FROM RESEARCH TO INDUSTRY

THE LIMITS OF VELOCITY EXTRACTION FROM LOW-SNR SPECTROGRAM

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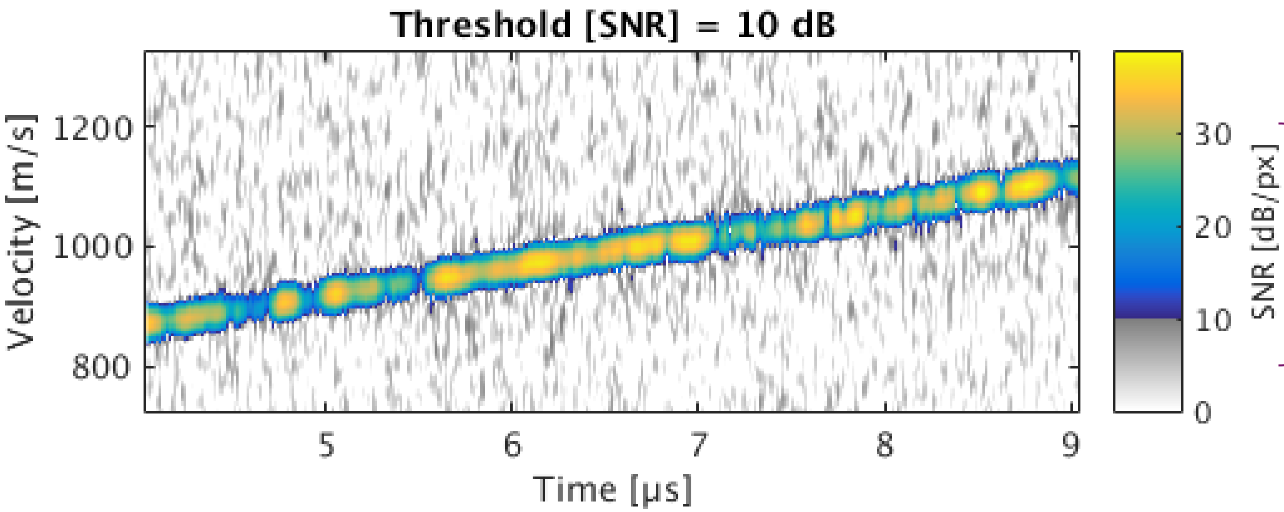
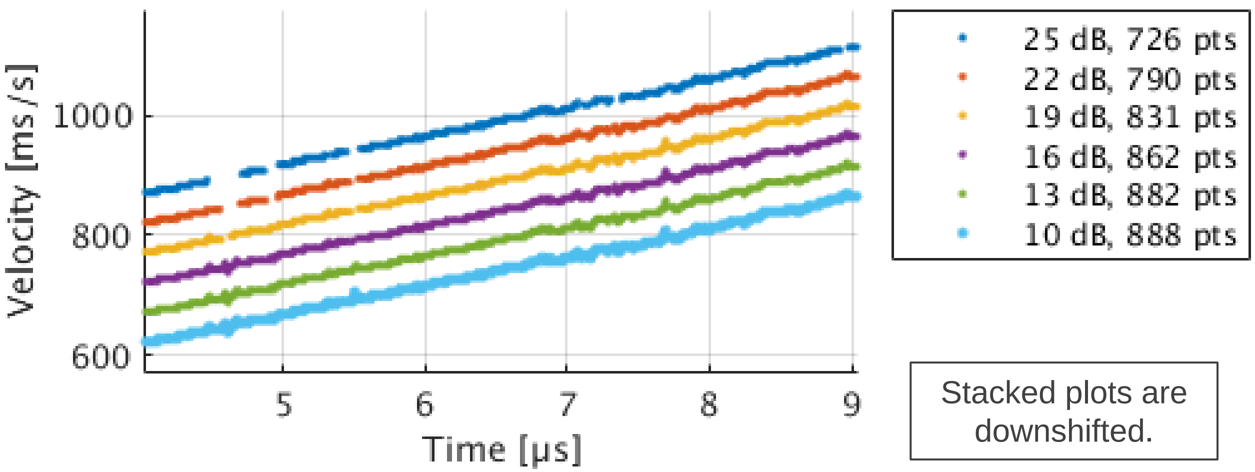
CEA, DAM, DIF

PDV USERS WORKSHOP, SANTA FE, MAY 2018

CONTEXT:
A GOOD SIGNAL-TO-NOISE RATIO (SNR), OPTIMAL SITUATION



PDV operator has to select a threshold to extract velocity.
Here: the threshold is sampled **between 10 to 25 dB (of SNR).**



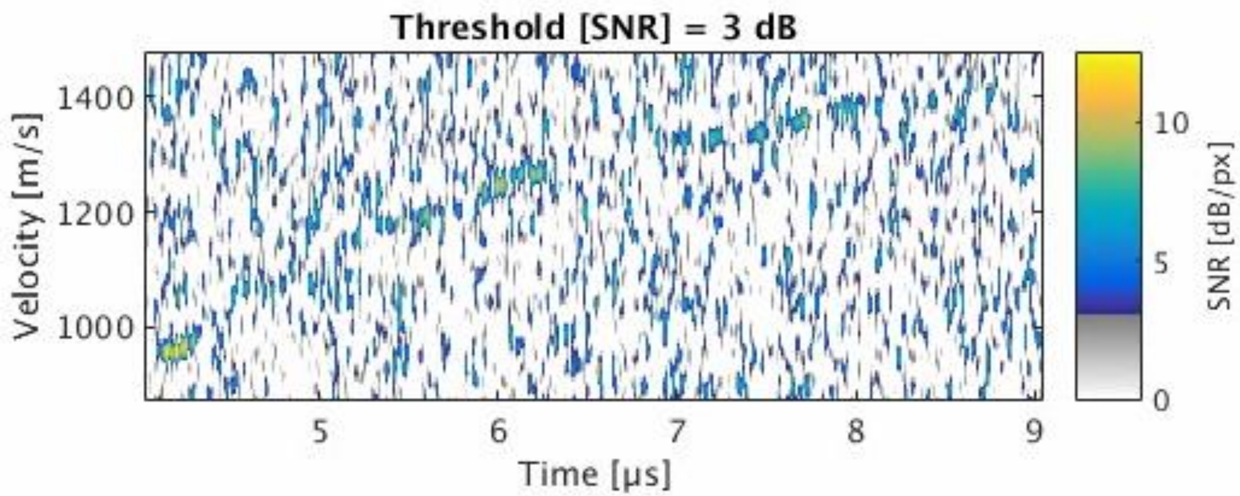
Stacked plots are downshifted.

Between 10 to 19 dB: no influence.

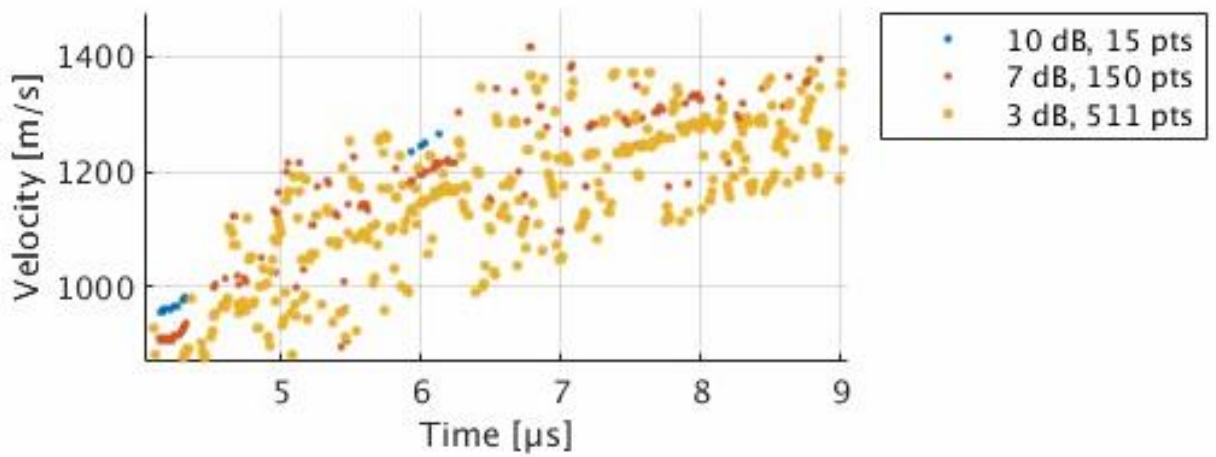
The choice of threshold is simple for the operator.

☺

CONTEXT:
THE OPPOSITE CASE, A VERY LOW SNR

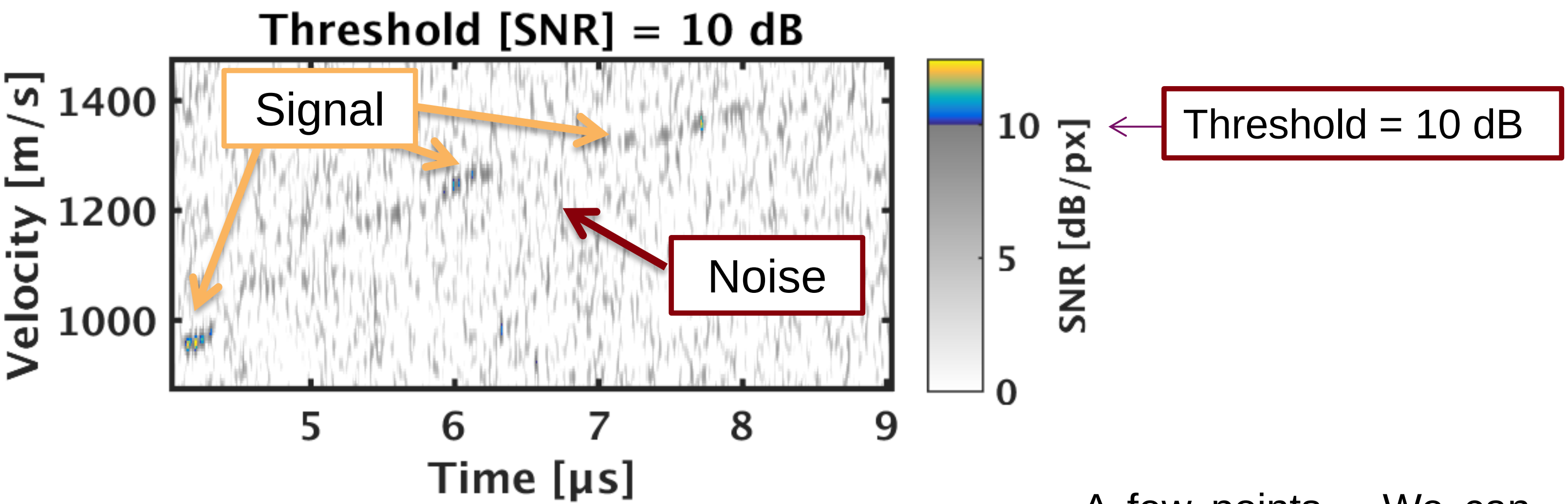


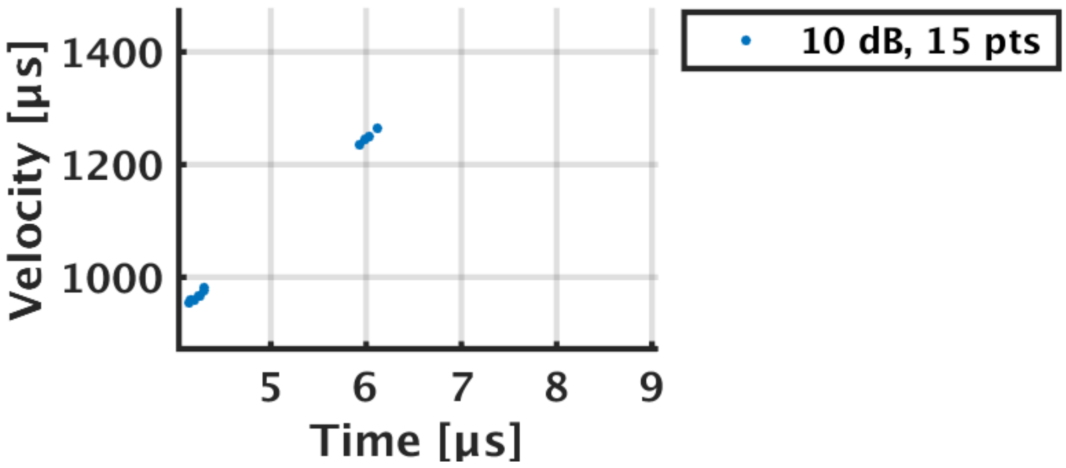
Here: the threshold is sampled **between 3 to 10 dB.**



A few points (10 dB)… or a lot of false-positives (3 dB).

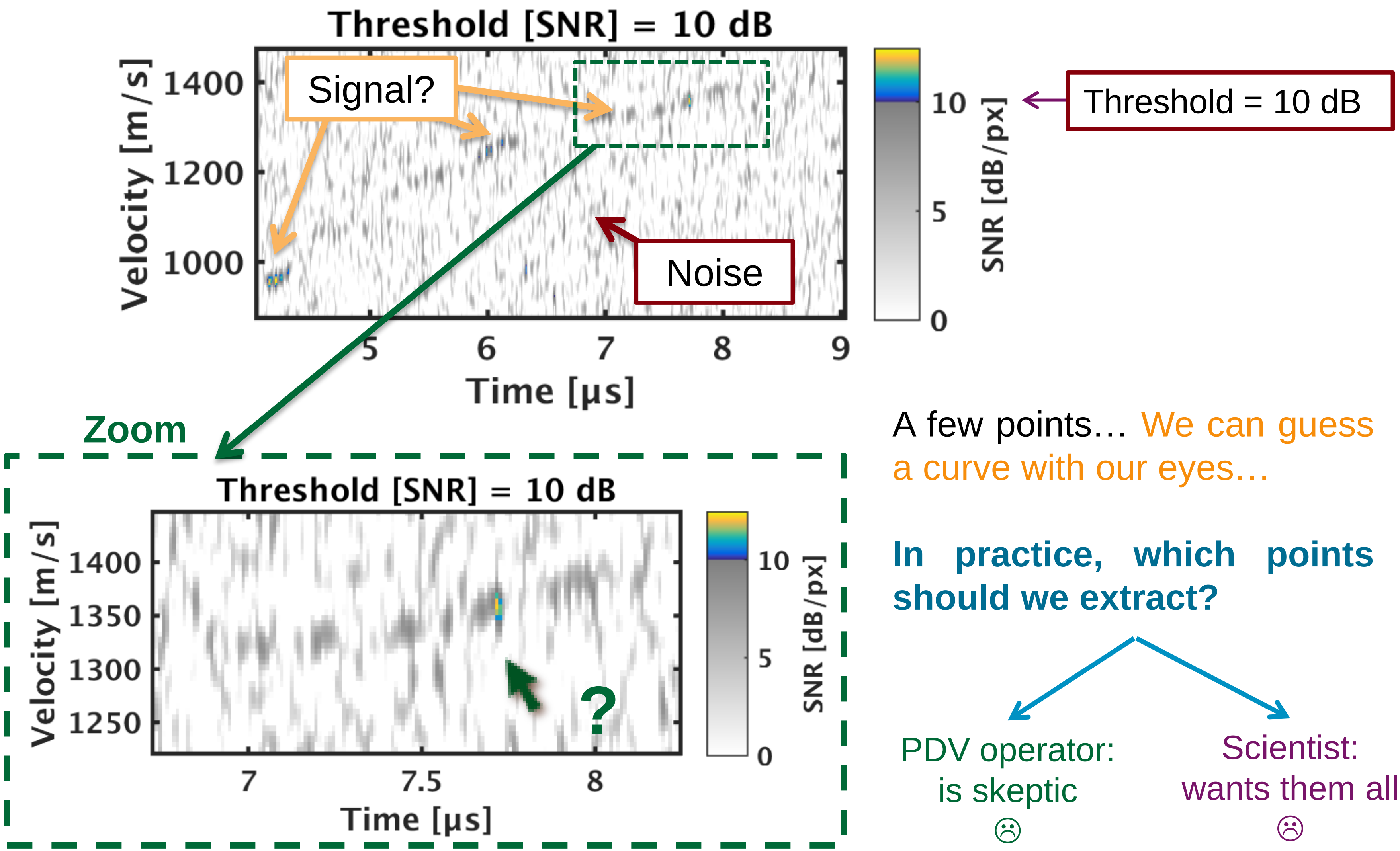
CONTEXT:
THE OPPOSITE CASE, A VERY LOW SNR, THRESHOLD = 10 DB





A few points… We can guess a curve with our eyes….

CONTEXT:
THE OPPOSITE CASE, A VERY LOW SNR, THRESHOLD = 10 DB



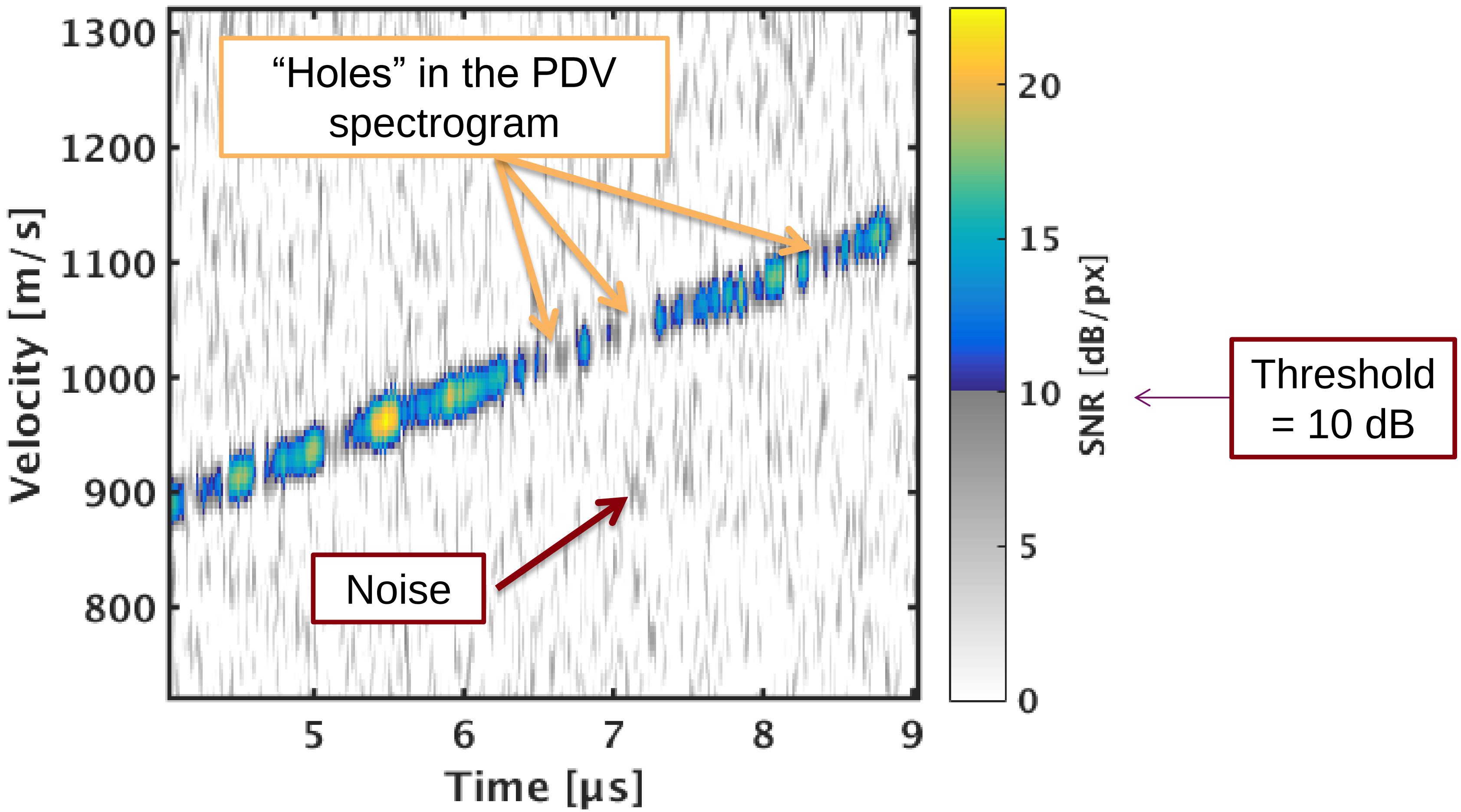
A few points… We can guess a curve with our eyes…

In practice, which points should we extract?

PDV operator: is skeptic ☹

Scientist: wants them all ☹

CONTEXT: LOW-SNR SITUATION



What is the optimal threshold to extract velocities?
This specific example will be used during my talk.

OUTLINE

- How risky is it?
  - ► False detection,
  - ► Uncertainties.

- How to extract velocity as a function of time for low-SNR spectrograms?

THE SIGNAL-TO-NOISE RATIO (SNR) OF A SPECTROGRAM

- In this talk, we define spectrogram as the amont of collected optical power by the PDV system[(1)].

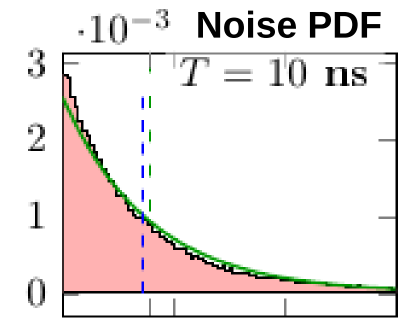
- **The SNR of the spectrogram is:**

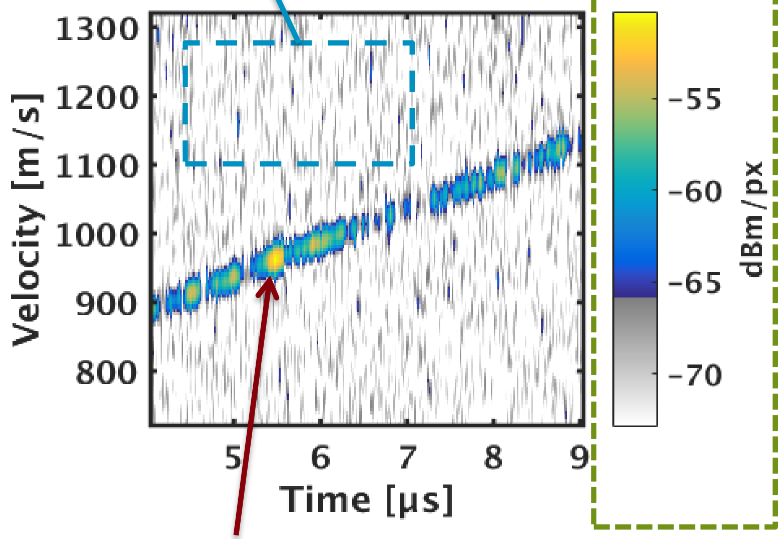
$$\mathrm{SNR_{dB}} = S_{\mathrm{dB}} - B_{\mathrm{dB}} = 10\,\log_{10}\left(\frac{S}{B}\right)$$

- **Extracted velocities are obtained by the local maximum of the spectrogram.**
  - ▶ Other methods exist (barycenter, interpolation, …).
  - ▶ We do not deal with them in this talk.

The background noise follow a exponential PDF.

B: mean value.





S: signal level.

Values are expressed in dBm.

(1) G. PRUDHOMME et al., AIP Proc., 2014

NOISE ON THE SIGNAL

- For a Noisy signal, with one velocity:

$$\boldsymbol{s(t) = s \cdot \cos(2kvt + \phi) + \underbrace{b_{\text{signal}}(t)}_{\text{noise}}}$$

- **The link between the SNR of signal and the one of the spectrogram is:**

$$\mathbf{SNR}_{\mathbf{spectrogram,dB}} = \mathbf{20}\ \log_{\mathbf{10}}\left(\frac{\boldsymbol{s}}{\boldsymbol{b}_{\mathbf{signal}}}\right) + \mathbf{10}\ \log_{\mathbf{10}}(G_i\ \boldsymbol{f}_{\mathbf{sampling}}\ \boldsymbol{W})$$

- **Short-Term Fourier Transform (STFT) increases the SNR.**
  - ► $G_i = 0.326$ is the incoherent gain of the padding Window,
  - ► f_{sampling} = 50 GHz,
  - ► $W = 50$ ns is the full width of the STFT window.

ESTIMATION OF THE PROBABILITY TO GET A FALSE POSITIVE

- The noise PDF on a PDV spectrogram is[2]:

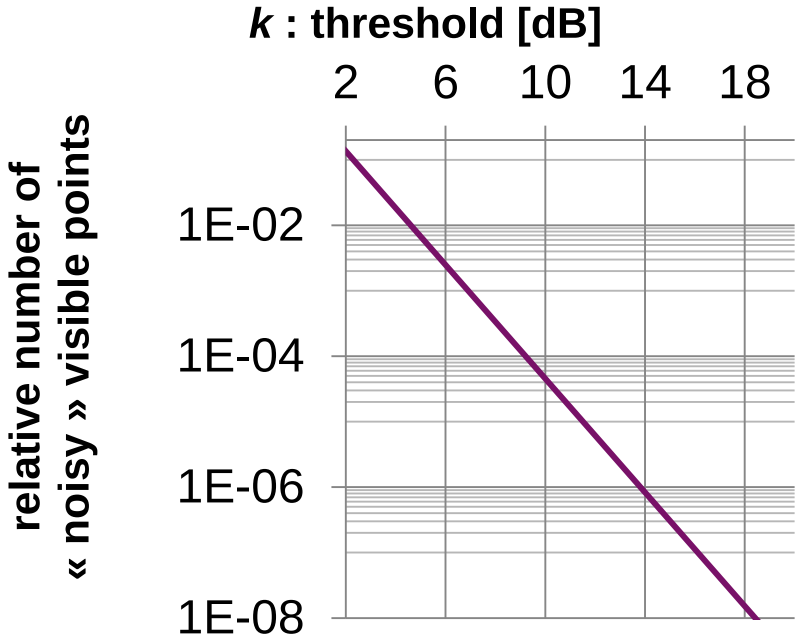
$$\mathbb{P}: s \mapsto \frac{1}{B}\exp\left(-\frac{s}{B}\right)$$

- False-positive probability with a threshold equal to $k \times SNR$:

$$F(k) = \int_0^{\text{threshold}} \mathbb{P}(s)\,\mathrm{d}s = \exp(-k)$$

- Example with $\text{threshold} = 10 \times SNR$, $k = 10$:

$$F(\text{threshold} = 10) = 6 \cdot 10^{-6}$$



(2) G. PRUDHOMME, PDV Workshop, 2016

HOW MANY FALSE POSITIVES?

- **Example with threshold $= 10 \times SNR$, $k = 10$: $F(10) = 6 \cdot 10^{-6}$.**

- Let's suppose that the operator selects a Region of Interest (ROI) on the spectrogram with a margin of ± 150 m/s.

20 px/columns: the probability to get a false positive (or more) in one column is:

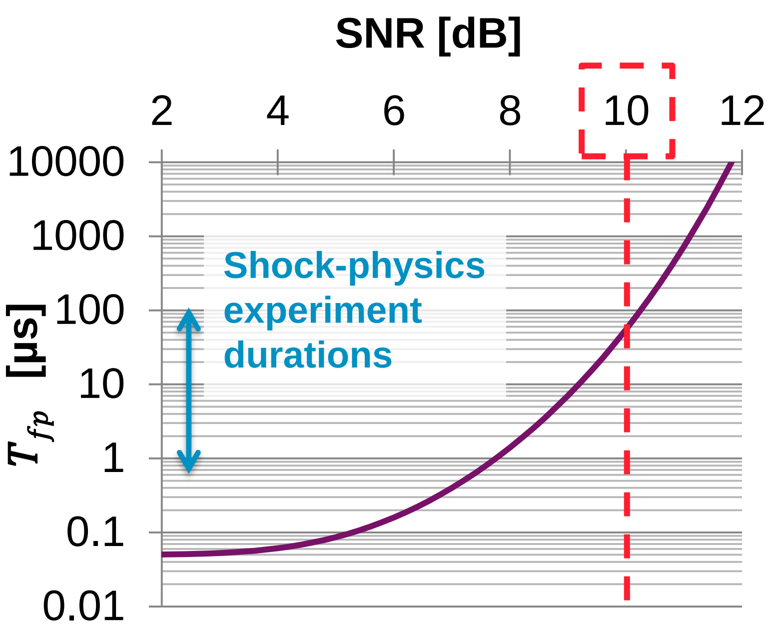
$$\mathbb{P}_2 = 1 - \prod_{i=1}^{20} \left(1 - F(10)\right) = 1.2 \cdot 10^{-4}$$

- The average time $\overline{T_{fp}}$ to have a false positive is:

$$\overline{T_{fp}} = \frac{W}{\mathbb{P}_2} = 420\ \mu s$$

$\overline{T_{fp}} \gg \text{experiment}$: very few false positives.

Average time to get one false positive (Number of pixels by column: 20)

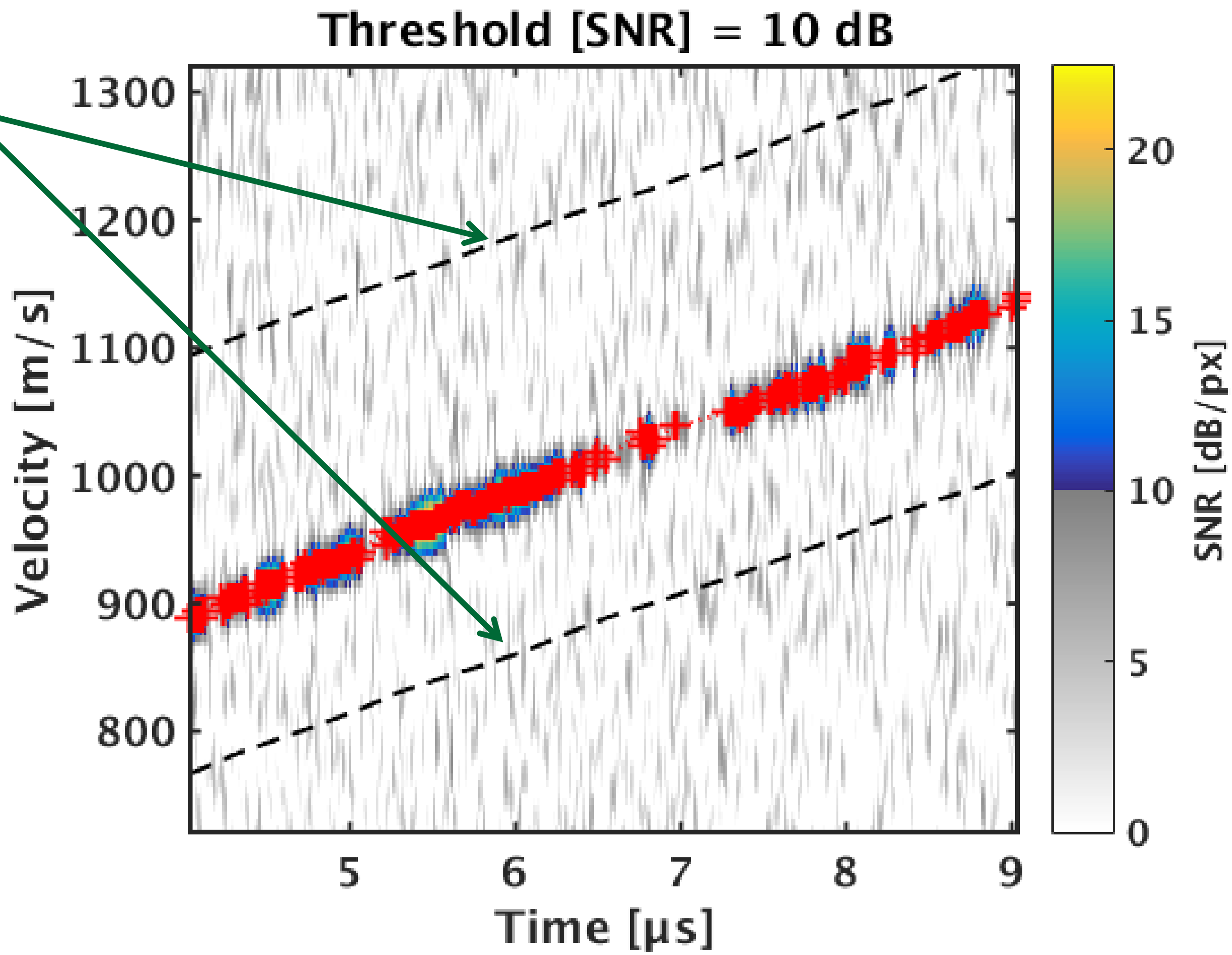


- $SNR \geq 10$ is required.
- $SNR \geq 7$ induced a false positive every 400 ns (in average):
  - ➢ **points must be filtered.**

EXAMPLE WITH *THRESHOLD* = 10 dB

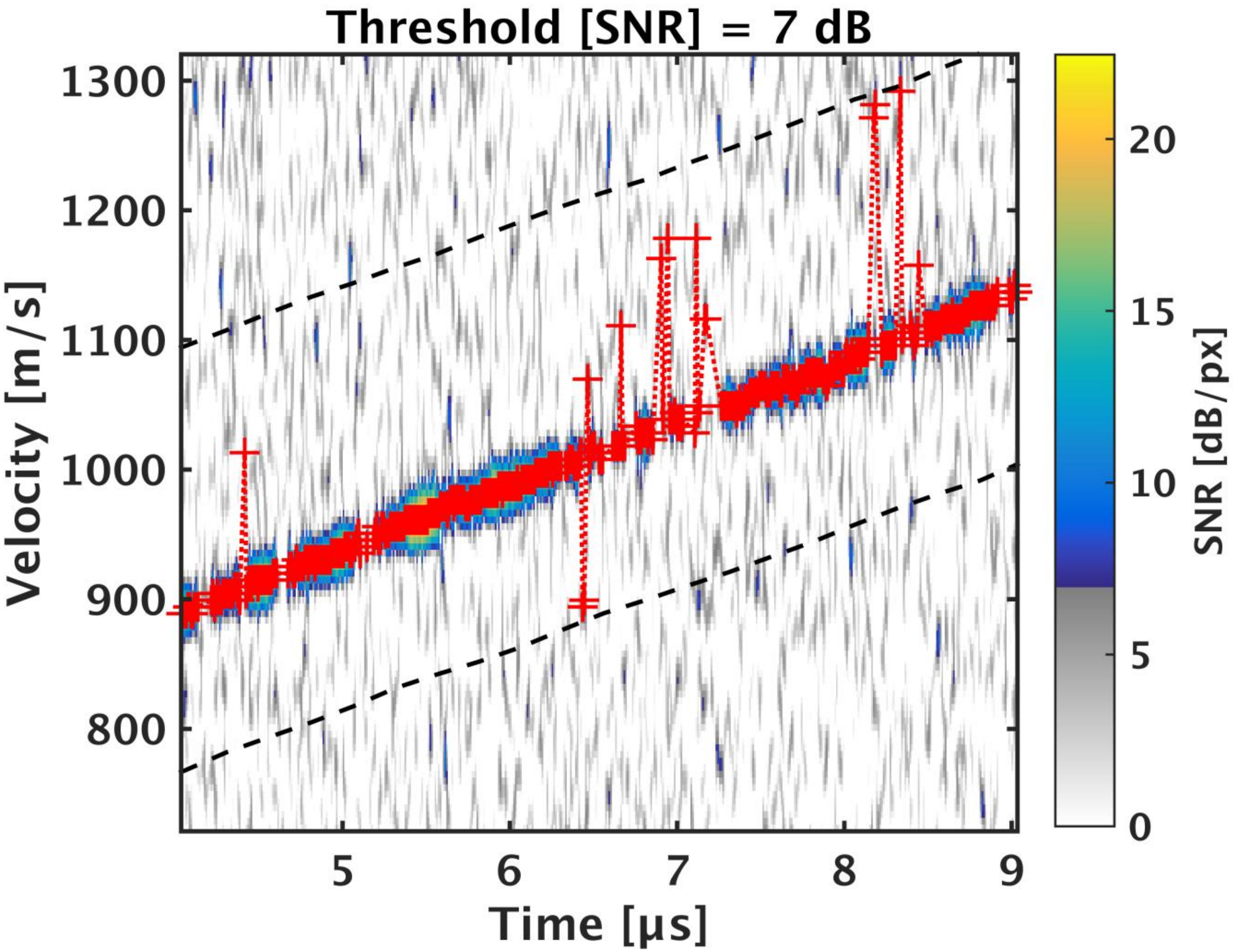
The region of Interest (ROI) could be:

- defined by a humain operator.
- derived from a simulation.
- derived from a first estimation of the velocities.



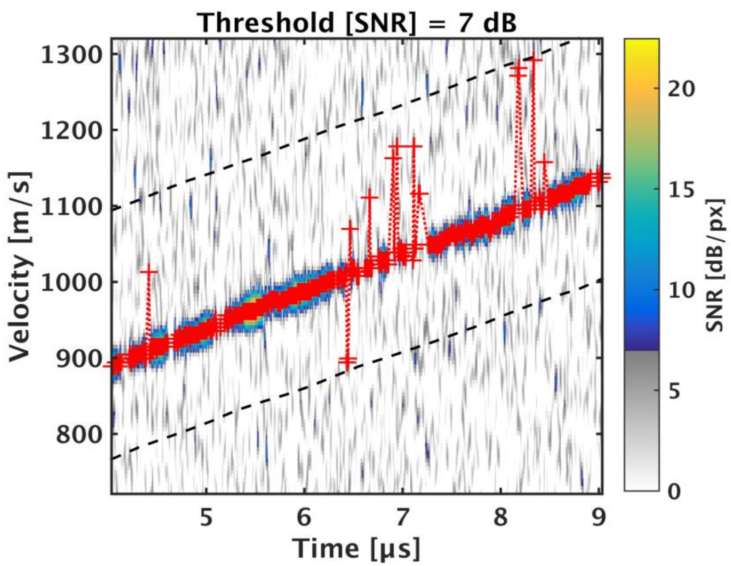
For a threshold of 7 dB, **the extracted velocities seem to be valid.**

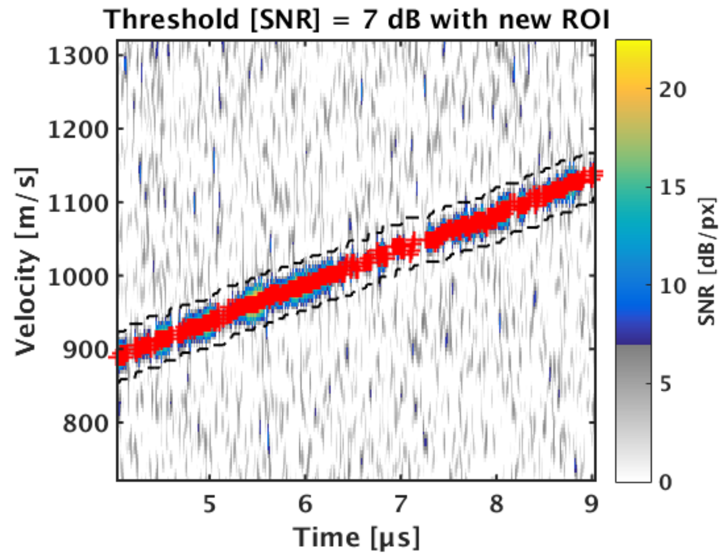
EXAMPLE WITH *THRESHOLD* = 7 dB



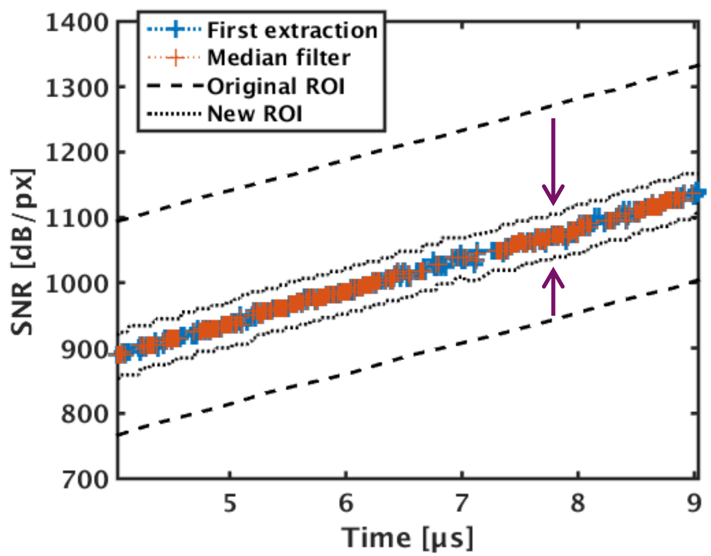
For a threshold of 7 dB, **several false positives appear.**

EXAMPLE OF ONE METHOD TO SUPPRESS SOME FALSE POSITIVES





- A median filter(*) is used on velocities (red curve, width: $4W$).
- The filtered velocities defined a new, thiner, ROI.
- This new ROI is use to extract velocities again.

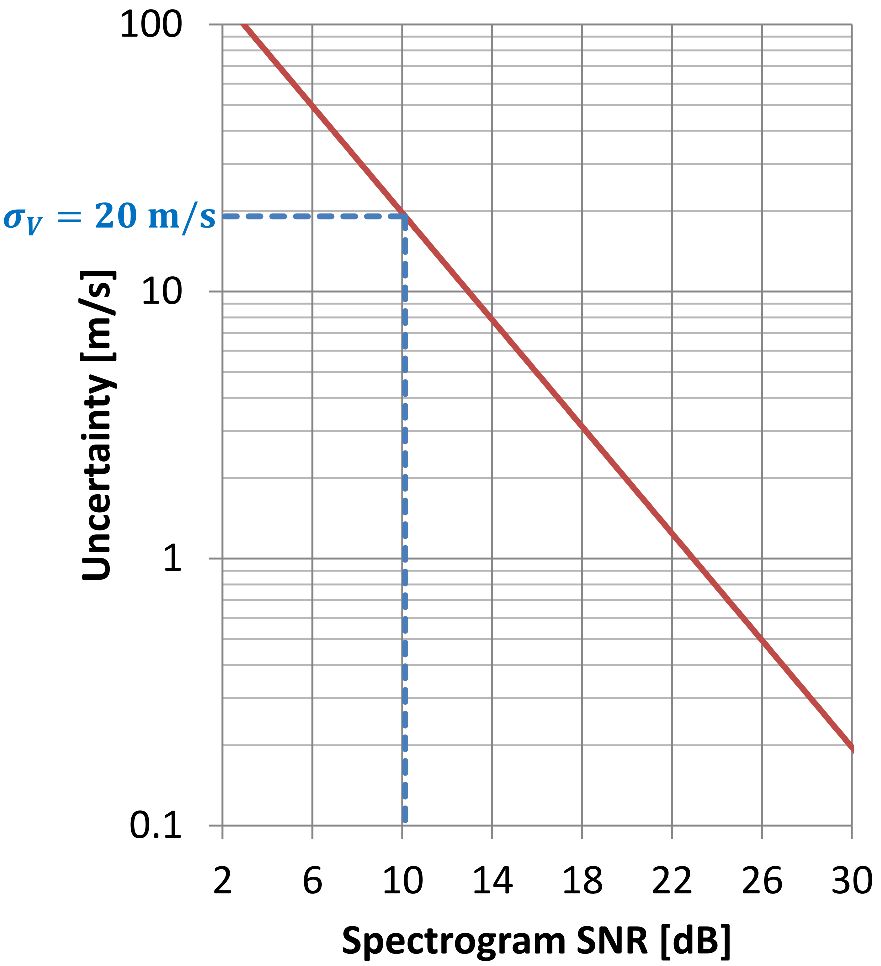


(*) *Median filter is robust again shock breakout.*

UNCERTAINTY INDUCED BY LOW-SNR

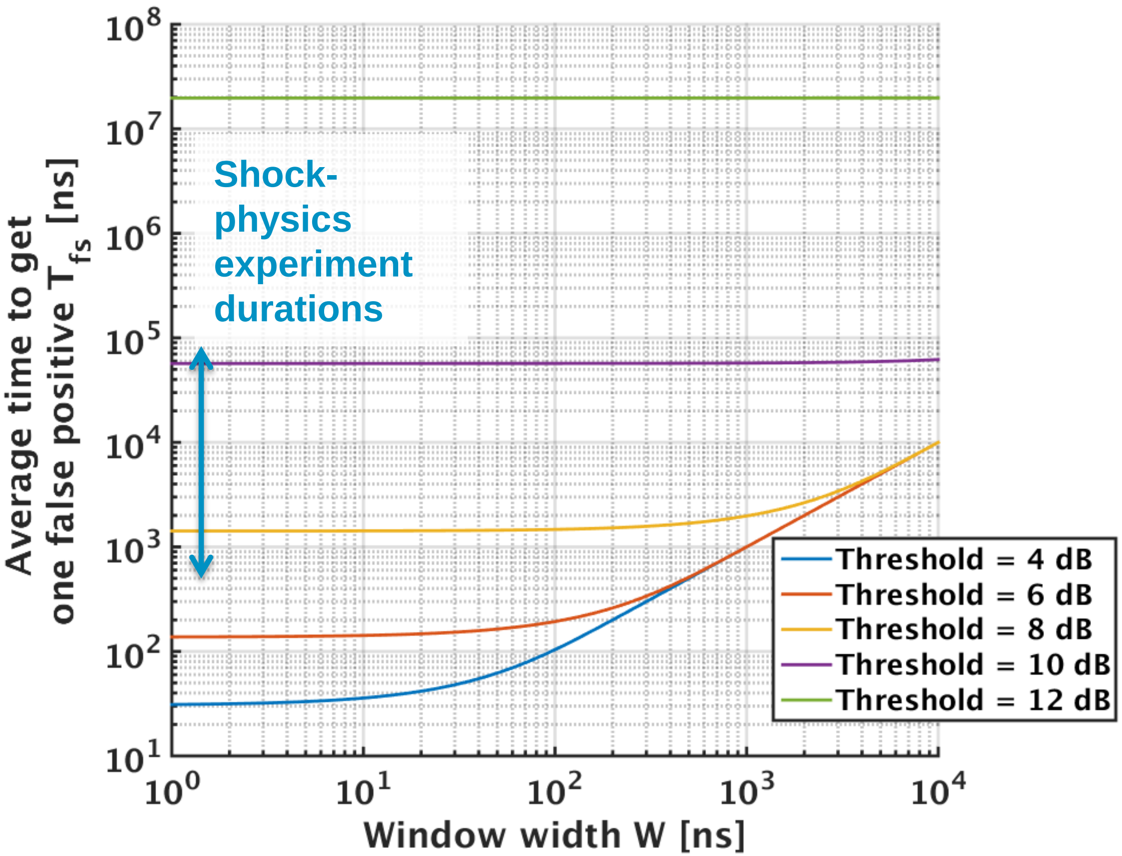
- This model is taken from D. H., Dolan[3] for the the Top-Hat Window case, and extrapolated for $SNR_{\text{signal}} < 3\ dB$.

- For an a threshold of 10 dB, we get a uncertainty larger than the pixel size :
  - $\sigma_V > \delta V = 15\ \text{m/s}$
  - *What is the meaning of this maximum-searching extraction if the uncertainty is higher than the pixel size?*



(3) D. H. Dolan., RSI, vol. 81, 053905, 2010

THE EFFECT OF THE WINDOW WIDTH *W*



- Fourier Windows has to be large enough (> 100 ns) to have a significative impact on the number of false positives.
- In most of the cases, a threshold above 10 dB is <u>still</u> required.

SUMMARY

- Extracting velocities from low-SNR (< 10 dB) data ($W = 50\,\mathrm{ns}$) induces:
  - Higher uncertainties, larger than the velocity sampling, *(maximum searching)*,
  - A large number of false positives.

- In this case, *explicit (software)* or *implicit* (our brain, our eyes: human factors) *a priori* is required.

- **A reasonable threshold seems to be at least 10 dB** (for a local maximum)**.**

- **Related question: why the amplitude of PDV signals is so fluctuating?**

  Speckle[4], polarization[5], …

(4) E.A. MORO, J. Phys Conf. Ser., 500, 142023
(5) J-E. FRANZKOWIAK et al., PDV Workshop, 2016

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BACK-UP SLIDES

POSSIBLE SOURCES OF FLUCTUATIONS (1/3): SPECKLE

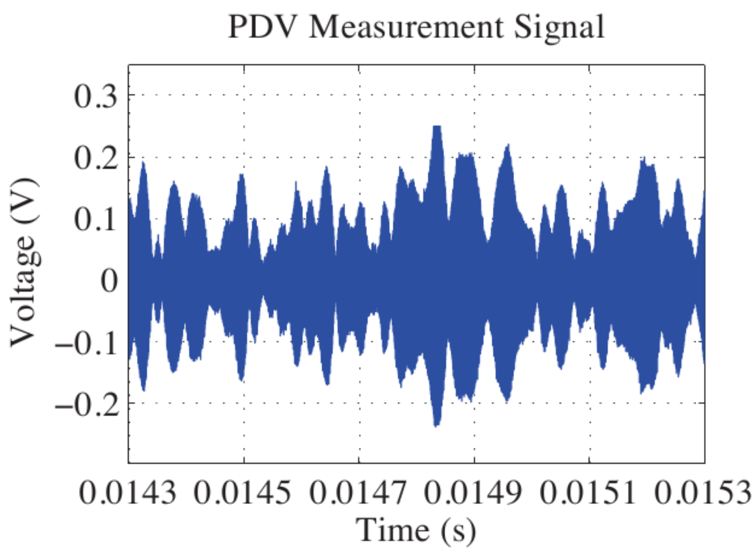


Figure 5. Speckle results in amplitude fluctuations in the measured PDV data.

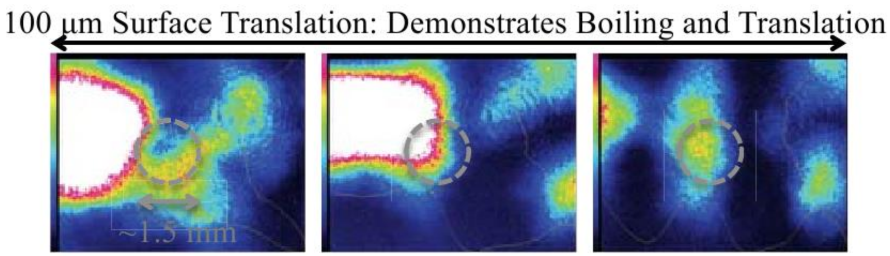


Figure 6. Speckle boiling is shown as a result of surface translation over 100 micrometers. The dashed circle indicates a region 1.5 mm in diameter, which indicates the probe's aperture at the image plane.

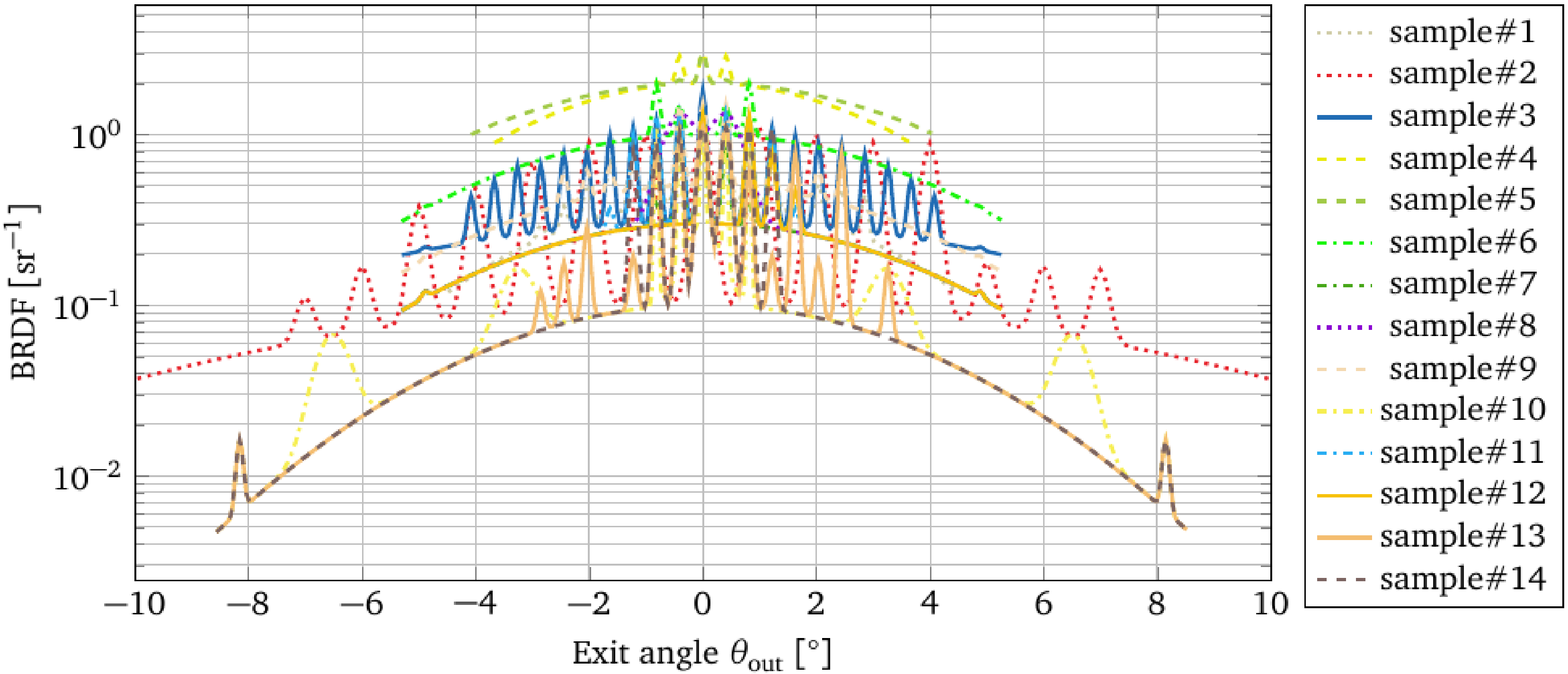
Measurements made by E.A. MORO and M.E. BRIGGS (5).

POSSIBLE SOURCES OF FLUCTUATIONS (2/3):
BRDF (BIDIRECTIONAL REFLECTANCE DISTRIBUTION FUNCTION)

- Examples of BRDFs for different materials (Al, Cu, Sn, Iron, …) and roughness.

(θ_{out} measured at $\lambda = 632$ nm, extrapolated for $\lambda = 1.55$ µm)

- BRDFs can vary up to 20 dB, for a few degrees inclination.

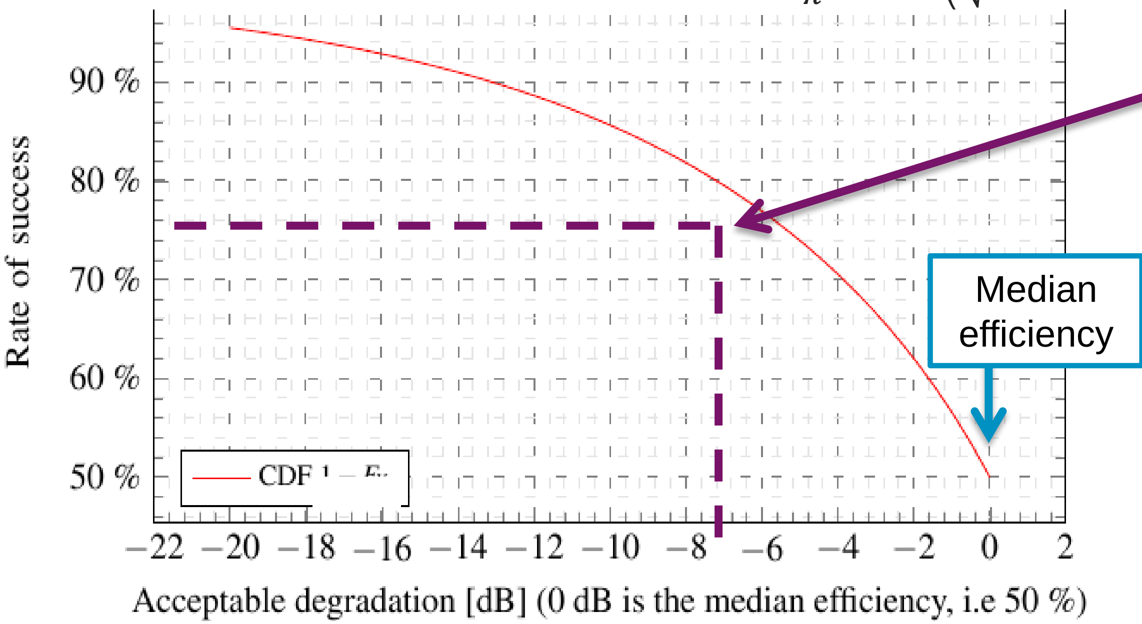


POSSIBLE SOURCES OF FLUCTUATIONS (3/3): THE ISSUE OF POLARIZATION WITH PDV

$$\varphi(t,v) = \frac{1}{2\,\mathcal{R}_{sys}^2\, P_{LO}\, \underbrace{\cos^2\theta_p}_{\text{pola.eff.}}} \frac{1}{\widehat{G}_i\, N^2} \left|\text{FFT}(U_e \cdot W^N)\right|^2_{f=\frac{2v}{\lambda}}$$

- The polarization efficiency could reduce the spectrogram by a factor $\cos^{-2}\theta$, with $\theta \sim \text{Uniform Distribution}(-{}^{\pi}/_{2}, +{}^{\pi}/_{2})$.
- The CDF of how many realizations are above than an “acceptable loss of SNR level” is:

$$CDF(level) = 1 - \frac{2}{\pi}\text{Arcsin}\left(\sqrt{{}^{level}/_{\text{max. level}}}\right)$$



In order to “ensure” 80% of PDV mesurements, we need to accept a reduction of 7.2 dB of the SNR.

This might explain why the visibility on PDV spectrogram of particle cloud is so variable.

*CDF: Cummulative Distribution Function

LINK BETWEEN THE INCOHERENT GAIN AND THE FULL-WIDTH OF THE FOURIER WINDOW

- $G_i = \tilde{G}_i W$
  - ▸ W is the full-width
  - ▸ G_i is the incoherent gain[(6)] of the window.
  - ▸ $\tilde{G}_i$ is a correction factor, which depends on the Window kind:
    Top-hat: $= 1$,
    Other window: ≈ 0.3.

- In this talk:
  - ▸ $\lambda_{\mathrm{PDV}} = 1.550\ \mu\mathrm{m}$,
  - ▸ $W = 50$ ns,
  - ▸ $\tilde{G}_i = 0.326$,
  - ▸ No zero-padding.
    Velocity sampling: $\delta_v = 15.5$ m/s.

Window name	$\hat{G}_i$
Rectangle	1,000
Hann	0,375
Hamming	0,397
Minimum 3s Blackman-Harris	0,306
3s Blackman-Harris	0,326
Minimum 4s Blackman-Harris	0,258
4s Blackman-Harris	0,290
Flat-Top *MS-5FT*	0,171

(6) F.J. HARRIS, Proc. IEEE., vol. 66, 1, 1978