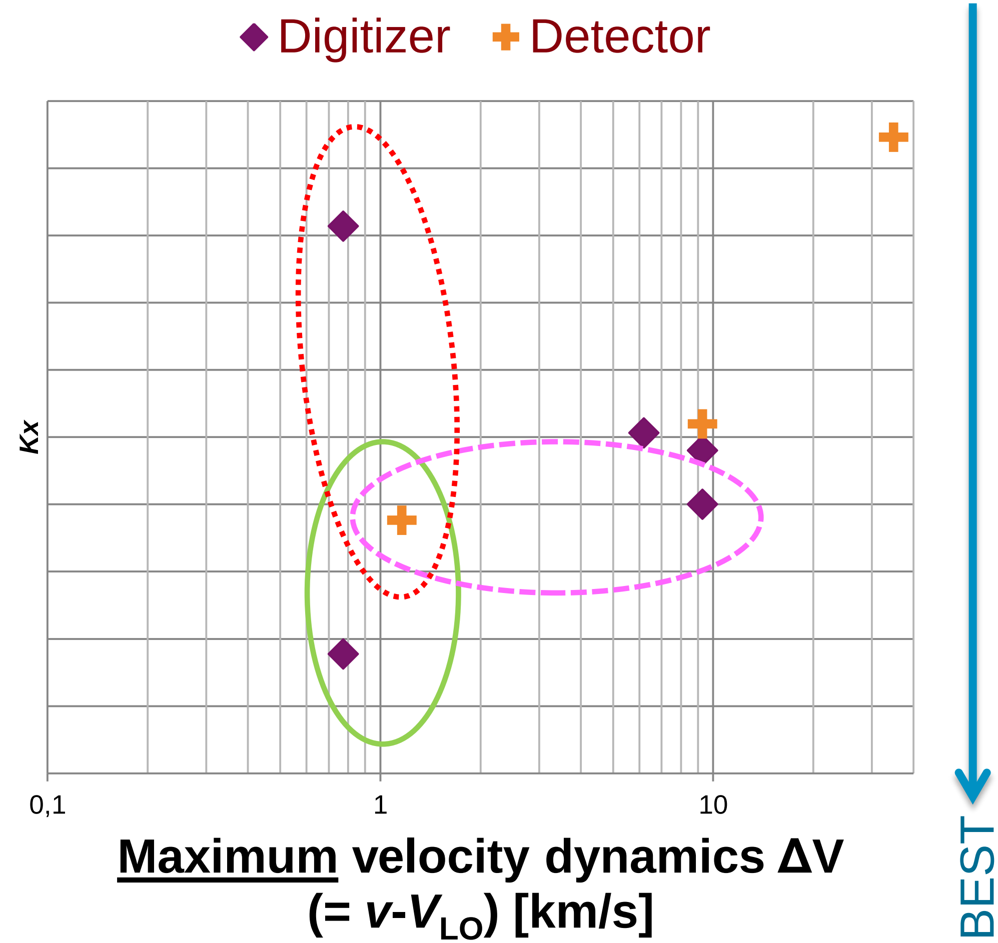
$$K_T^2 = K_Q^2 + K_D^2 + K_{ph}^2 + K_S^2$$

You can't act on it !
Changing P_{LO}? → Backup

The detector may be changed

The digitizer may be changed

How to select a couple of {detector + digitizer}?



NOISE SOURCE LEVELS (8-GHZ CEA PDV SYSTEM)

Example with a 8-GHz bandwidth, CEA PDV system ($\alpha_{LO}P_{LO} = 250$ µW).

1. Photonic noise – 1.1 µW

For low reflected power ($P_o < 10\ \mu W$), it depends only on $\alpha_{LO}P_{LO}$ and detector efficiency.

2. Internal detector noise – 2.1 µW

Depends on the detector.

3. Quantification noise – 2.3 µW

Depends on the total reflected power and on the digitizer.

4. AES noise – 0 µW

Very spectral-narrow lasers and no optical amplifier.

So, $B_{det} = \sqrt{1.1^2 + 2.1^2 + 2.3^2} = 3.3\ \mu W$

Curves of noise sources as a function of the vertical resolution (backup)

WHY HETERODYNING IS SO SENSITIVE ?

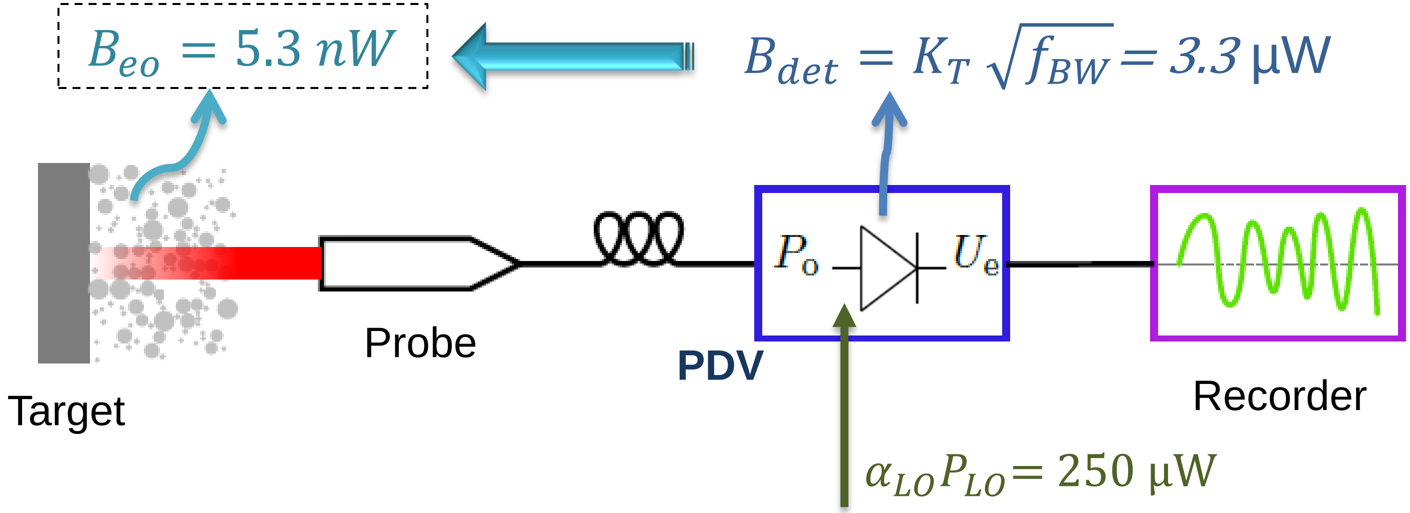
Let's define a "NEP" in object space : "object-equivalent noise" …(see my thesis)

$$B_{oe} = \frac{K_T^2 f_{BW}}{8\alpha_o \alpha_{LO} P_{LO}} = 5.3 \text{ nW} \quad (\alpha_{LO} P_{LO} = 250 \text{ µW})$$

Another reinterpretation of the high detectivity of heterodyne systems:

$B_{oe} \ll B_{det}$, thanks to the $\sqrt{P_{LO} P_P}$ term.

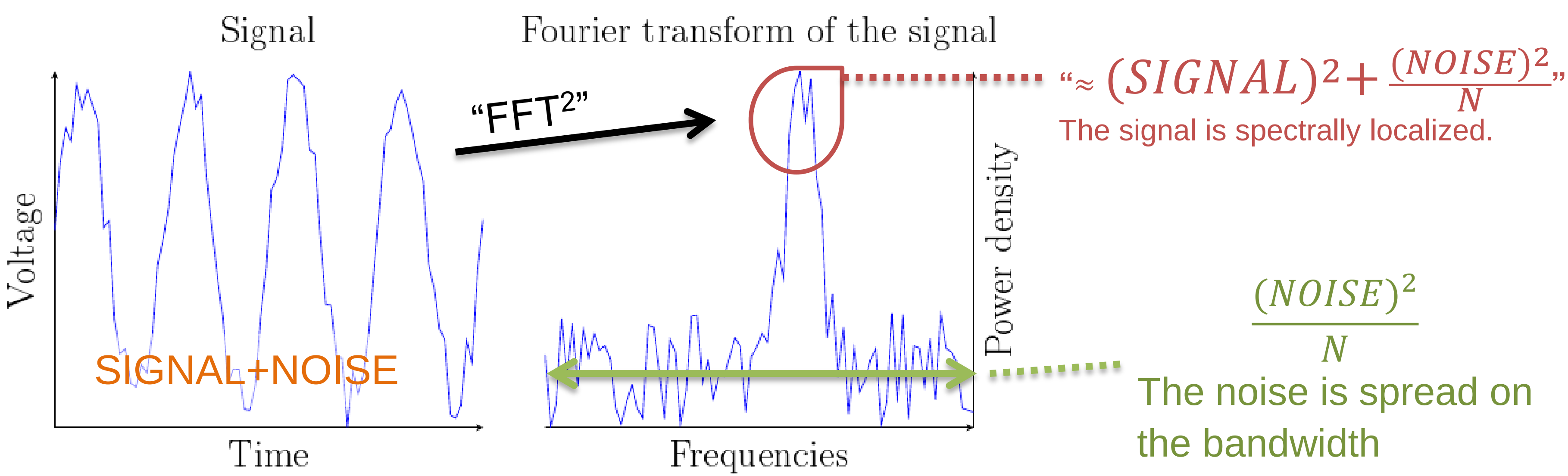
"Only objects, of which reflected and collected power is above B_{oe}, could be measured (using the raw signal)"



THE EFFECTS OF STFT TRANSFORM ON NOISE

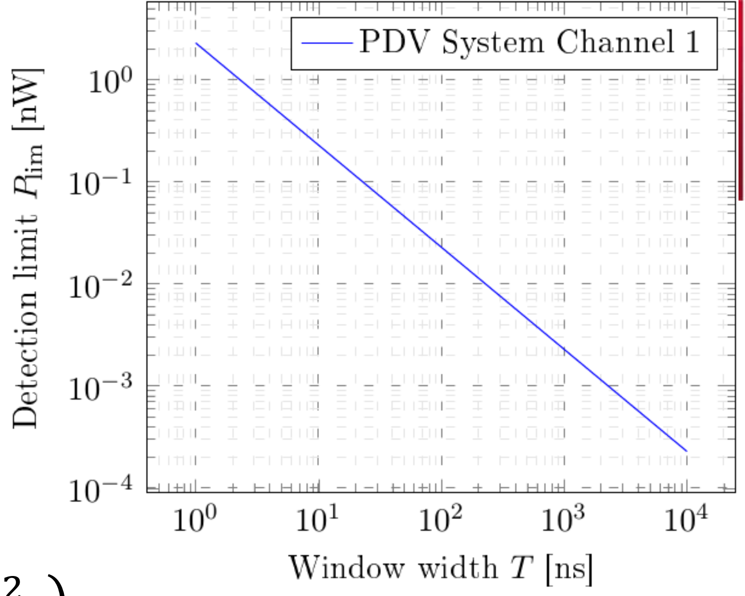
WHY {HETERODYNING+STFT} ARE REALLY SO SENSITIVE?

- Fourier transform consequences:
  - Mono frequency base: $e^{4i\pi\frac{v_m t}{\lambda_s}}$ -> monokinetic contributors are identified for each velocity (v_m).
  - Noise is spread on all the bandwidth: after "FFT^2", noise mean level is reduced by N.



For example, see Harris, 1978, Proc IEEE

NOISE ON THE SPECTROGRAM: B_φ



Taking into account FT, noise level on the spectrogram is

$$B_\varphi = \frac{1}{\hat{G}_i\, T f_{\text{BW}}} B_{eo}$$
$$= \frac{1}{\hat{G}_i\, T} \frac{1}{8\alpha_o \alpha_{LO} P_{LO}} \underbrace{\left(K_Q^2 + K_D^2 + K_{ph}^2\right)}_{\text{noise levels}}$$

$\hat{G}_c$ is a window correction factor (≈ 0.2).

STFT makes detection limit independent of bandwidth (if Nyquist criteria is respected).

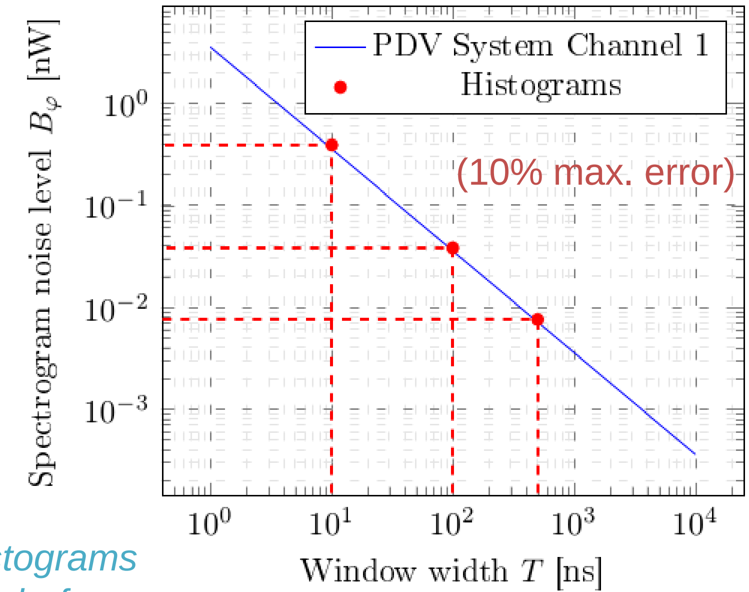
- but K_X are hardware-depending.
- It is useless to pre-filter the signal with a low-pass filter (maybe to reduce the number of samples).

NOISE ON SPECTROGRAMS: COMPARISONS

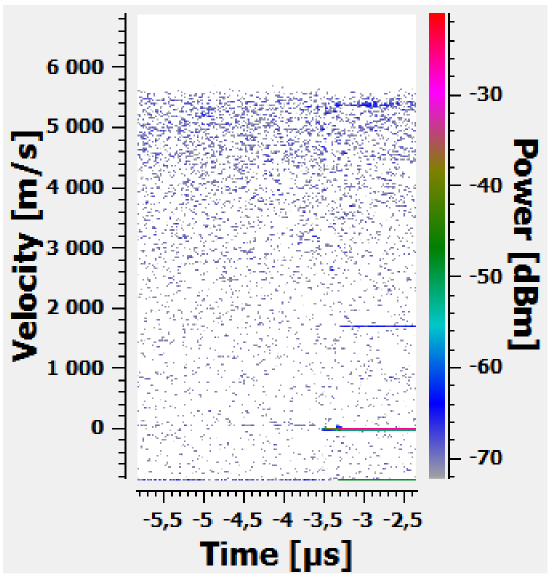
$$B_\varphi = \frac{1}{\hat{G}_i \, T f_{\mathrm{BW}}} \quad B_{eo} = \frac{1}{\hat{G}_i \, T} \frac{1}{8\alpha_r \alpha_o P_r} \underbrace{\left(K_Q^2 + K_D^2 + K_{ph}^2\right)}_{\text{noise levels}}$$

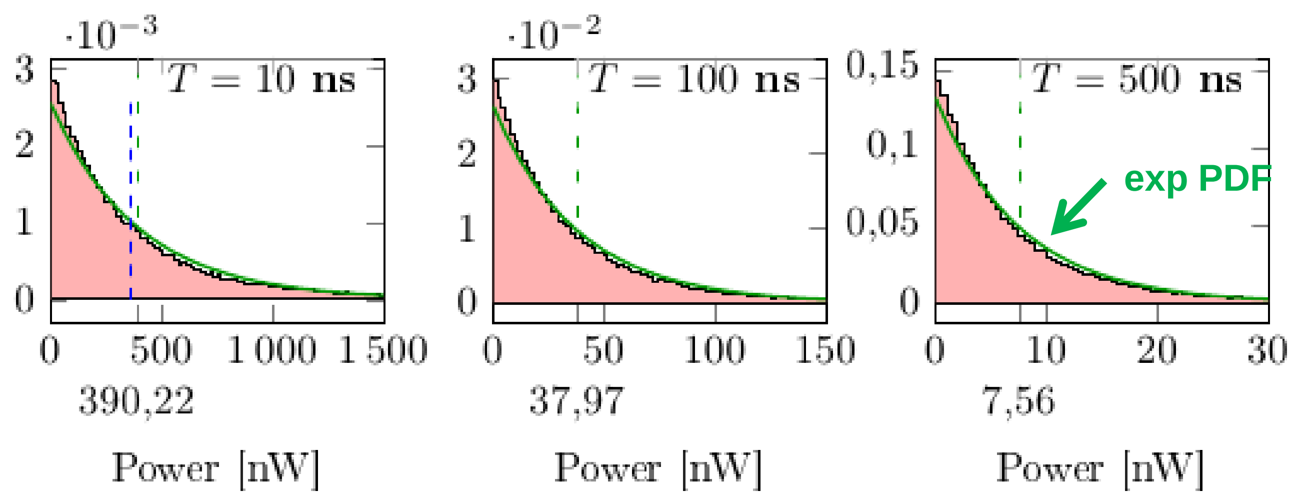
- When no signal is present, **the noise PDF* is exponential**.
- B_φ is both the bias and the dispersion of the background noise (The mean is equal to the standard deviation).

**PDF: Probability Density Function*



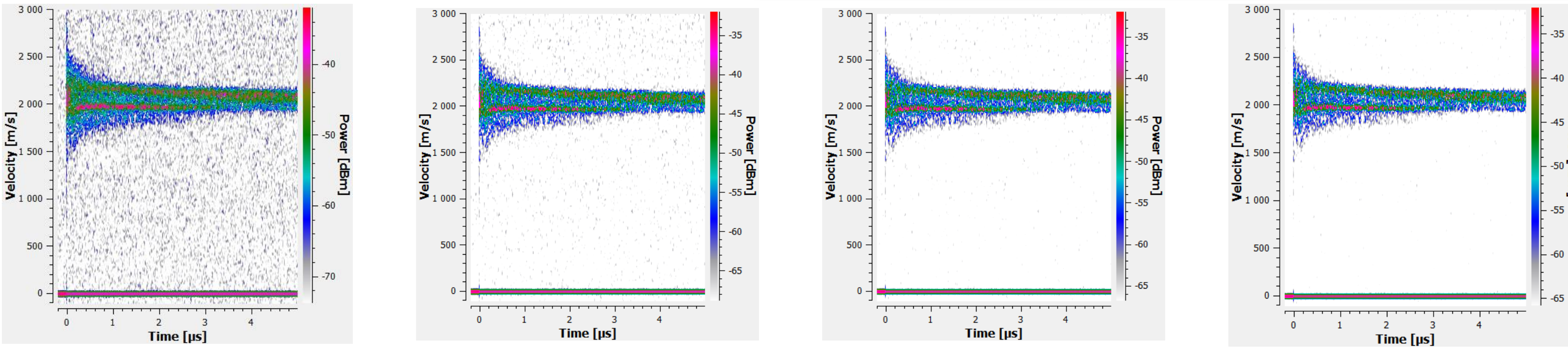
For 3 windows of T width, histograms computed from spectrograms before shock break-out are analyzed.





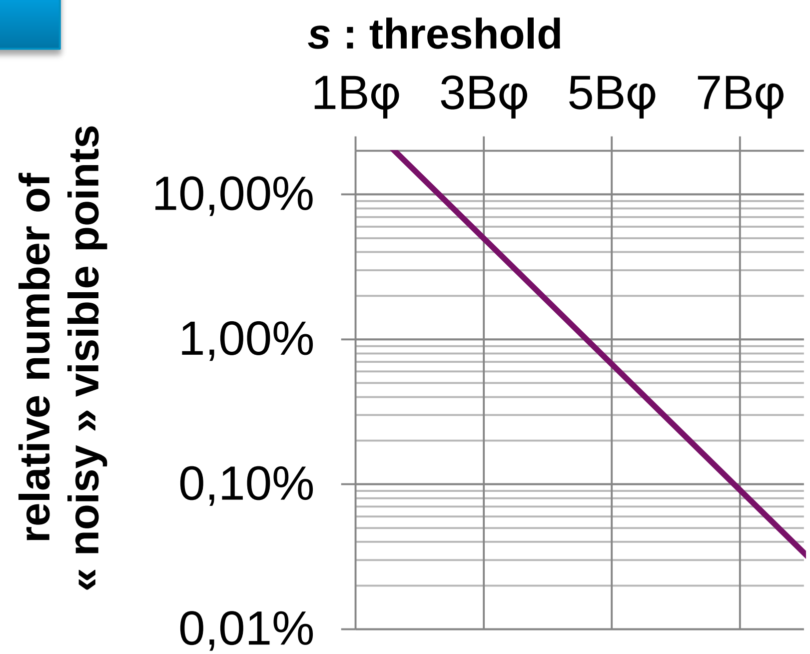
HOW TO IMPROVE SPECTROGRAM DISPLAYS?

Which threshold should be set to display spectrogram?

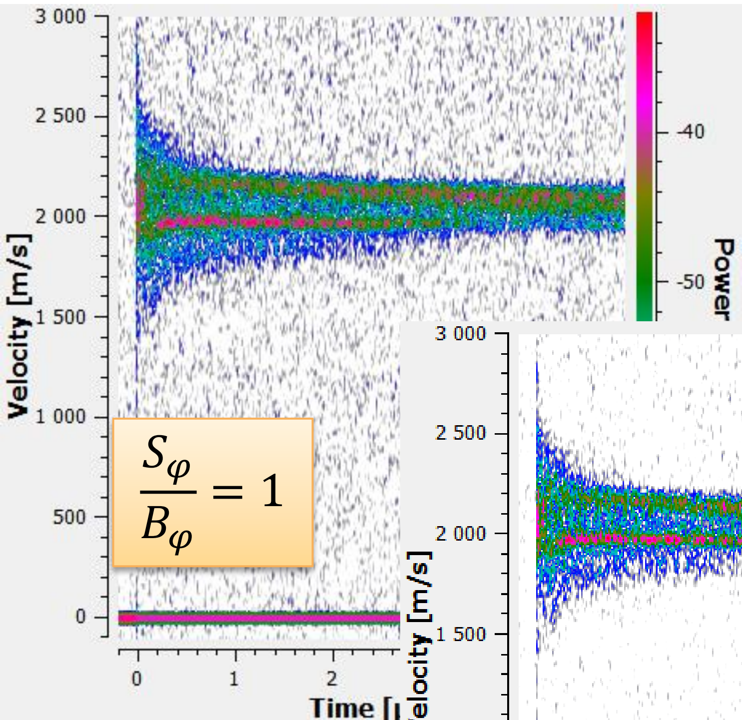


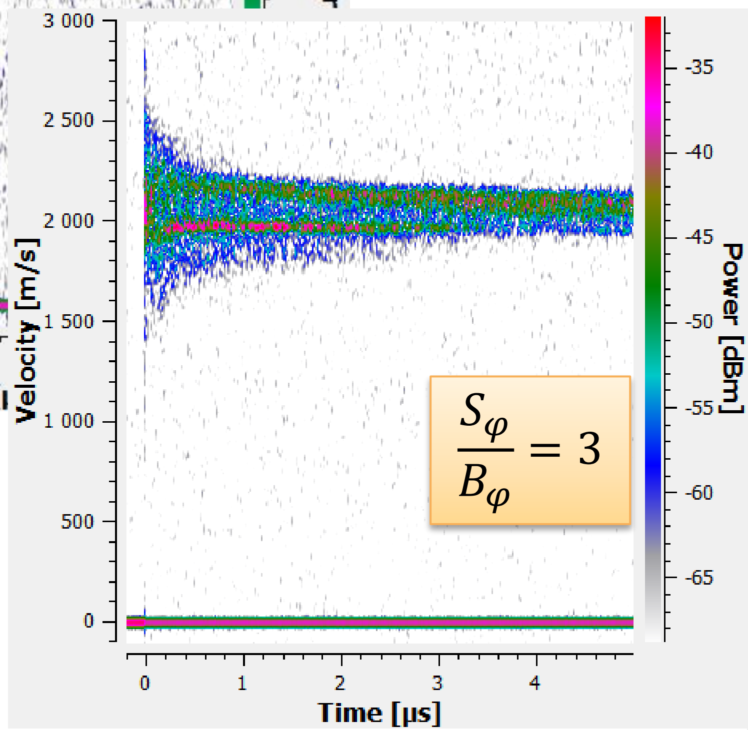
How many “blue points” do we accept?

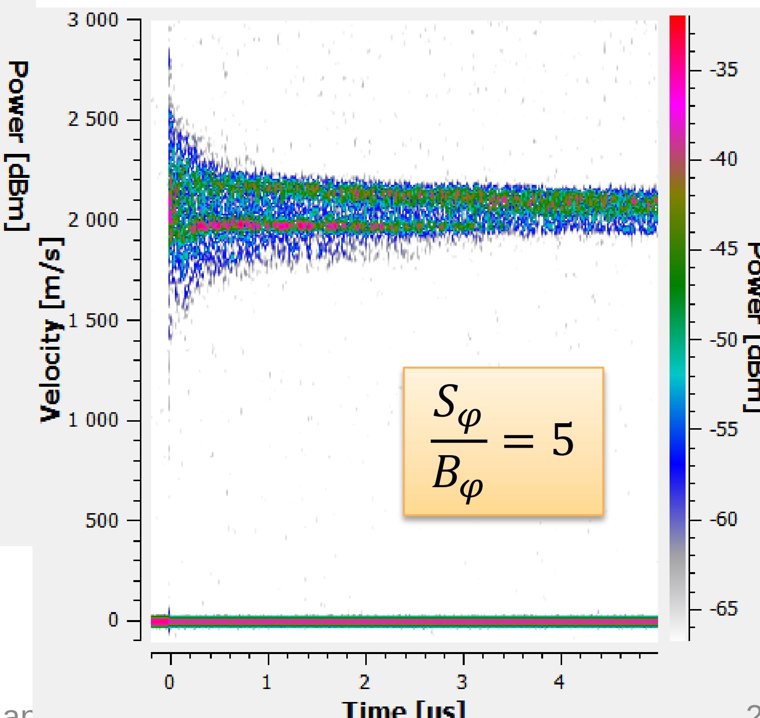
- exp PDF
  - $z \rightarrow 1 - \exp\left(-\frac{z}{B_\varphi}\right)$ CDF : relative number of background values below z.
- So, $s \rightarrow \exp\left(-\frac{s}{B_\varphi}\right)$ gives the relative number of « noisy » visible points.
- Example : $T = 50\ ns$ @ 50 GS/s,
  - $N = 2500$
  - $S_\varphi = 5B_\varphi \rightarrow \approx 0.67\ \%$
  - 17 expected noisy points in each column.



HOW TO IMPROVE SPECTROGRAM DISPLAYS? EXAMPLES







$\frac{S_\varphi}{B_\varphi}$	$\exp\left(-\frac{S_\varphi}{B_\varphi}\right)$
1	36 %
3	5,0 %
5	0,67 %
7	0,091 %

SUMMARY

We present rules for PDV design in order to decrease noise:

- Contributors are quantified.
  Major ones: detector background and digitizer noise.
  Other source noises? (non-linear effect, backscattering inside the fiber, …)
- Our model provides a good estimation of B_φ, the spectrogram mean noise level.

We analyzed the impact of noise on STFT spectrograms

- Noise level depends on noise density (“NEP”) and not on bandwidth,
  - Reducing digitizer bandwidth will not decrease B_φ (as long as Nyquist is respected).
- We are able to threshold the spectrogram as a function of the number of noisy samples.

THANK YOU FOR YOUR ATTENTION

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BACKUP

P_{LO} MAY BE INCREASED?

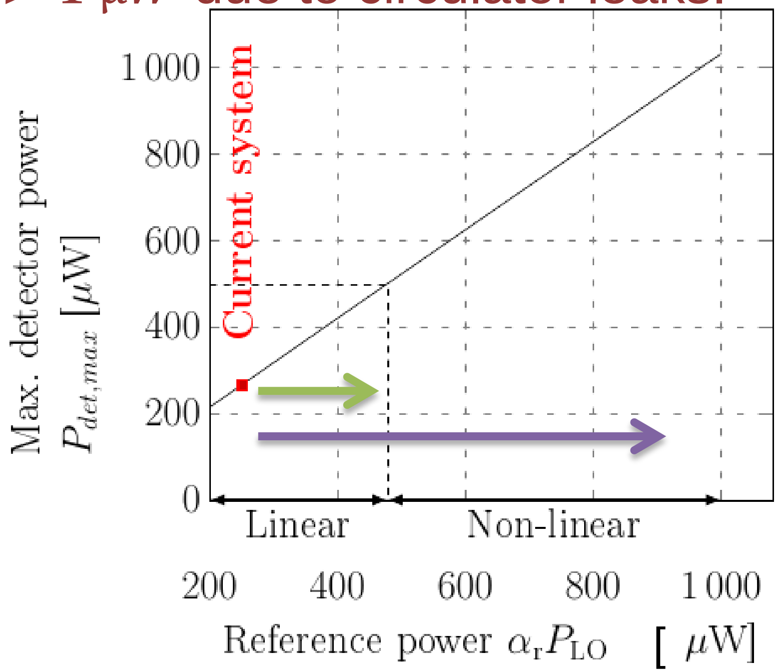
- Why? $P_{LO} \nearrow$, gain $\nearrow$ ($\sqrt{P_{LO}}$), detectivity $\nearrow$ because CEA PDV system is not photonic-noise limited.

- Detector limitation:
  - 1 mW for saturation.
  - $\gtrsim 500$ µW for non-linear effects, but may be corrected by post-processing (required P_{LO} or P_{dect} knowledge).

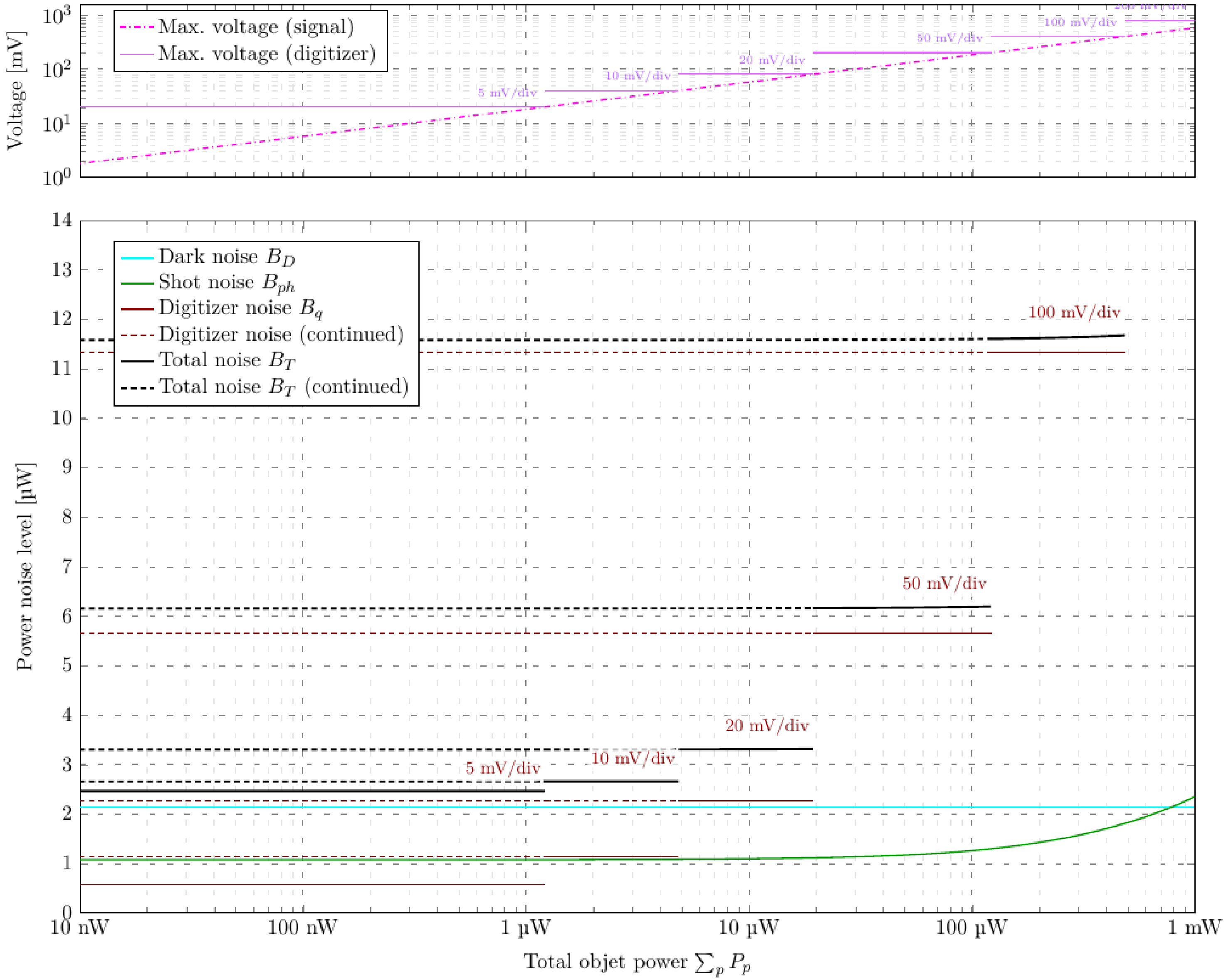
- $P_{det,\max} \approx \underbrace{\alpha_{LO}}_{\approx 2\%} P_{LO} + \sqrt{\alpha_{LO} \underbrace{\alpha_o}_{\approx 1} P_{LO} P_o}$, $P_o > 1\ \mu W$ due to circulator leaks.

- Up to 4x P_r increasing is relevant ($K_{ph} = K_q$).
  - 2x: linear domain.
  - 4x: non-linear domain.

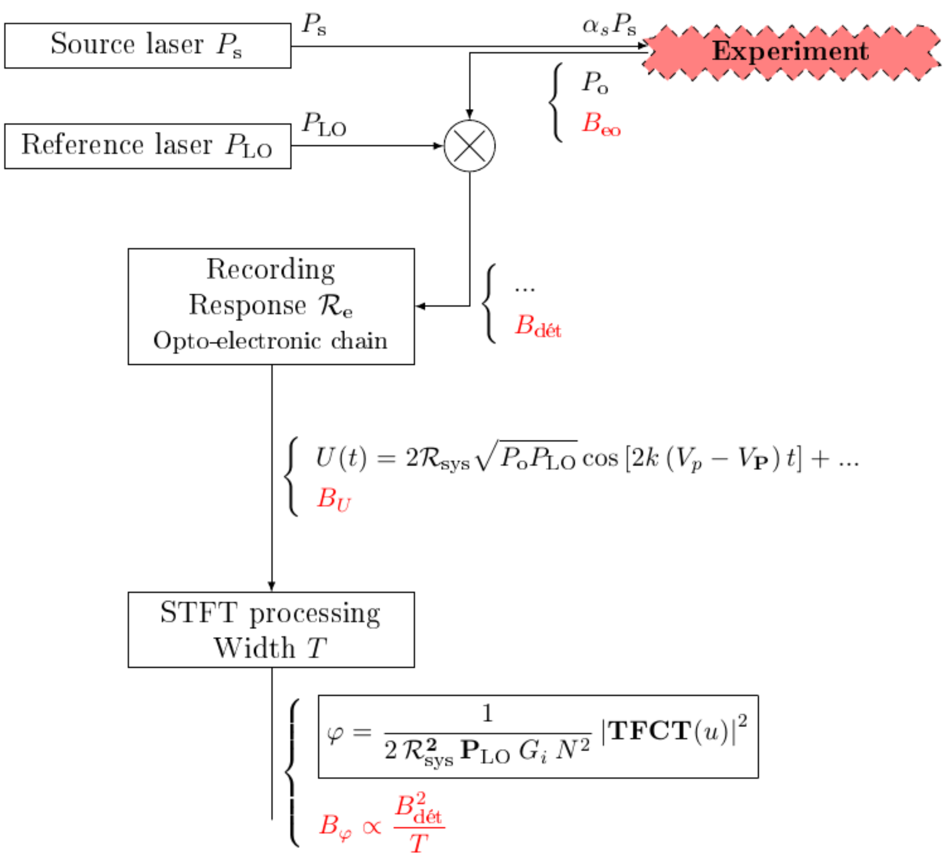
Technical development required.



NOISE AS A FUNCTION OF DIGITIZER V. RESOLUTION (CEA PDV SYSTEM)



"PDV RADIOMETRIC SCHEME"



Radiometric PDV equation of the spectrogram:

$$\varphi(t,v) = \frac{1}{2\,\mathcal{R}_{sys}^2\,P_{LO}}\,\frac{1}{\widehat{G}_i\,N^2}\left|\mathrm{STFT}\underset{f=2v/\lambda}{t}(u)\right|^2$$

$\mathcal{R}_{sys}^2$: system response,

P_{LO} : local oscillator power,

$\widehat{G}_i\,N^2$: FT normalization, ≈ bin number of the window (N).

- ➢Equals to the power distribution over the velocities.
- ➢Leads to the definition of a spectrogram mean noise level B_φ.
- ➢Do not take into account the polarization-efficiency $(\vec{e}_o \cdot \vec{e}_{LO})$ dot factor. Let's include it into $\mathcal{R}_{sys}$. (see *J-E. FRANZKOWIAK's* talk).

The point is:

The dB-expressed DSP is equal to the optical power distribution (with an offset).