

# Broadband Laser Ranging: Distance Uncertainties due to Noise in the Time Domain Data

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# Outline

Introduce analytic estimate for distance uncertainty

Present 4 different BLR systems

Build up mock data with 4 different amounts of noise

Extract data using QuickView

Calculate the scatter in the data

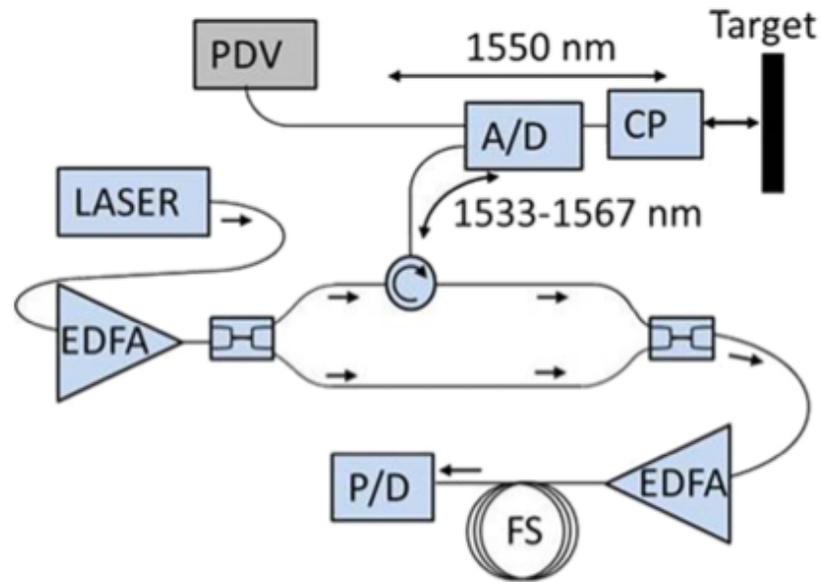
Use Standard Deviation as a measure of uncertainty

Compare to analytic expression



# Sketch of Broadband Laser Ranging Diagnostic

(Figure from La Lone, et al., *Rev. Sci. Instrum.* **86** (2015) 023112.)



The BLR is multiplexed with a PDV in this figure.

The laser is short pulse, broadband.

One leg of the Mach-Zehnder (MZ) varies in length as the surface moves closer to the probe.

The long fiber spool (FS) provides the necessary dispersion.



## Analytic Estimate of Peak Position Uncertainty

Dan Dolan derived an expression for frequency (velocity) uncertainty

(“Accuracy and precision in photonic Doppler velocimetry,” *Rev. Sci. Instrum.* **81** (2010) 053905)

$$\Delta f = \sqrt{\frac{6}{N}} \frac{\sigma}{\pi\tau}$$

where:

N = number of samples in the FFT window

$\sigma$  = noise fraction (inverse of SNR)

$\tau$  = time duration of FFT window

Brandon La Lone modified it for distance uncertainty

$$\delta z_{peak} = \sqrt{\frac{6}{N}} \frac{\sigma\lambda_0^2}{2\pi\Delta\lambda}$$

where:

N = number of samples in the FFT window

$\sigma$  = noise fraction (inverse of SNR)

$\lambda_0$  = laser wavelength

$\Delta\lambda$  = linewidth of laser

We will look at the dependence on N and  $\sigma$ .

Note: Uncertainty is linear with  $\sigma$  and 1/root(N).

Note: Uncertainty is NOT a function of distance.



# Final BLR parameters and comparison with an actual system

(This table is from “Deriving BLR Parameters from First Principles” in this workshop.)

Parameter	WG	This Study				La Lone
		OTS1	OTS2	OTS3	OTS4	
Digitizer (GS/s)		50	50	50	50	50
Digitizer (ps/pt)		20	20	20	20	20
N(fft)		1024	2048	4096	8192	
Laser wavelength (nm)	1560	1550	1550	1550	1550	1570
Laser linewidth (nm)	20	20	20	20	20	17
Data interval (ns)	50	20.48	40.96	81.92	163.84	80
Laser rep rate (MHz)	20	48.8	24.4	12.2	6.1	12.5
Temporal spread (ns)		20.48	40.96	81.92	163.84	65
Sensitivity (mm/GHz)		1.23	2.46	4.92	9.84	4.5
Fiber length (km)	200	59	118	237	474	200

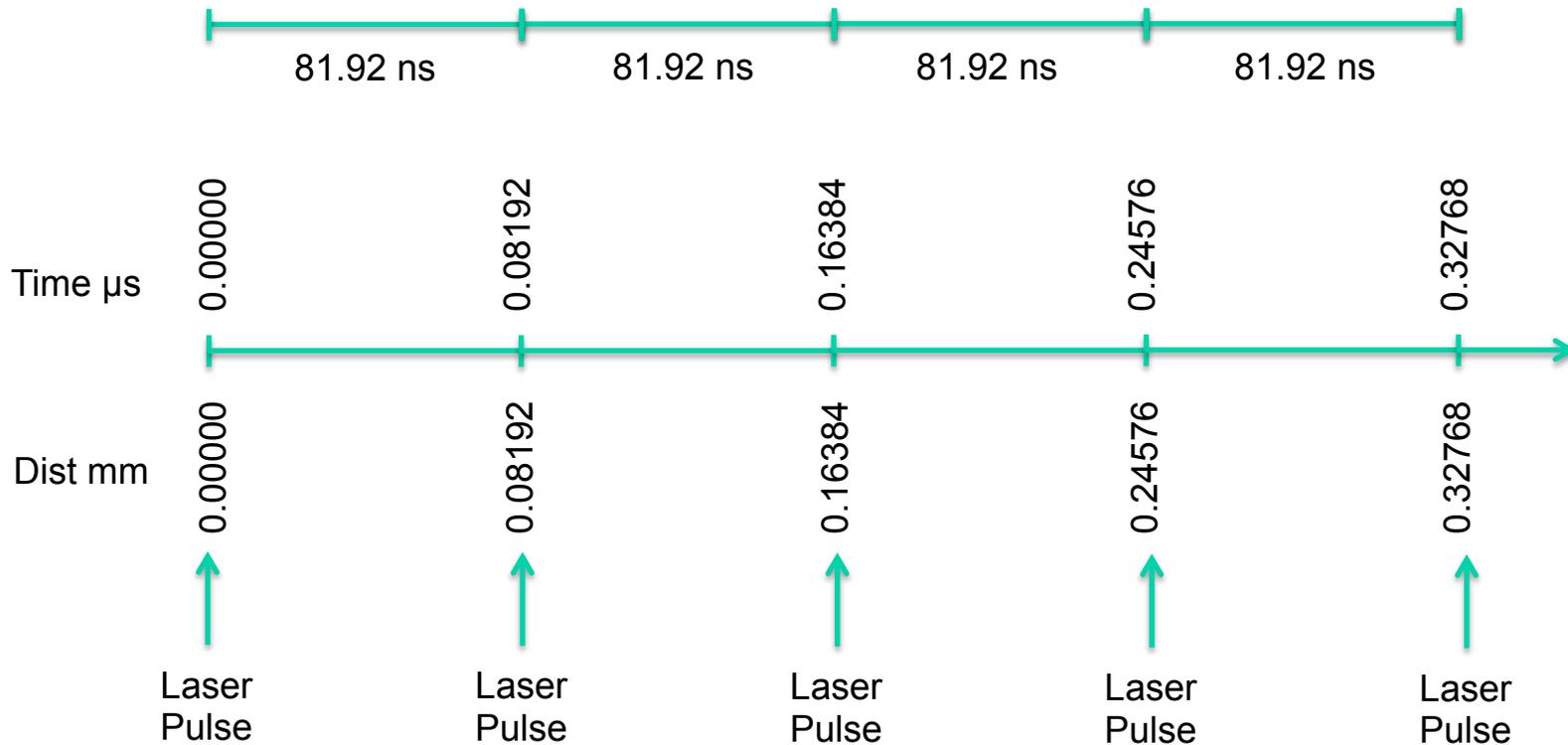
Note: Chirp was not considered in this study.

(red indicates calculated values)



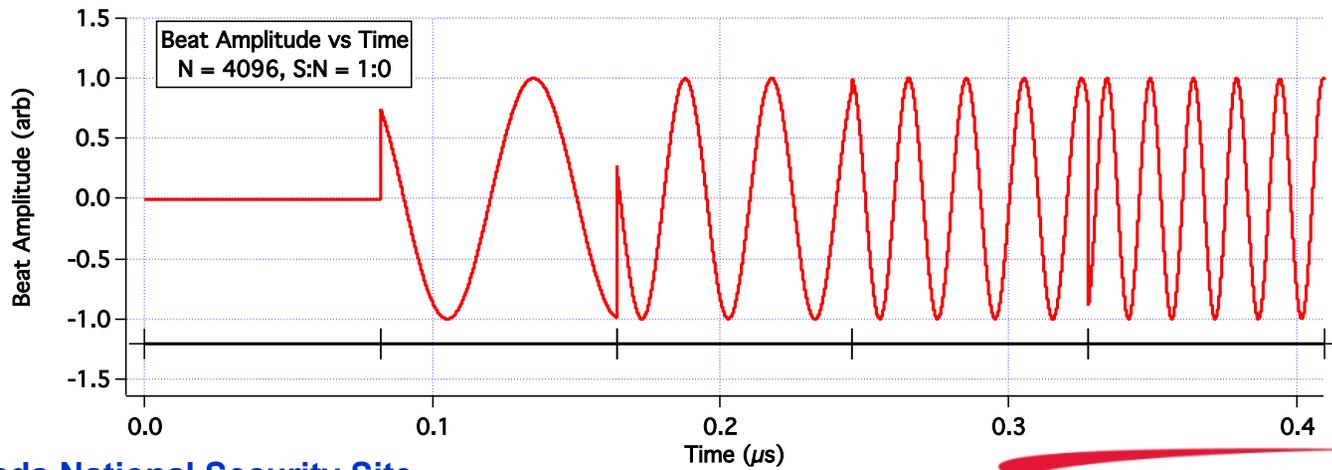
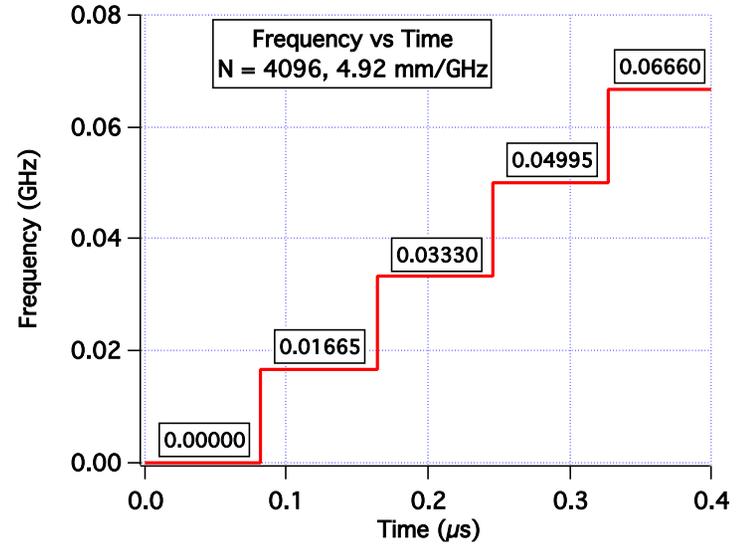
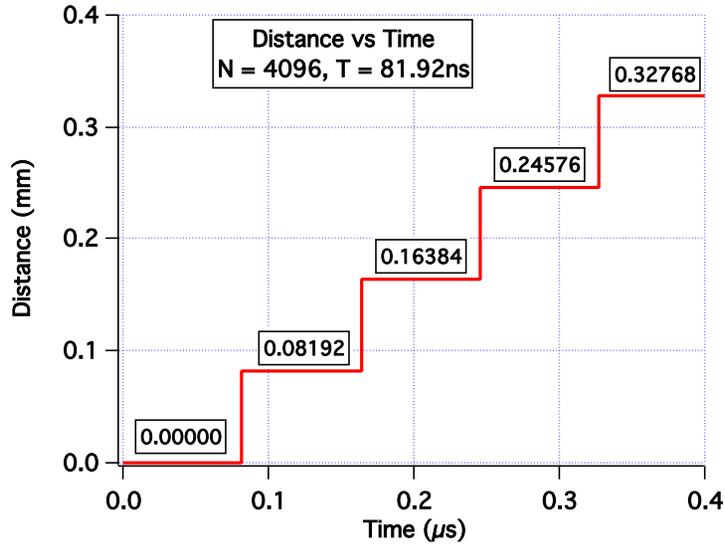
# We are ready to build the BLR waveforms. Assume a velocity of 1 mm/ $\mu$ s.

This is the timing for N = 4096:  
Step Size = 81.92 ns with V = 1 mm/ $\mu$ s.



# We are ready to build the beat waveforms. Assume a velocity of 1 mm/ $\mu$ s, N = 4096.

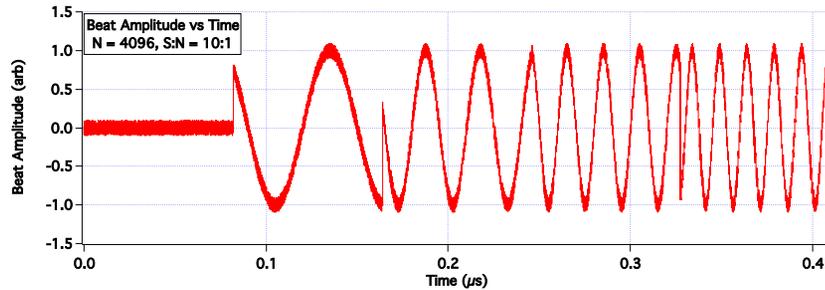
Convert distance to frequency using sensitivity of 4.92 mm/GHz.



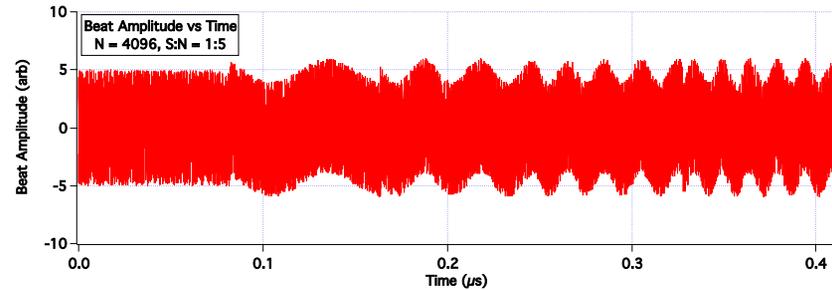
# Add noise to the BLR waveforms, N = 4096

S:N = 10:1, 1:1, 1:5, 1:10  
Noise fraction = 0.1, 1, 5, 10

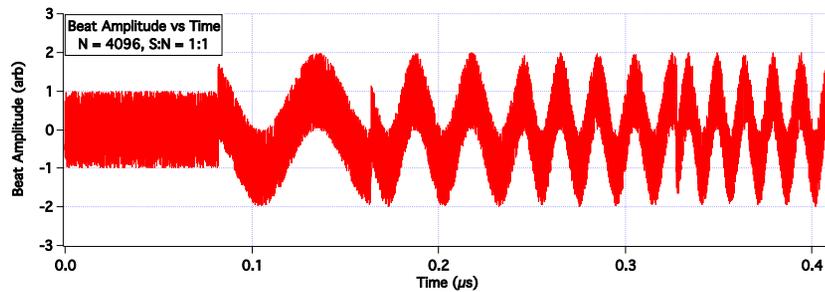
S:N = 10:1, sigma = 0.1



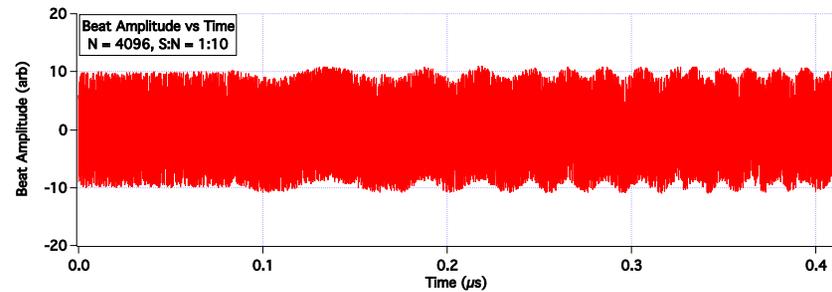
S:N = 1:5, sigma = 5.0



S:N = 1:1, sigma = 1.0



S:N = 1:10, sigma = 10.0



## We want to study the dependence of uncertainty vs N and $\sigma$

We will use standard deviation as our measure of uncertainty.

N		1024	2048	4096	8192
Step size (ns)		20.48	40.96	81.92	163.84
S:N	$\sigma$				
10:1	0.1	StdDev	StdDev	StdDev	StdDev
1:1	1	StdDev	StdDev	StdDev	StdDev
1:5	5	StdDev	StdDev	StdDev	StdDev
1:10	10	StdDev	StdDev	StdDev	StdDev

We will generate 16 BLR waveforms, process with Fourier transform, extract the data, and look at the scatter via the standard deviation. We want to maintain the same statistics in each case, so we need to build the BLR waveforms with the same number of data points:

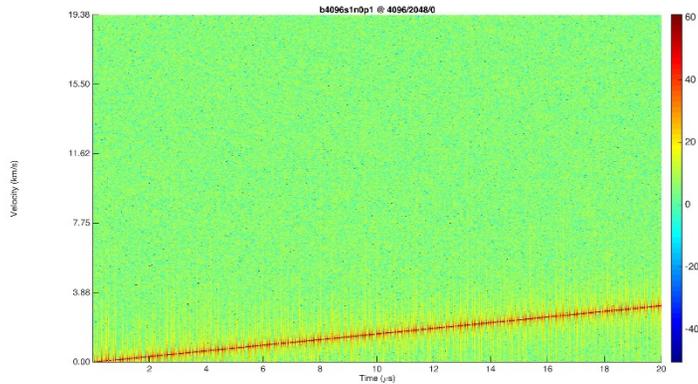
N	Length ( $\mu$ s)	#data pts
1024	5	244
2048	10	244
4096	20	244
8192	40	244



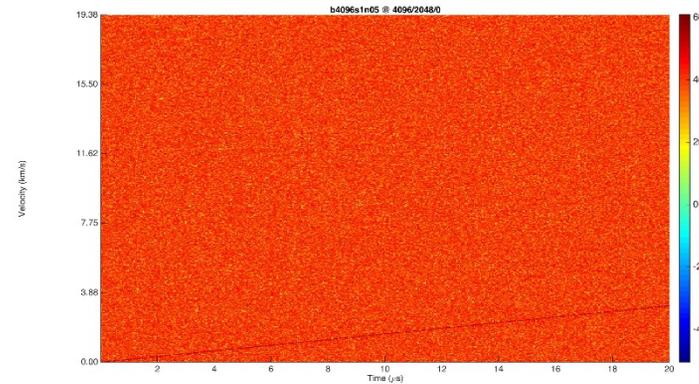
# Calculate the spectrograms and extract the d vs. t data

N = 4096 for these spectrograms

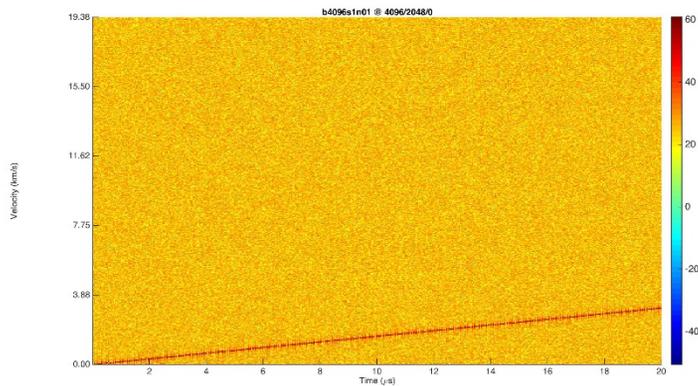
S:N = 10:1,  $\sigma = 0.1$



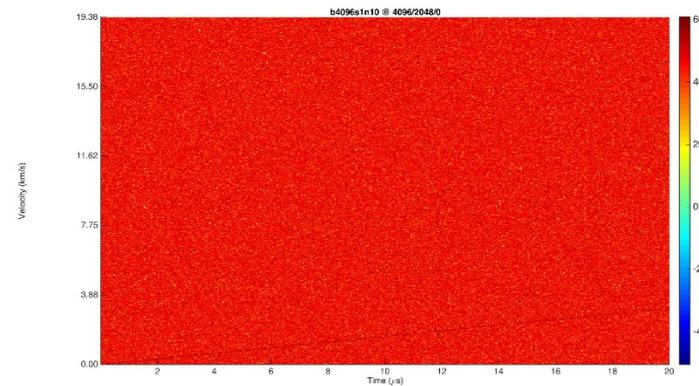
S:N = 1:5,  $\sigma = 5$



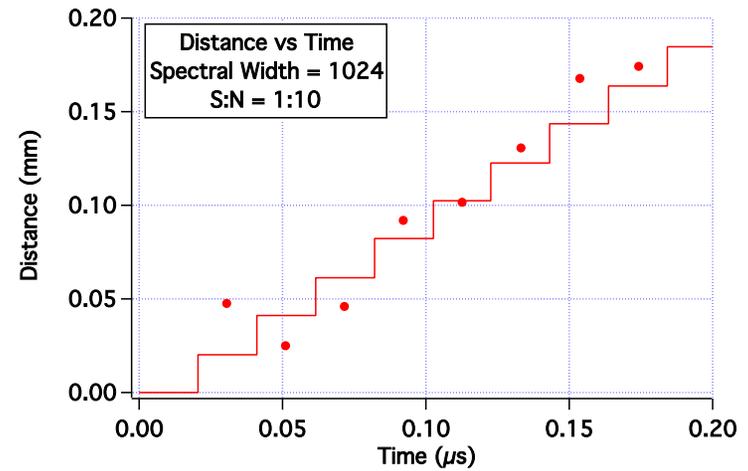
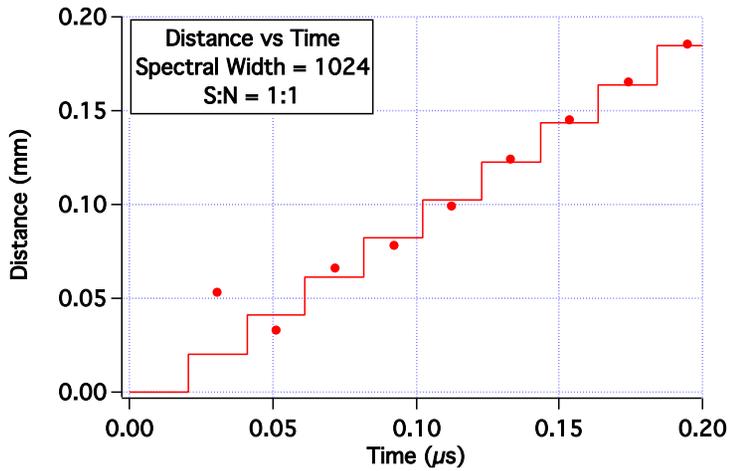
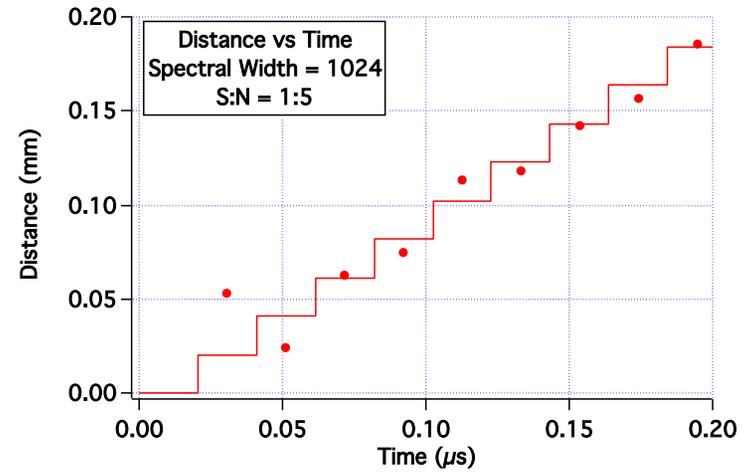
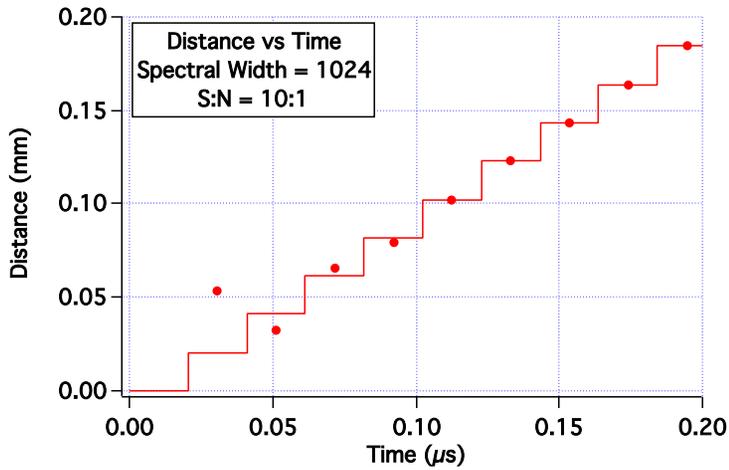
S:N = 1:1,  $\sigma = 1$



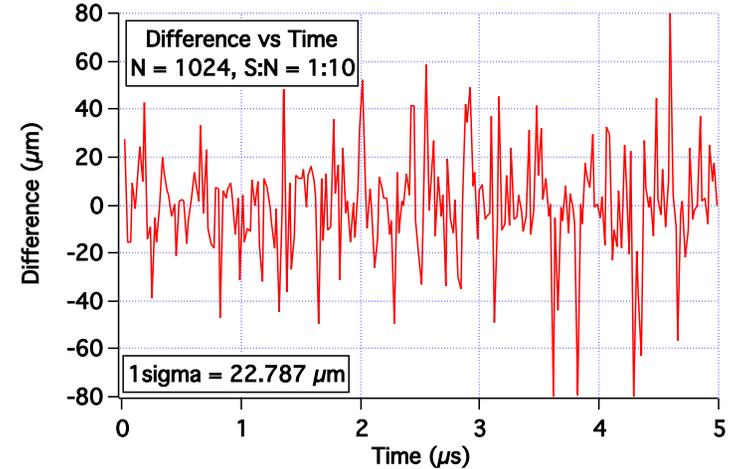
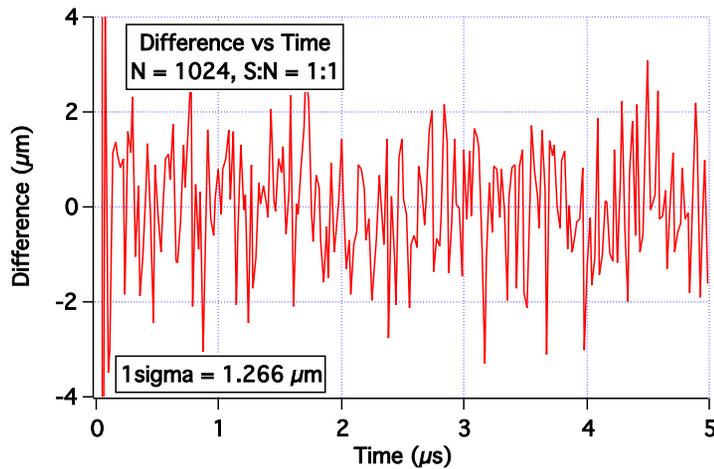
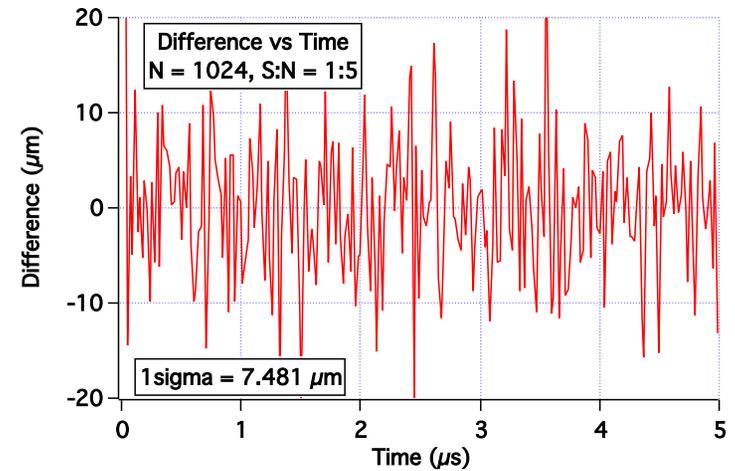
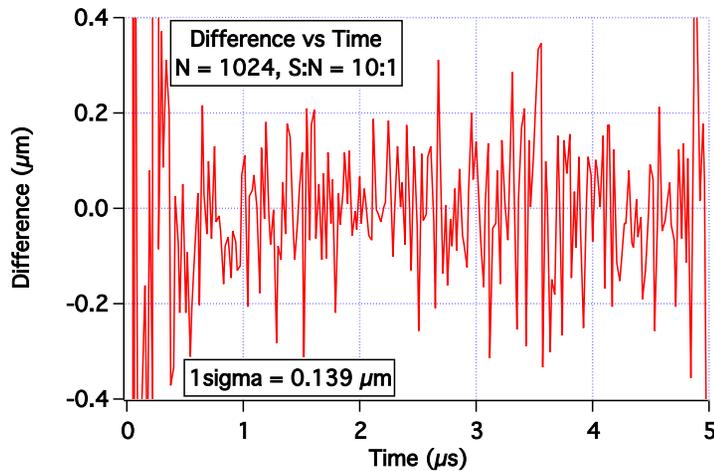
S:N = 1:10,  $\sigma = 10$



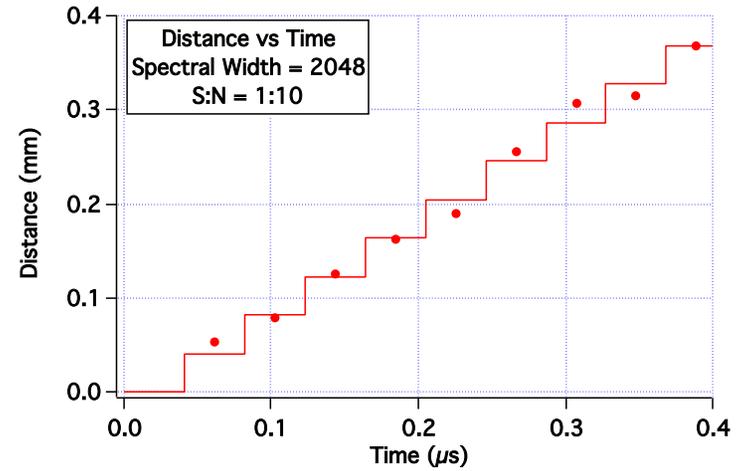
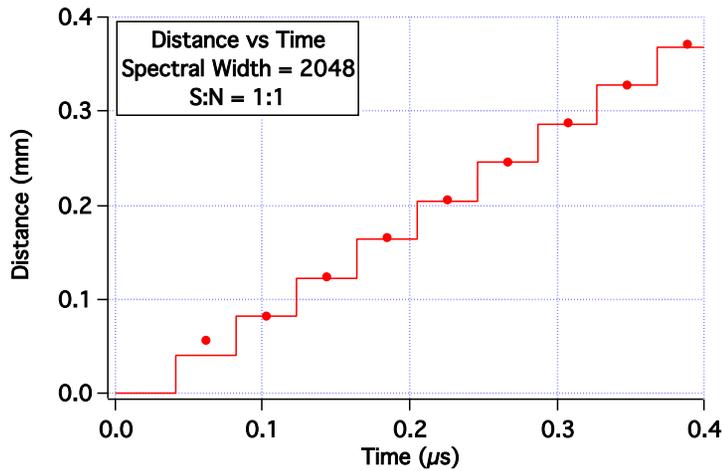
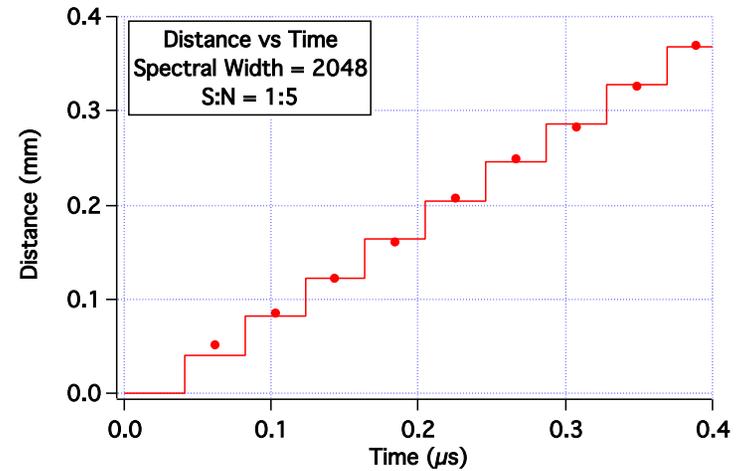
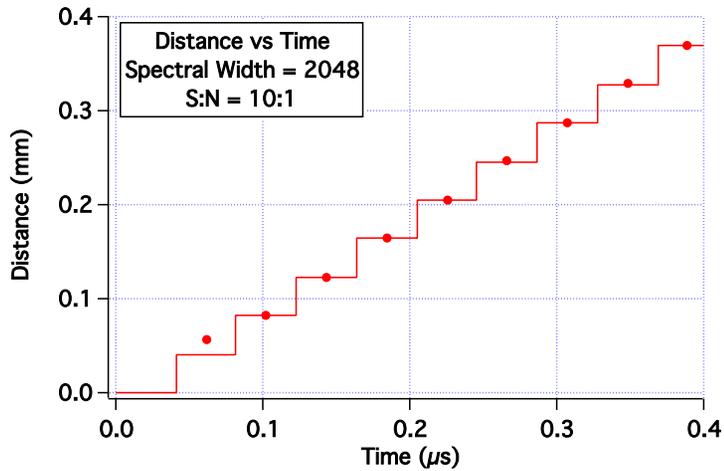
## Distance results for extracted data N = 1024, Step Size = 20.48 ns



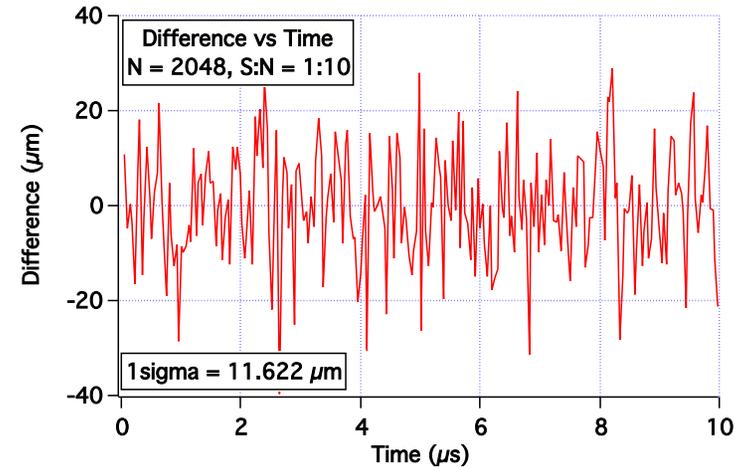
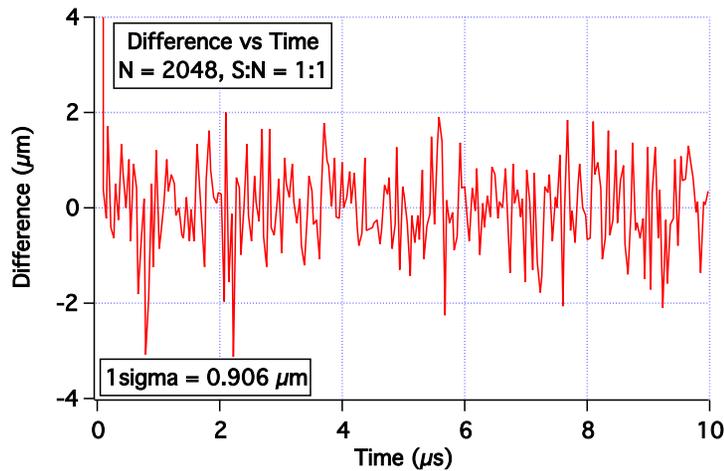
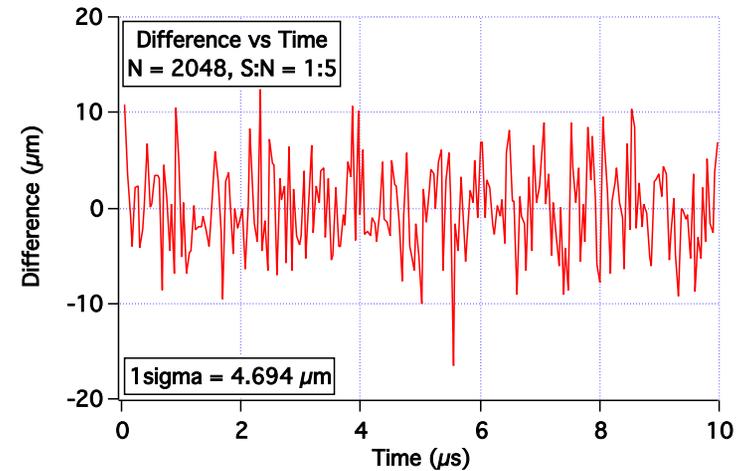
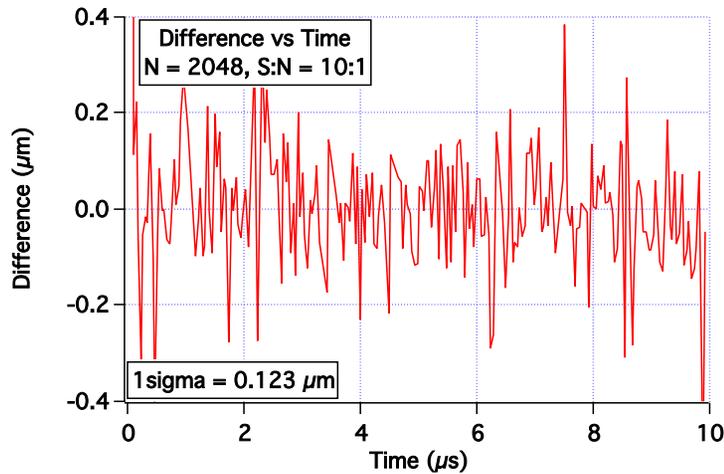
## Differences between extracted data and quadratic fit to extracted data ( $\langle \text{mean} \rangle = 0$ ) N = 1024, Step Size = 20.48 ns



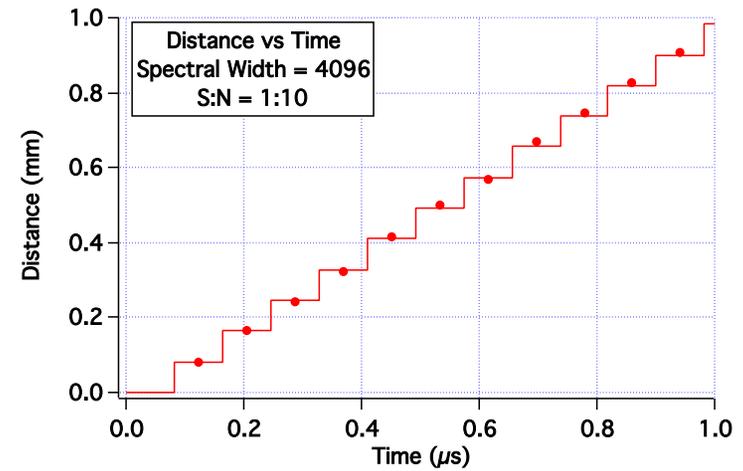
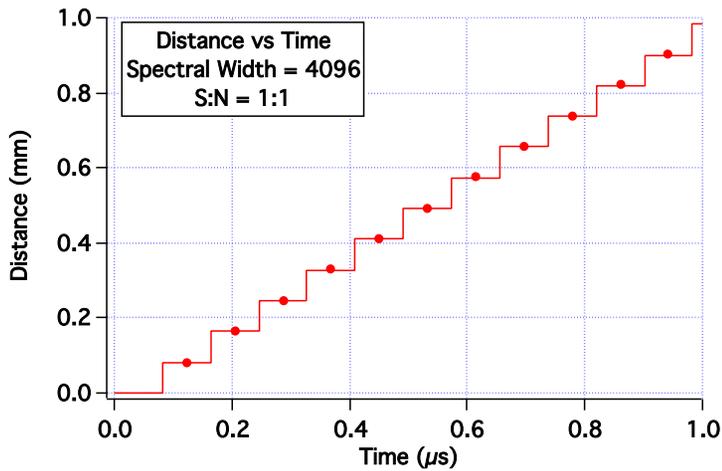
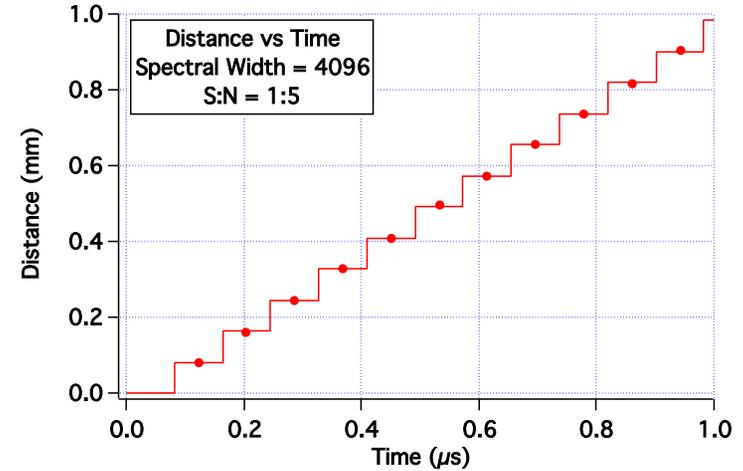
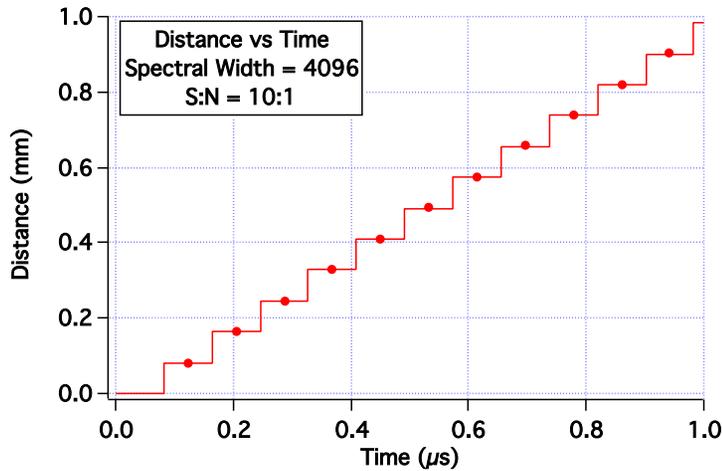
## Distance results for extracted data N = 2048, Step Size = 40.96 ns



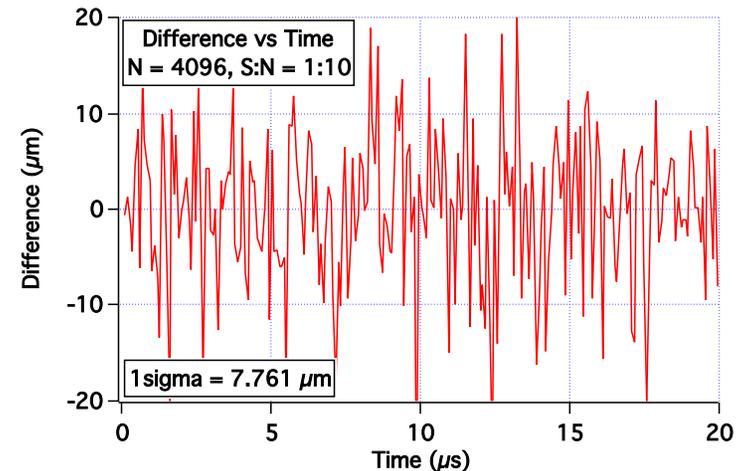
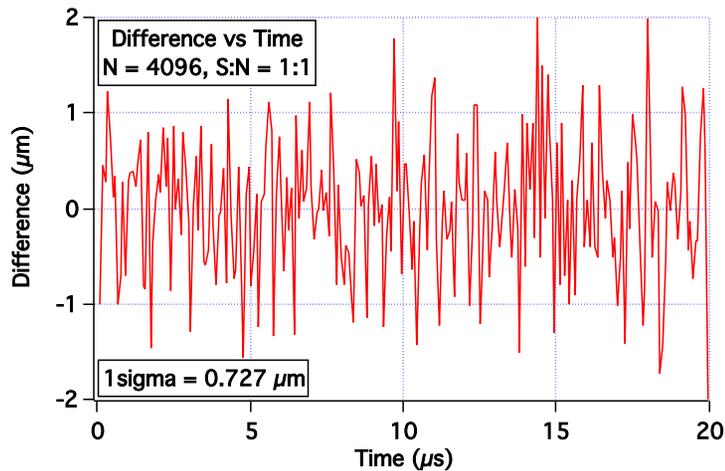
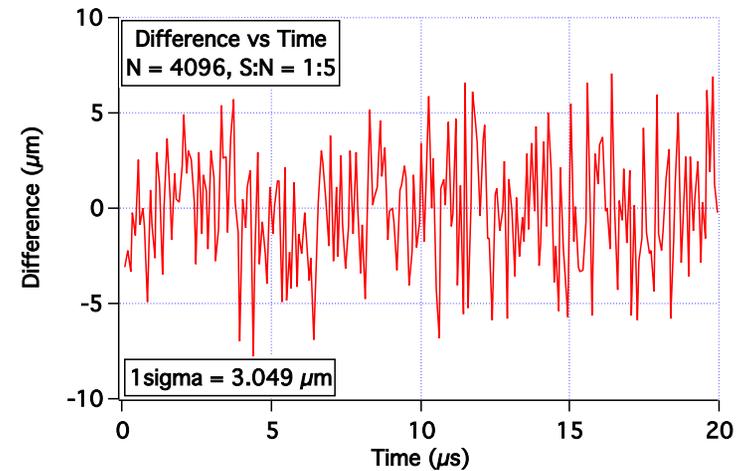
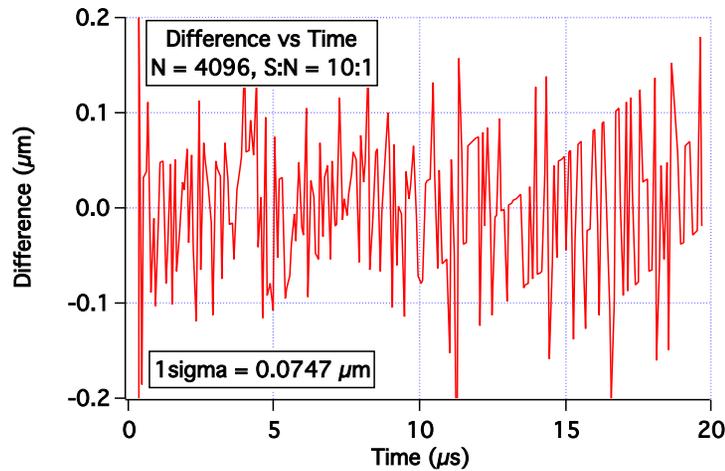
## Differences between extracted data and quadratic fit to extracted data ( $\langle \text{mean} \rangle = 0$ ) N = 2048, Step Size = 40.96 ns



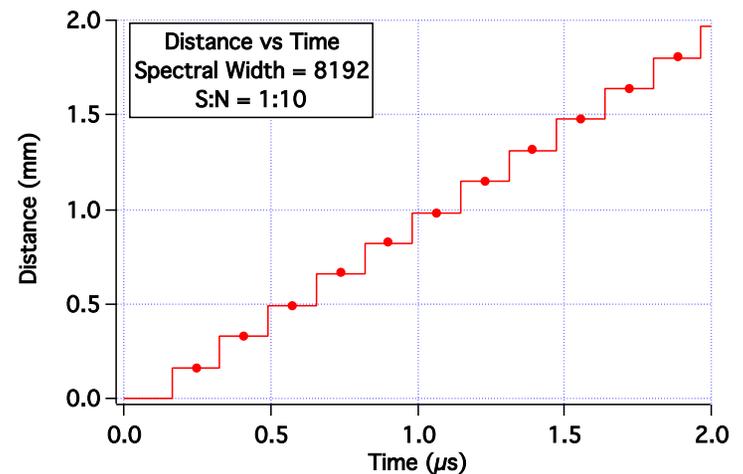
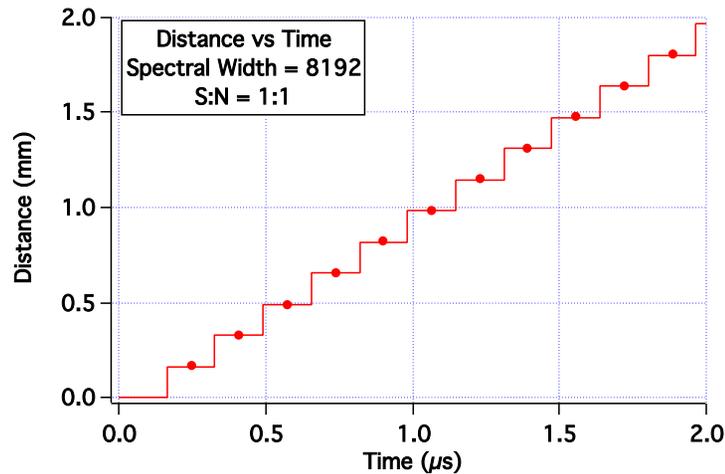
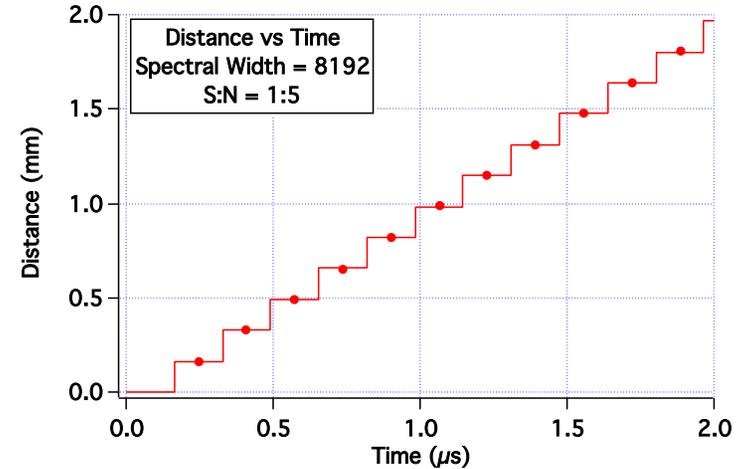
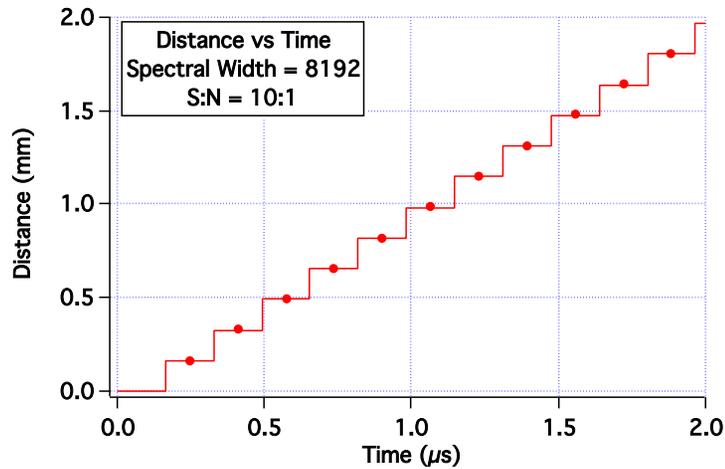
## Distance results for extracted data N = 4096, Step Size = 81.92 ns



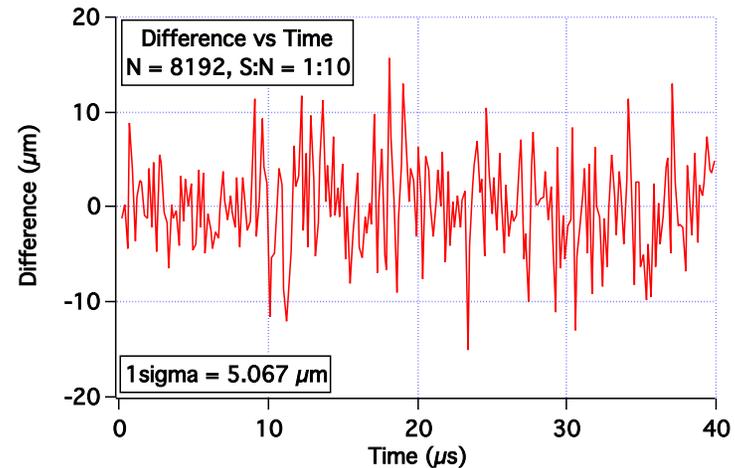
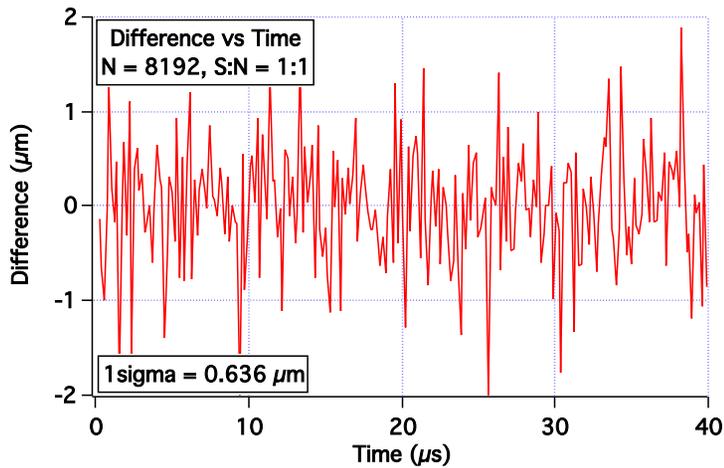
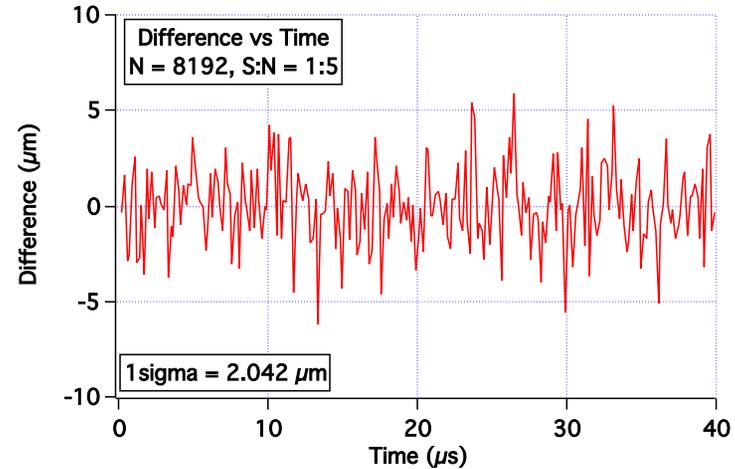
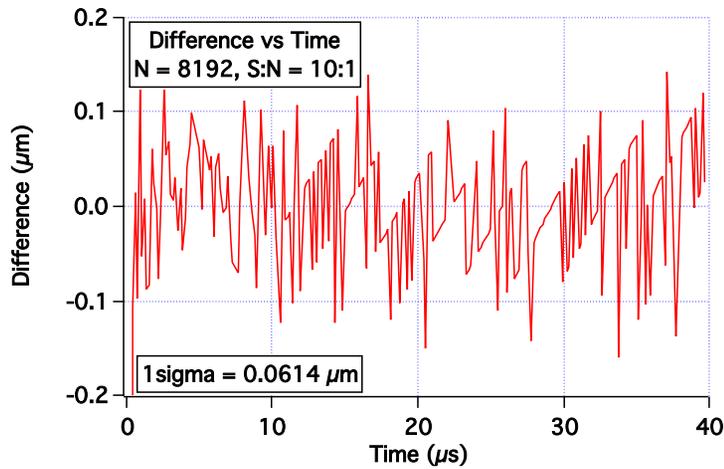
## Differences between extracted data and quadratic fit to extracted data ( $\langle \text{mean} \rangle = 0$ ) $N = 4096$ , Step Size = 81.92 ns



## Distance results for extracted data N = 8192, Step Size = 163.84 ns



## Differences between extracted data and quadratic fit to extracted data ( $\langle \text{mean} \rangle = 0$ ) N = 8192, Step Size = 163.84 ns



## This summarizes the results of standard deviation vs. N(fft) and Noise Fraction

N		1024	2048	4096	8192
Step size (ns)		20.48	40.96	81.92	163.84
S:N	$\sigma$				
10:1	0.1	0.14	0.12	0.075	0.061
1:1	1	1.27	0.91	0.73	0.64
1:5	5	7.48	4.69	3.05	2.04
1:10	10	22.79	11.62	7.76	5.07

**Model Results**  
Matrix elements are  
1 standard deviation  
in  $\mu\text{m}$ .



## Analytic expression results and % error.

### Analytic Estimate

Matrix elements are position uncertainty in  $\mu\text{m}$ .

N		1024	2048	4096	8192
Step size (ns)		20.48	40.96	81.92	163.84
S:N	$\sigma$				
10:1	0.1	0.15	0.10	0.073	0.052
1:1	1	1.46	1.04	0.73	0.52
1:5	5	7.32	5.17	3.66	2.59
1:10	10	14.64	10.35	7.32	5.17

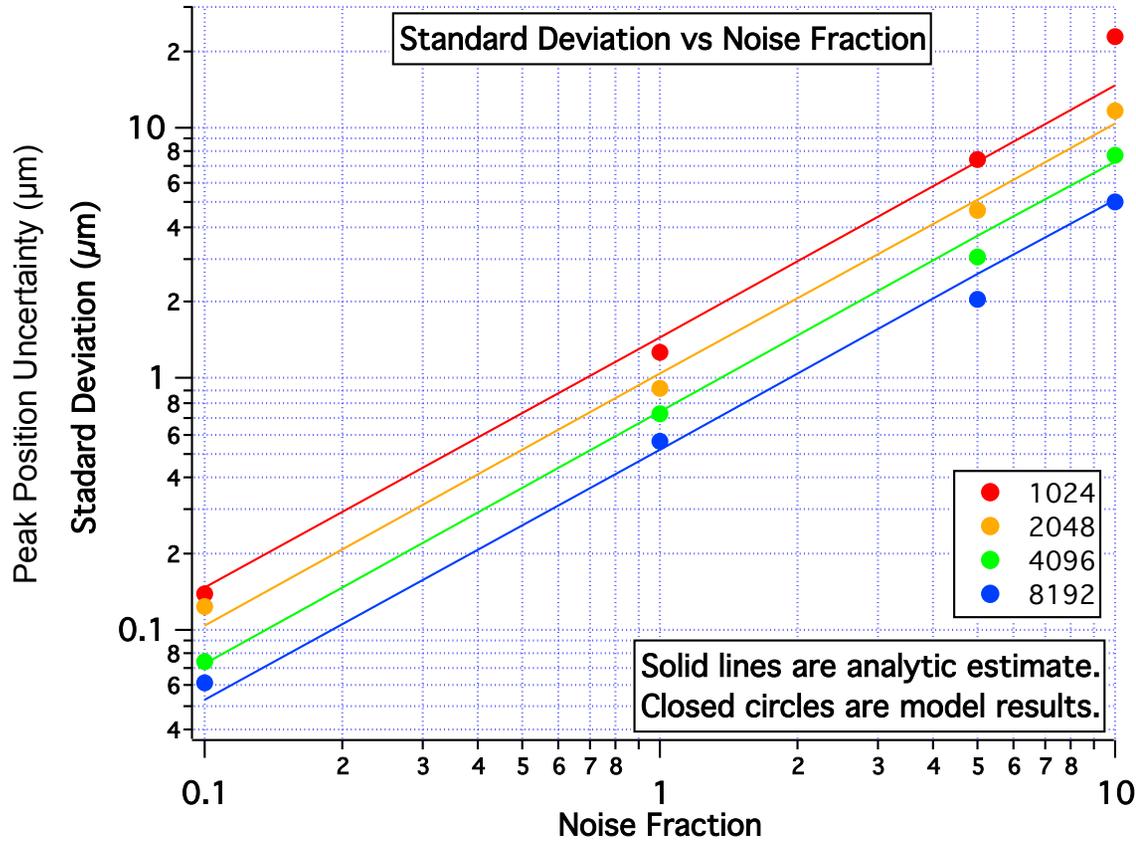
### % Error

Matrix elements are errors in %.

N		1024	2048	4096	8192
Step size (ns)		20.48	40.96	81.92	163.84
S:N	$\sigma$				
10:1	0.1	-5.0	18.9	2.1	18.6
1:1	1	-13.5	-12.5	-0.7	8.0
1:5	5	2.2	-9.3	-16.7	-21.1
1:10	10	55.7	12.3	6.1	-2.1



# This summarizes the results of standard deviation vs. N(fft) and Noise Fraction



Modeling agrees well with analytic estimate.

Model is high for N = 1024,  $\sigma = 10$ .



## Summary

La Lone/Dolan derived an analytic expression for uncertainty:

- Linear with noise fraction,
- Inverse as the square root of  $N(\text{fft})$ .
- Independent of distance.

Generated BLR waveforms using derived sensitivities.

- Added different amounts of noise.
- Processed with different values of  $N(\text{fft})$ .
- Calculated standard deviation of scatter in extracted data.

Standard deviations from this model are in good agreement with the analytic expression.

Did not consider chirp in this study.

