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A Study of PDV Data: Errors and Uncertainties versus Acceleration and Noise

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Introduction

Topics

Consider a range of accelerations and noise in PDV data.

Look at errors and uncertainties in velocity.

Look at errors and uncertainties in acceleration.

Method

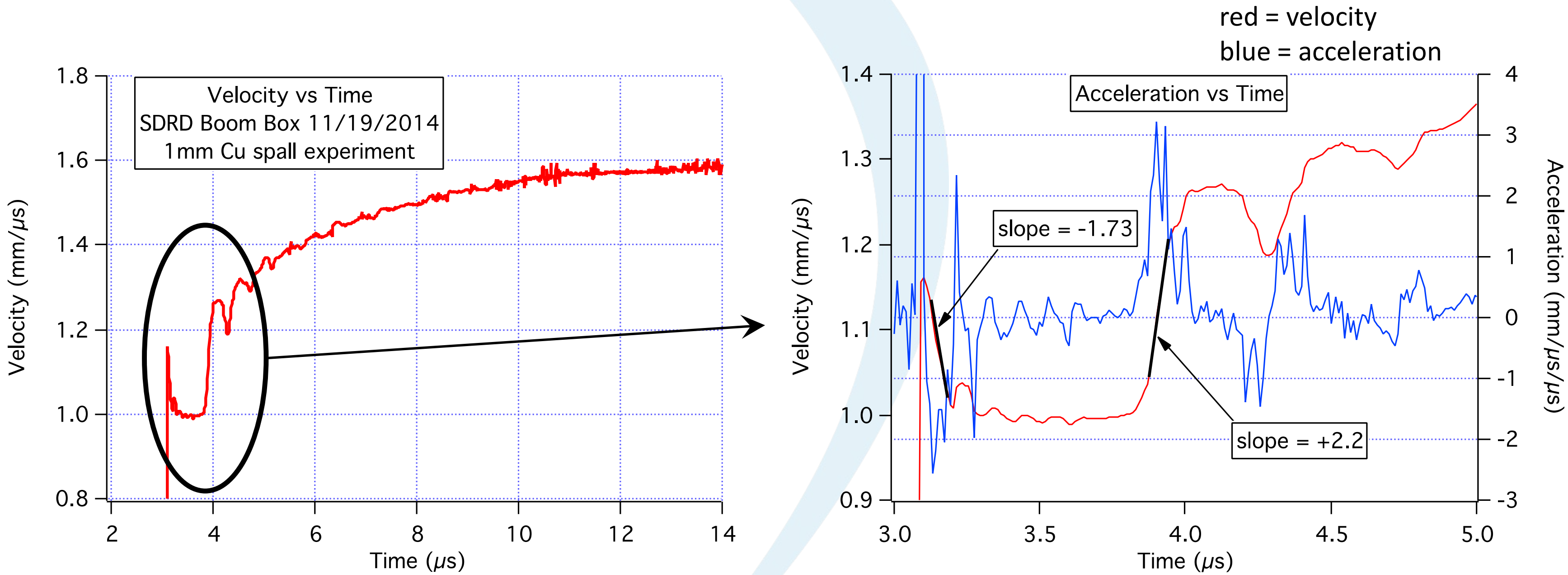
Determine a range of accelerations in actual shock physics experiments.

Build velocity profiles with appropriate ranges of acceleration.

Build beat waveforms with different amounts of noise.

Perform Fourier transforms, extract data, and compare to input values.

Look at range of accelerations in HE-driven data

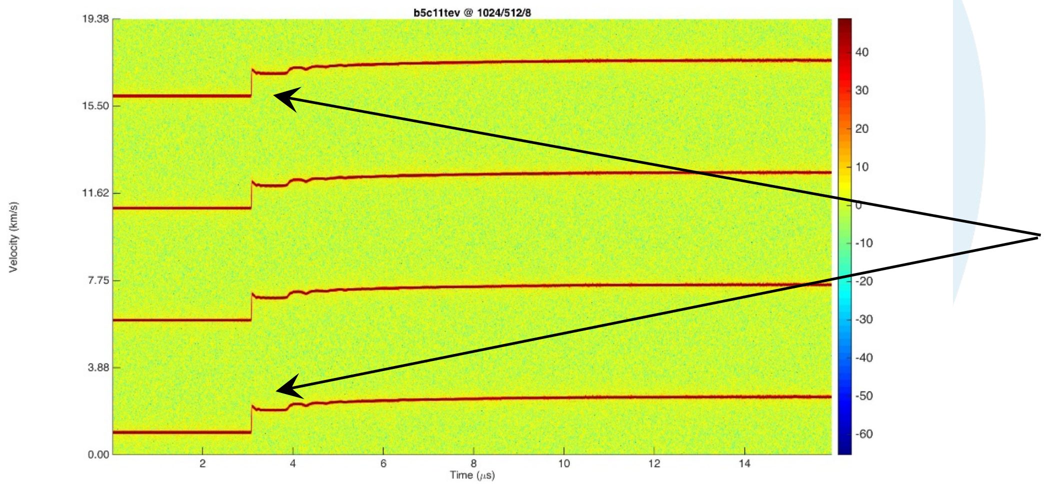


Accelerations range from –2 mm/μs/μs to +2 mm/μs/μs.
Linear fits to velocity look good ≥ constant acceleration.

Many thanks to Ed Daykin (MSTS) for permission to use this data.

Other considerations for this study

Data could be shifted to different velocities in the spectrogram (frequency multiplexing)



Are the errors and uncertainties in the upper part of the spectrogram the same as in the lower part?

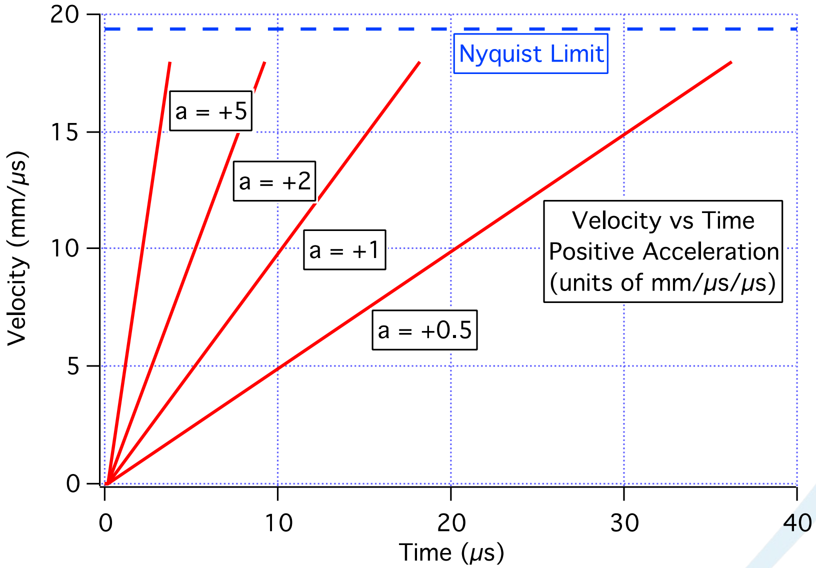
Summary: Acceleration range of –2 to +2 mm/μs/μs.
Constant acceleration.
Data could be frequency shifted.

Conclusion: Choose accelerations that range from –5 to +5 mm/μs/μs.
Limit study to constant accelerations.
Examine entire velocity range from 0 to near Nyquist.

Build velocity profiles with constant accelerations (a = –5, –2, –1, –0.5, +0.5, +1, +2, +5 mm/µs/µs)

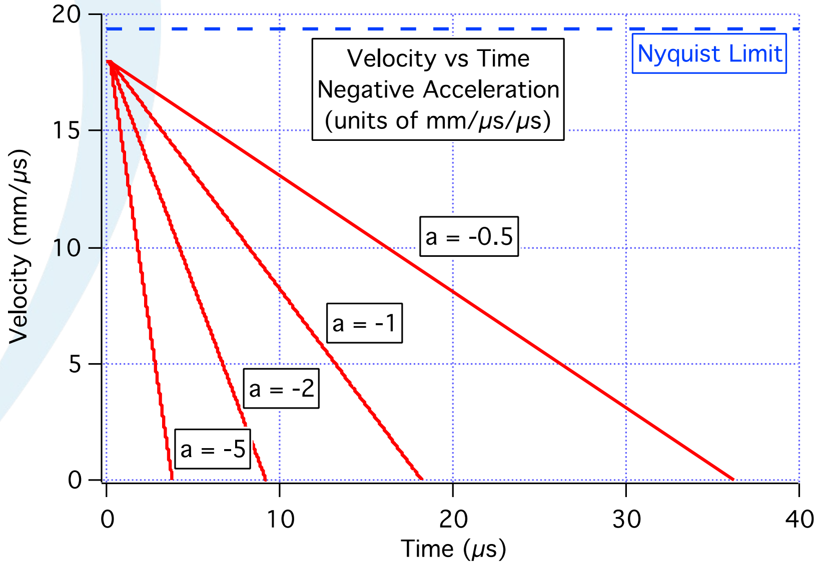
Accelerations have units of mm/µs/µs = 10^9 m/s^2 = 10^8 g

Positive Accelerations



initial velocity = 0 mm/µs
final velocity = 18 mm/µs

Negative Accelerations



initial velocity = 18 mm/µs
final velocity = 0 mm/µs

(Nyquist limit is 19.375 mm/µs for digitizer sample rate = 50 GS/s.)

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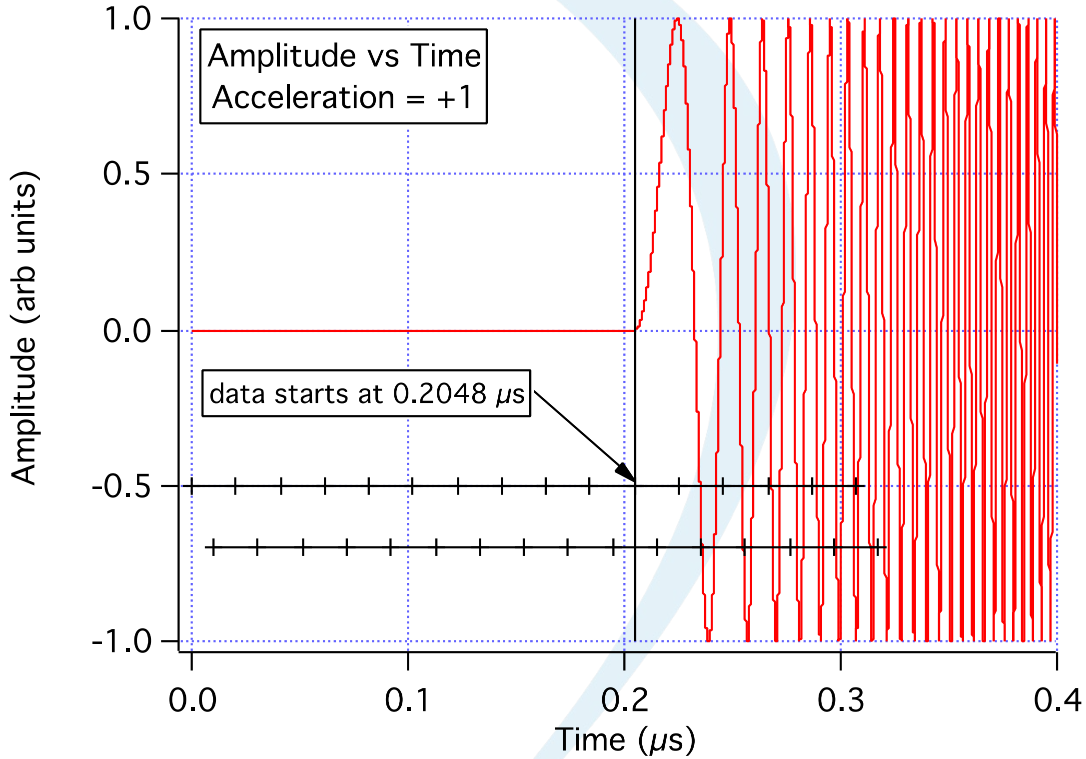
Build beat waveforms using 8 different accelerations



Beat waveform parameters:

50 GS/s = 20 ps/pt

Lambda = 1554.13 nm (ITU29)

Process data using 1024/512 = 0.02048 μs FFT windows

Use QuickView to process data.

Start data at 0.2048 μs (21st FT window)

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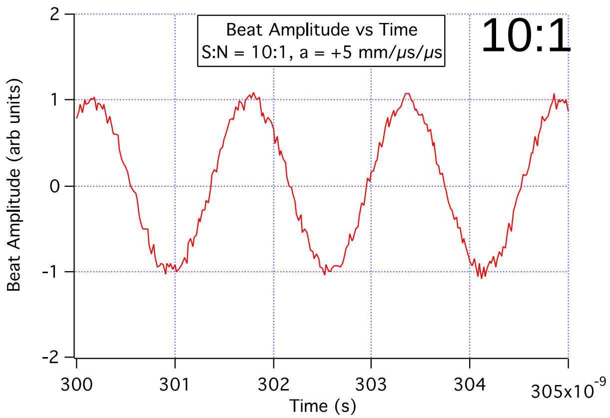
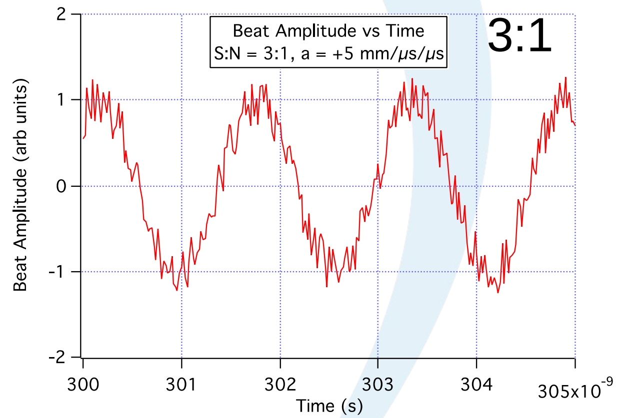
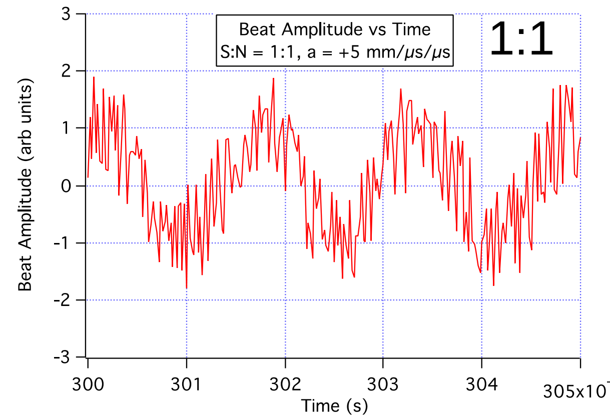
And 5 different SNR values = 10:1, 3:1, 1:1, 1:3, 1:10

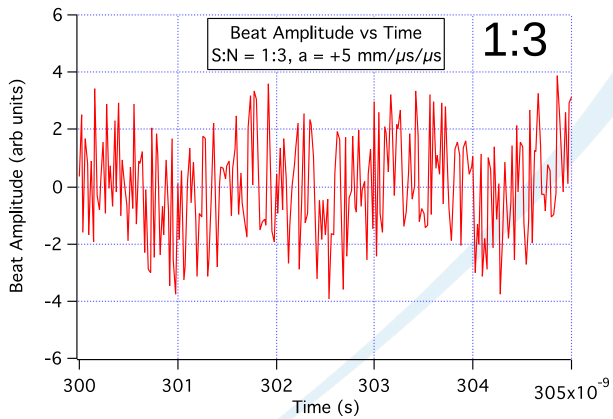
These beat waveforms are for acceleration = +5 mm/µs/µs.

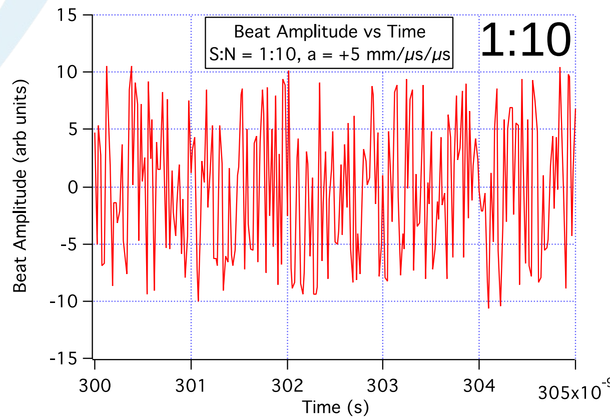
Igor: b01ev = sin(2*pi*d01ev*1e6/777.067)+enoise(0.1)











We have 8 accelerations and 5 SNRs for each study

Velocity Error
Velocity Uncertainty
Acceleration Error
Acceleration Uncertainty

Accelerations (mm/µs/µs)	Signal-to-Noise				
	10:1	3:1	1:1	1:3	1:10
–5					
–2					
–1					
-0.5					
+0.5					
+1					
+2					
+5					

We will be looking at 160 values.

Look at the change in frequency for each FT window

Constant accelerations ≥ linear velocities

$$v = at + b$$

Beat Frequency:

$$f_b = \frac{2f}{c} v = \frac{2}{\lambda}(at + b)$$

$$\frac{df_b}{dt} = \frac{2a}{\lambda}$$

$$df_b = \frac{2a}{\lambda} dt$$

λ = 1.55413e-3 mm
dt = 0.02048 µs

a (mm/µs/µs)	df_b (MHz)
0.5	13
1	26
2	53
5	132

Larger accelerations broaden the frequency peaks in the spectrogram.
Does this increase the error or uncertainty of the extracted data?

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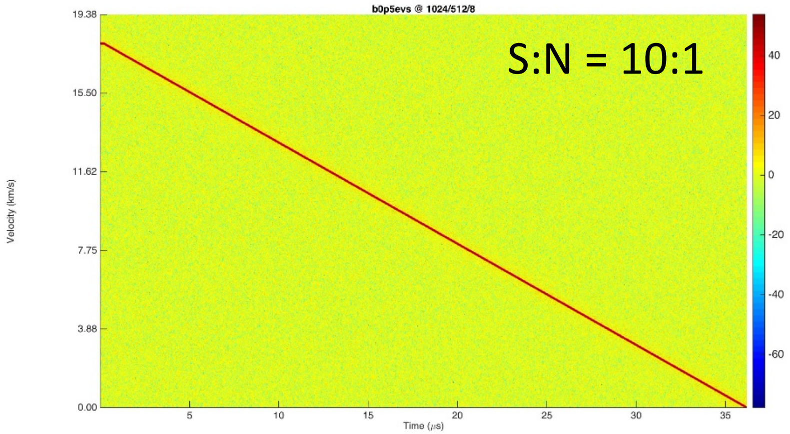
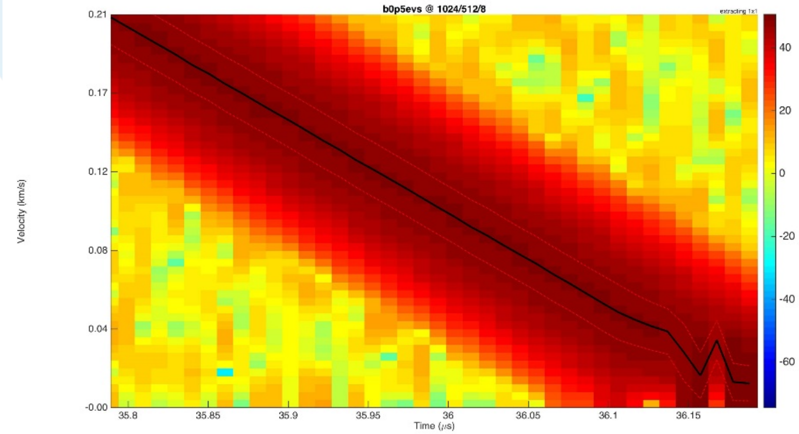
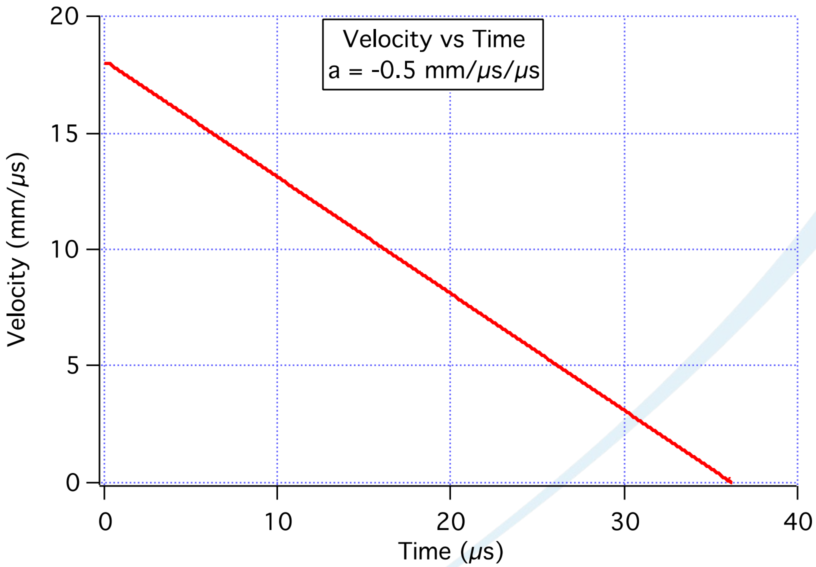
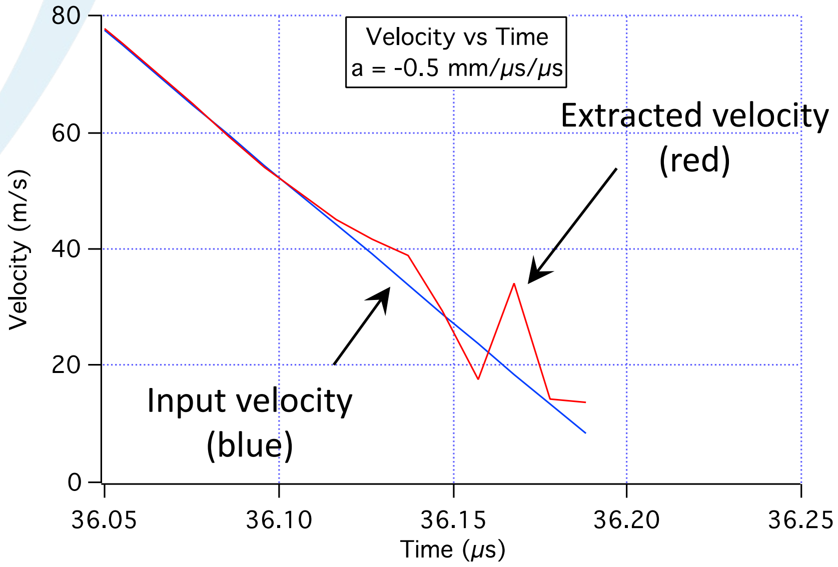
Example of negative acceleration (–0.5 mm/μs/μs)



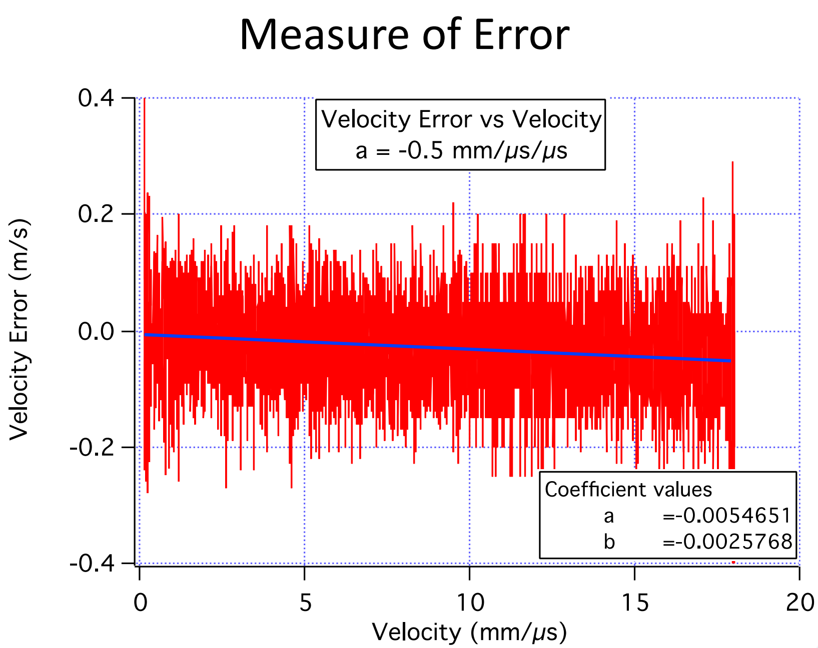


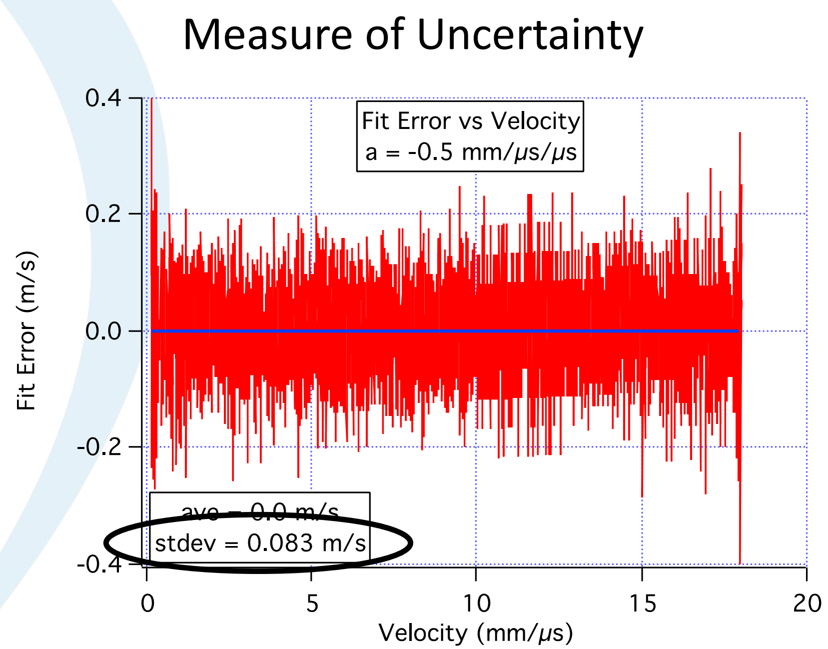




Define velocity error and uncertainty

This example is S:N = 10:1, Acceleration = –0.5 mm/μs/μs





Velocity Error = (extracted velocity) – (actual velocity)
Use a linear fit to the velocity error as a measure of error.

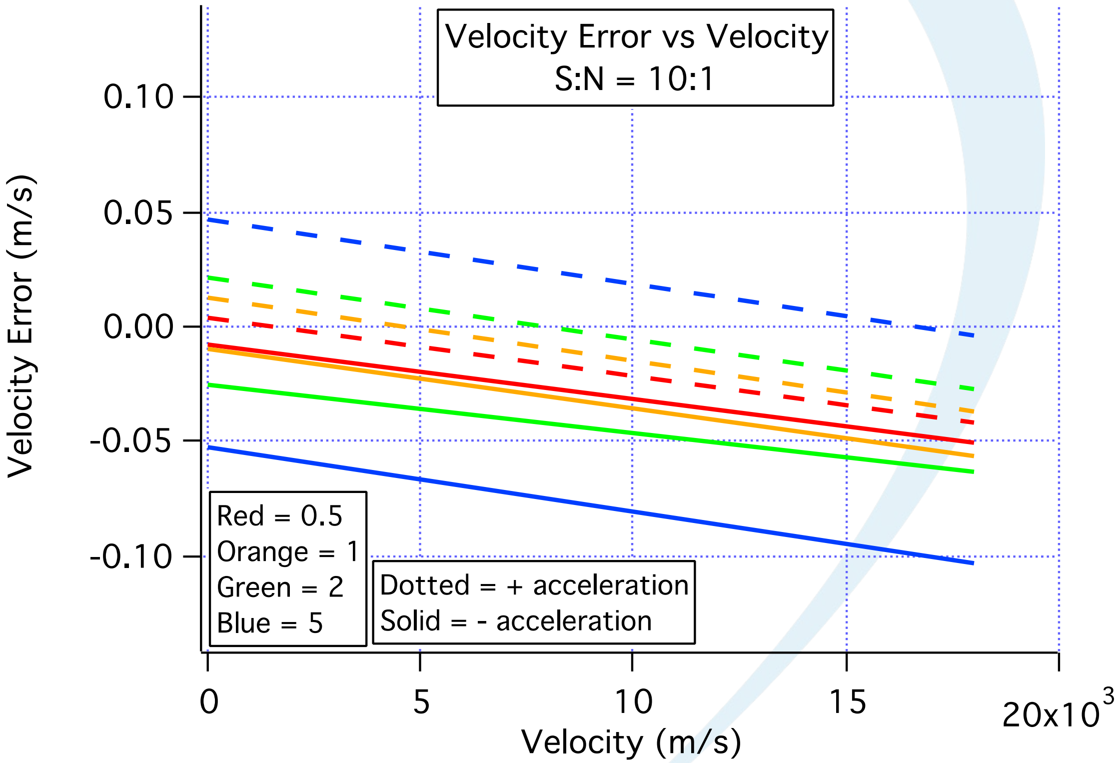
Fit Error = (Velocity Error) – (Fit to Velocity Error)
Use 1 standard deviation of the fit error as a measure of uncertainty.

Velocity Errors

Velocity Errors

Velocity errors are functions of velocity

This example: SNR = 10:1 and Acceleration = ±0.5, ±1, ±2, ±5 mm/µs/µs.



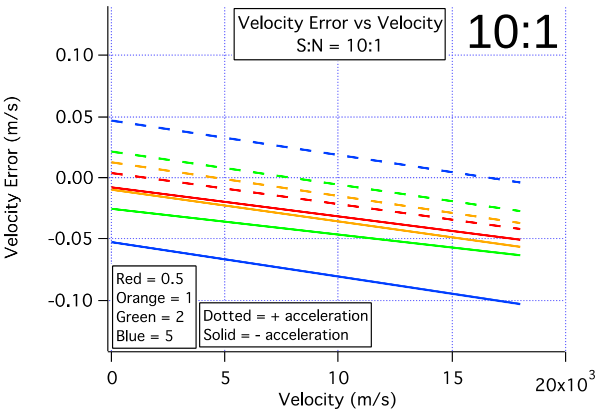
Accelerations have units of mm/µs/µs.

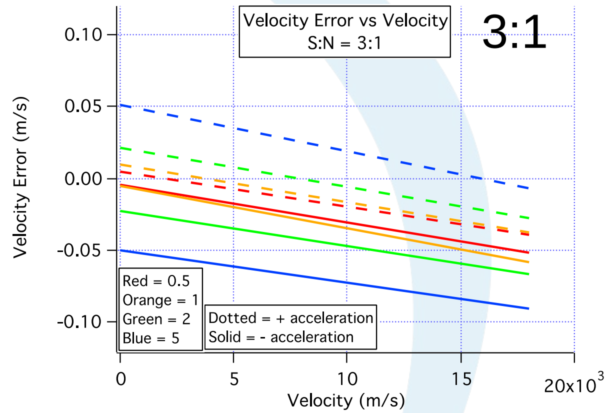
Notes for this case (SNR = 10:1):

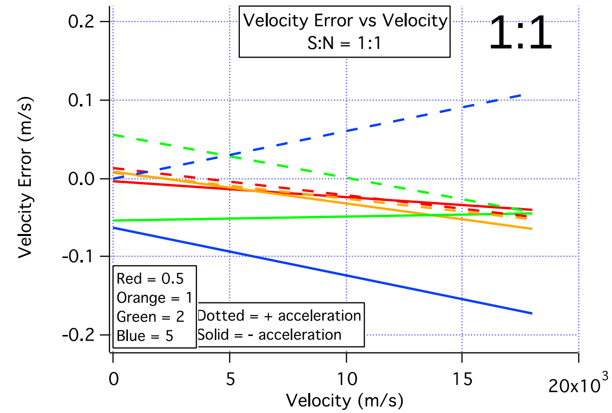
All slopes are negative.

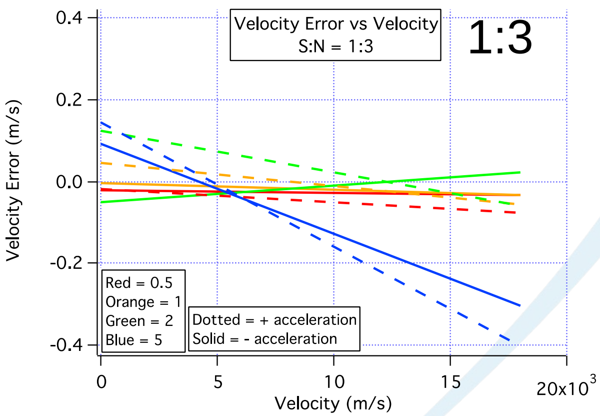
For negative accelerations, all errors are negative for all velocities.

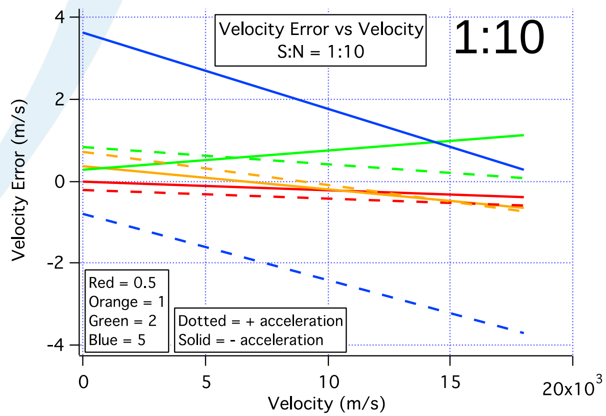
Summary of velocity errors versus velocity











Could in principle determine velocity error for any velocity
with constant acceleration –5 to +5 mm/µs/µs and SNR = 10:1 to 1:10.

Consider the magnitude of the velocity errors

(Would this make a good rule of thumb?)

There is no noticeable pattern to the velocity errors—especially as the SNR gets worse.

On the previous slide, notice the range of the vertical scales for the different SNRs.

Except for σ = 10, the ranges are all less than 0.4 m/s. Perhaps in most cases, it is good enough to know the maximum absolute value of the velocity errors.

Error <0.5 m/s is easy to remember.

Noise Fraction

$$\sigma = \frac{1}{SNR}$$

Magnitude of Velocity Errors (m/s)

σ	Max ABS(Error)
0.1	0.1
0.3	0.1
1	0.2
3	0.4
10	4

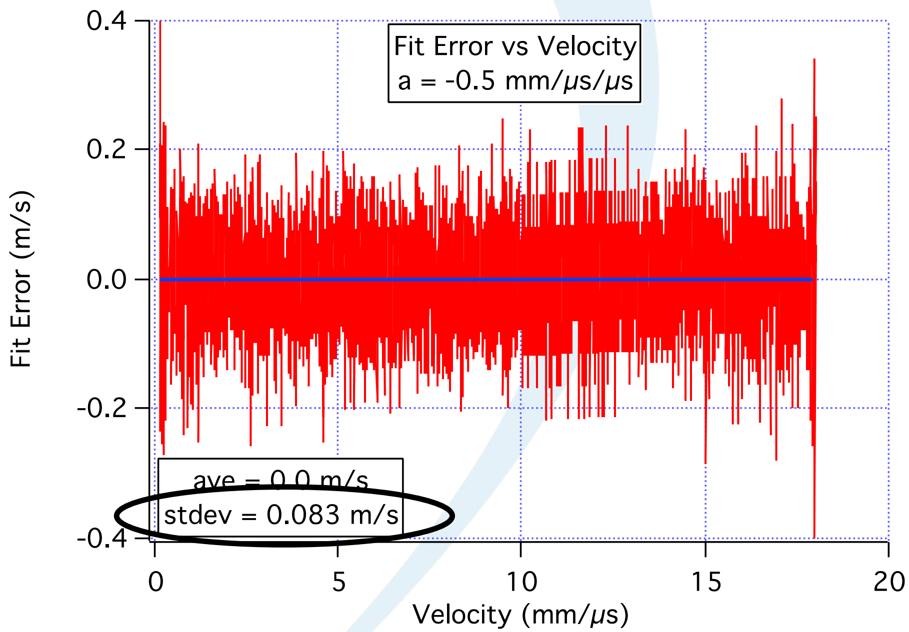
Constant accelerations
-5 to +5 mm/μs/μs

Velocity Uncertainties

Velocity Uncertainties

Velocity uncertainties are independent of velocity

This example shows:
Acceleration = –0.5 mm/µs/µs, SNR = 10:1



Note:

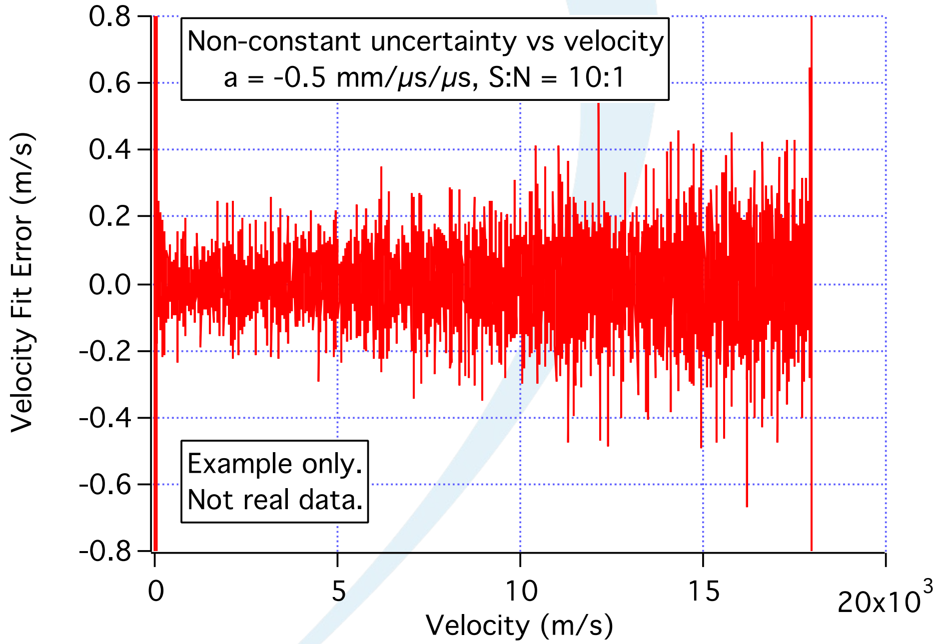
After looking at all fit errors, it appears that the uncertainties are not functions of velocity.

Measure of Uncertainty

Fit Error = (Velocity Error) – (Fit to Velocity Error)
Use 1 standard deviation of the fit error as a measure of uncertainty.

What would non-constant uncertainty look like?

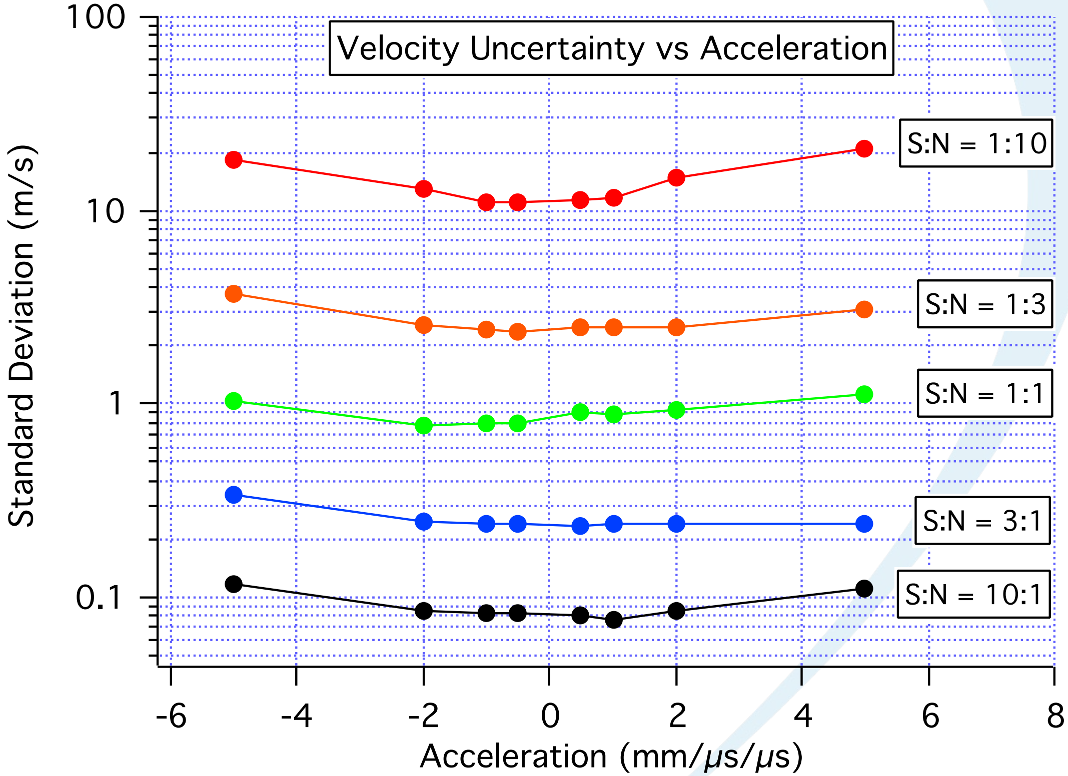
I modified the amplitude of the following case:
Acceleration = –0.5 mm/μs/μs, SNR = 10:1



This example shows increasing uncertainty at higher velocities.
None of the actual uncertainty plots looked like this.

Summary of Velocity Uncertainty versus Acceleration

Uncertainty does not vary with velocity.



σ = 10

σ = 3

σ = 1

σ = 0.3

σ = 0.1

Noise Fraction

$$\sigma = \frac{1}{SNR}$$

Notes for velocity uncertainties:

Uncertainties are constant with velocity.

Uncertainties increase with increasing noise fraction.

Uncertainties increase weakly with acceleration.

Velocity Uncertainty (m/s) ≈ Noise Fraction

Interesting coincidence!

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Velocity uncertainty is related to the noise fraction

from Dan Dolan's paper:
(eqn. 9)

REVIEW OF SCIENTIFIC INSTRUMENTS **81**, 053905 (2010)

Accuracy and precision in photonic Doppler velocimetry

D. H. Dolan[a)]
Sandia National Laboratories, Albuquerque, New Mexico 87185-1195, USA

$$\Delta f = \sqrt{\frac{6}{N}}\frac{\sigma}{\pi\tau} \quad \Longrightarrow \quad \Delta v = \left(\frac{\lambda}{2}\sqrt{\frac{6}{N}}\frac{1}{\pi\tau}\right)\sigma$$

where:

Δv = velocity uncertainty
λ = laser wavelength = 1554.13 nm
N = FFT sample number = 1024
σ = noise fraction = inv(SNR)
τ = FFT window length = 0.02048 μs

Many thanks to Ed Daykin for reminding me of Dolan's analytic expression.

We obtain: Δv(m/s) = 0.93σ ≈ σ

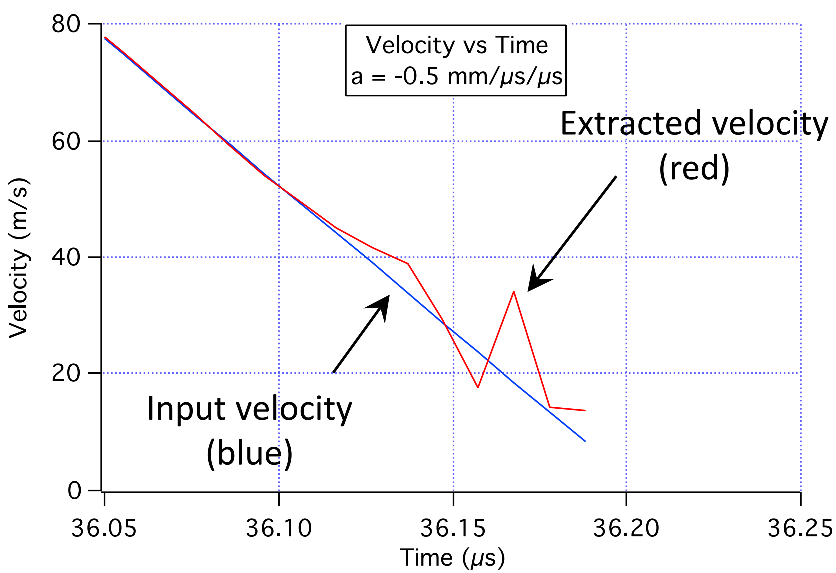
(For this study with λ = 1554.13 nm, N = 1024, τ = 0.02048 μs)

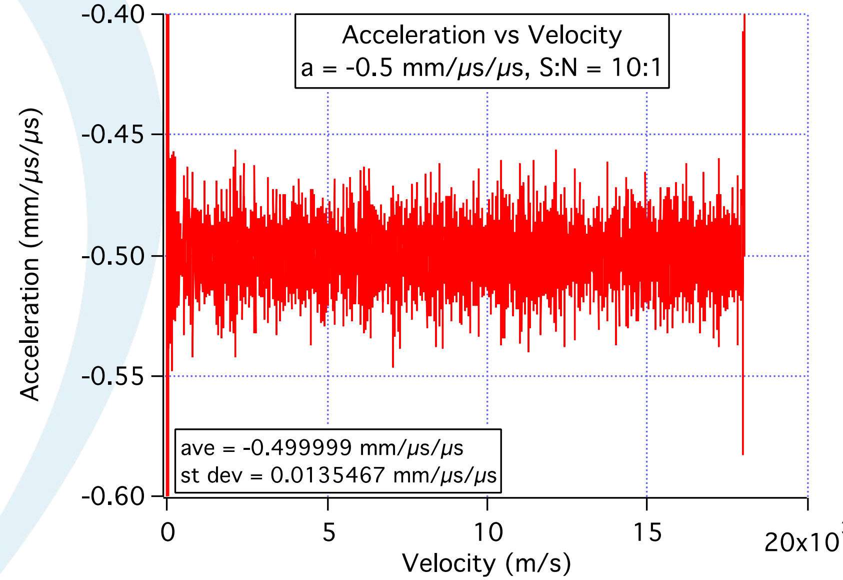
Acceleration Errors

Acceleration Errors

Differentiate the extracted velocity profiles to obtain the acceleration profiles

This example is for acceleration = –0.5 mm/µs/µs, S:N = 10:1.





Acceleration = point-by-point derivative of the extracted velocity

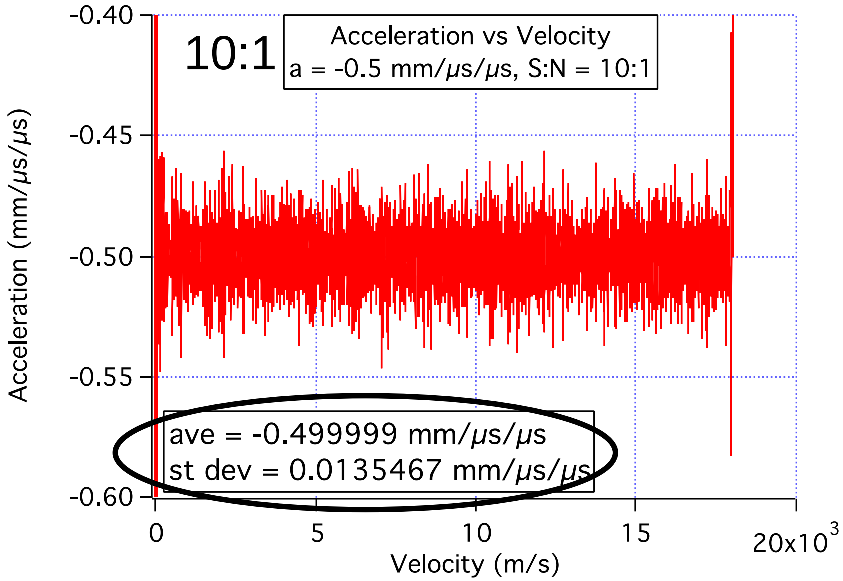
$$a_n = \frac{v_n - v_{n-1}}{t_n - t_{n-1}}$$

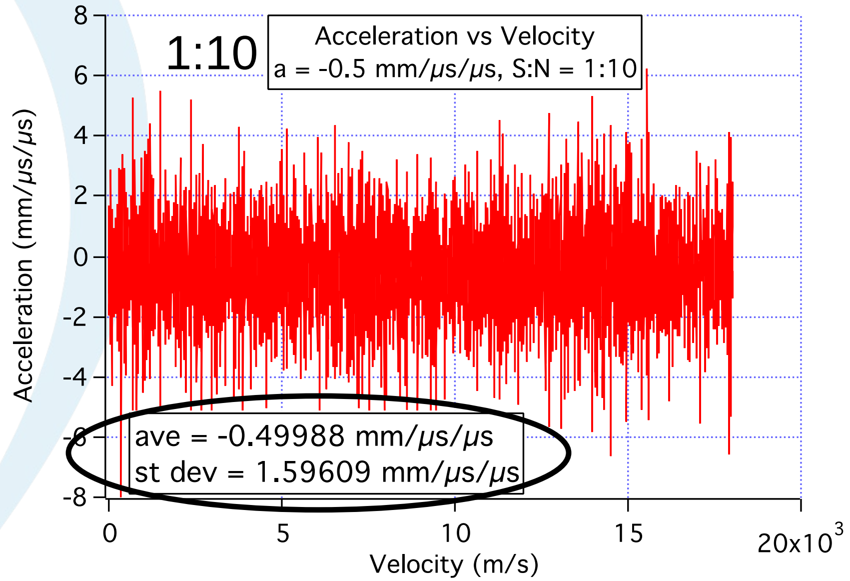
a = a(mm/µs/µs)

I used Excel to calculate the accelerations.

Acceleration errors and acceleration uncertainties are constant with velocity

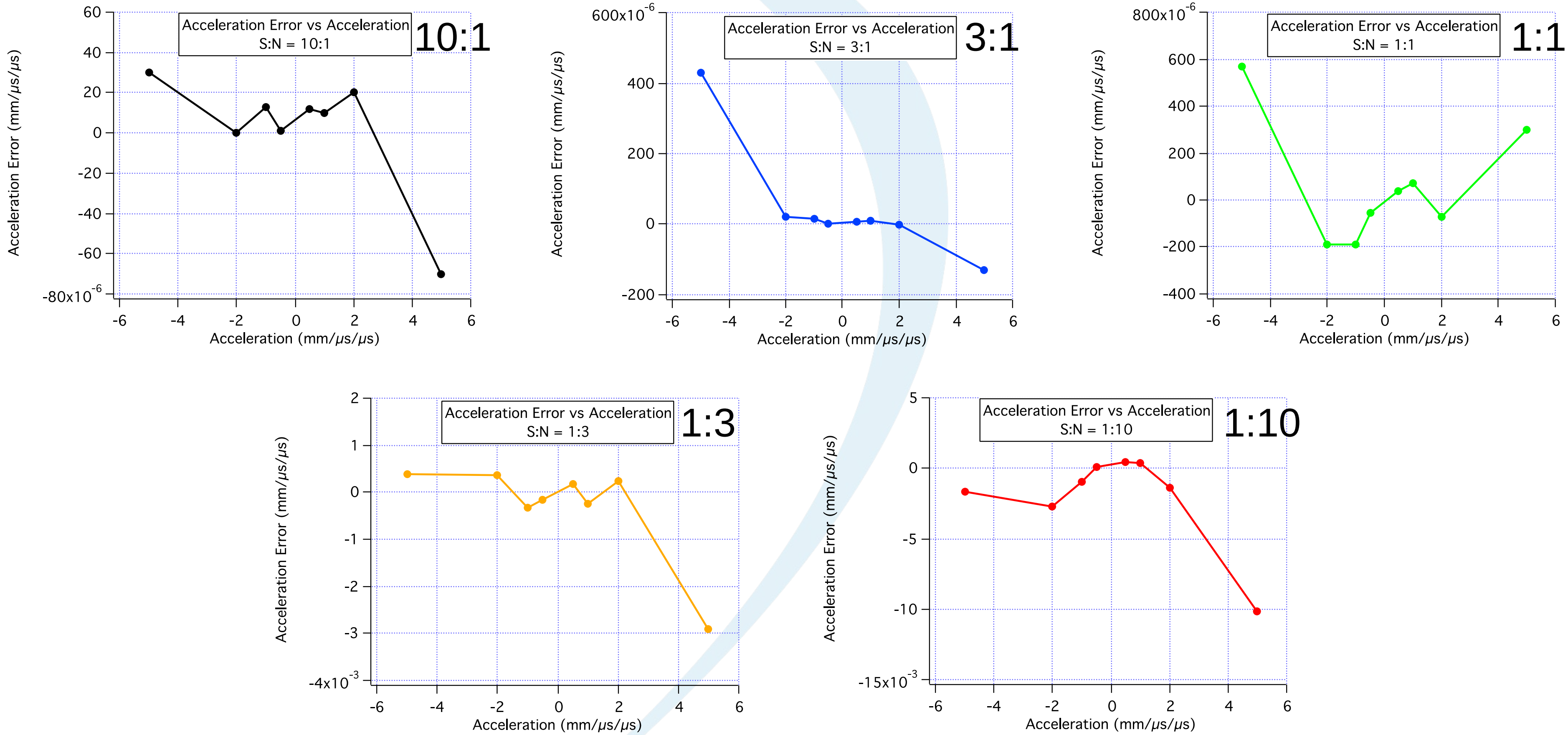
This example is for a = –0.5 mm/µs/µs.





Acceleration error = average acceleration – input acceleration
Acceleration uncertainty = 1 standard deviation of average

Summary of acceleration errors versus acceleration



Acceleration errors are positive and negative, but extremely small.
Look at magnitude of errors (as we did with velocity errors).

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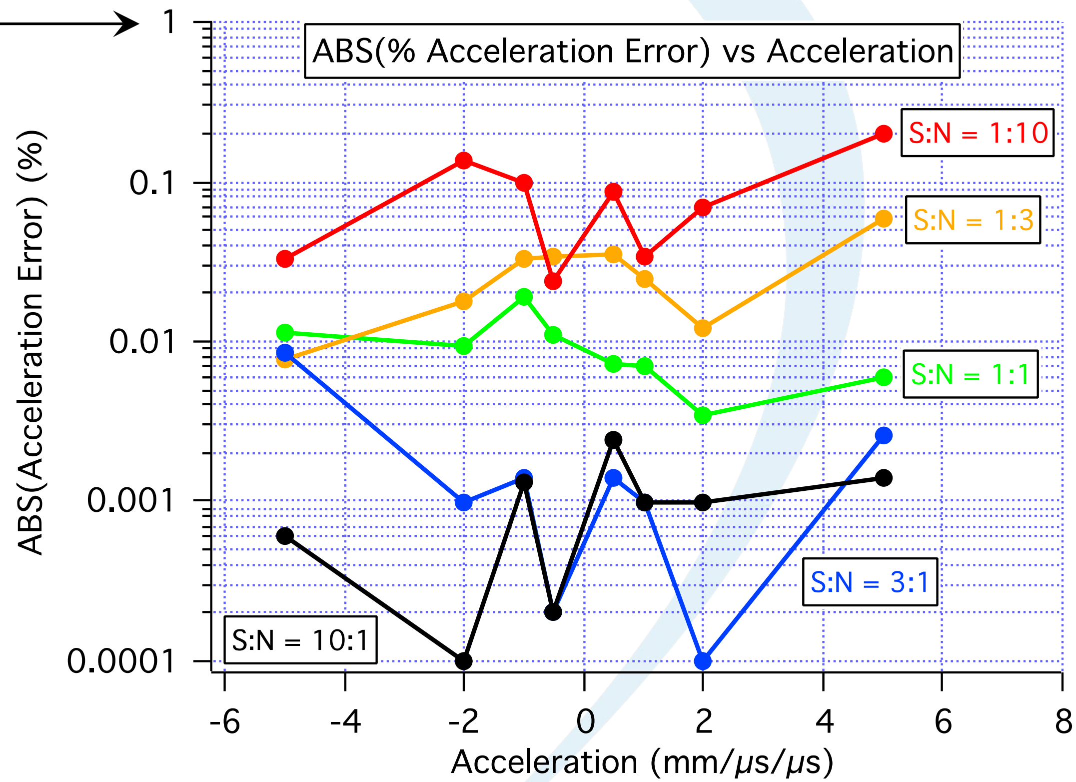
The magnitudes of acceleration error are negligible

1% error →



Notes for acceleration errors:

Errors are constant with velocity, but not constant with acceleration.

Errors increase with increasing noise fraction.

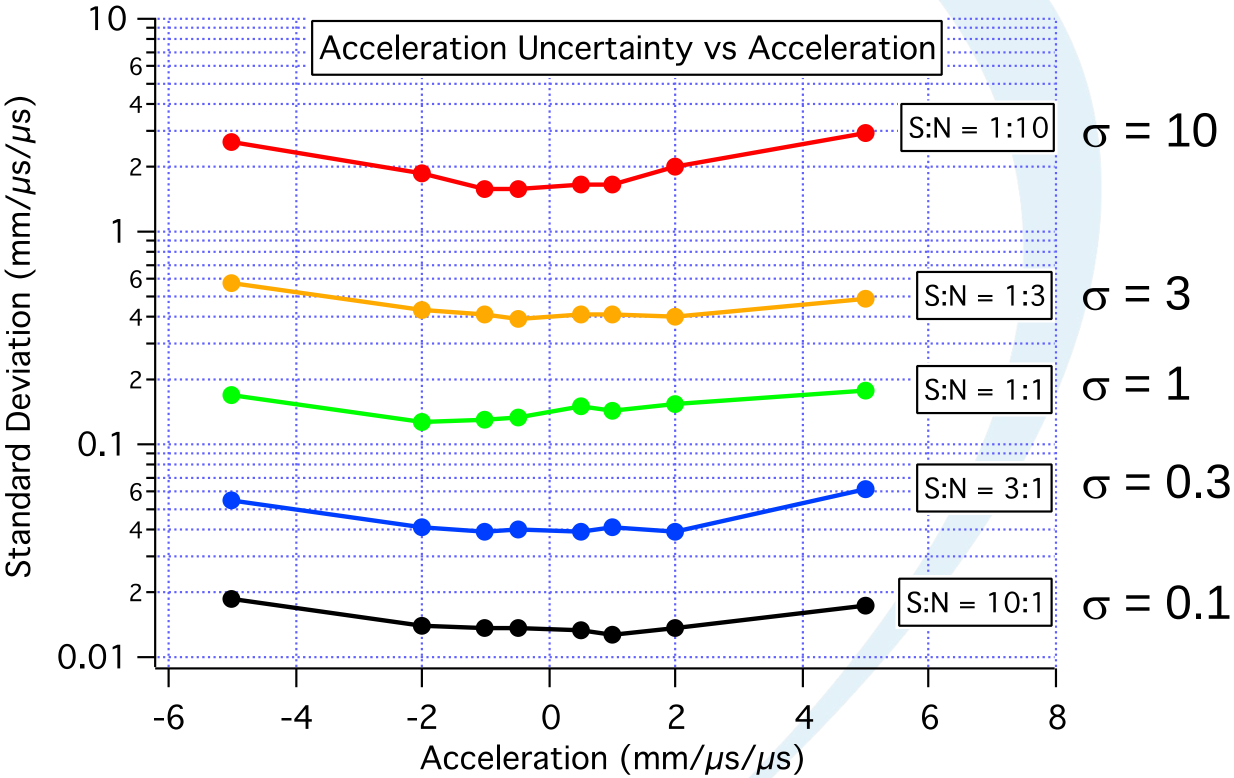
Errors are extremely small—especially with small noise fraction.

Acceleration errors are generally less than 0.1% for constant accelerations = –5 to +5 and σ = 0.1 to 10.

Acceleration Uncertainties

Acceleration Uncertainties

Summary of acceleration uncertainties versus acceleration



Noise Fraction

$$\sigma = \frac{1}{SNR}$$

Notes for acceleration uncertainties:

Uncertainties are constant with velocity.

Uncertainties increase with increasing noise fraction.

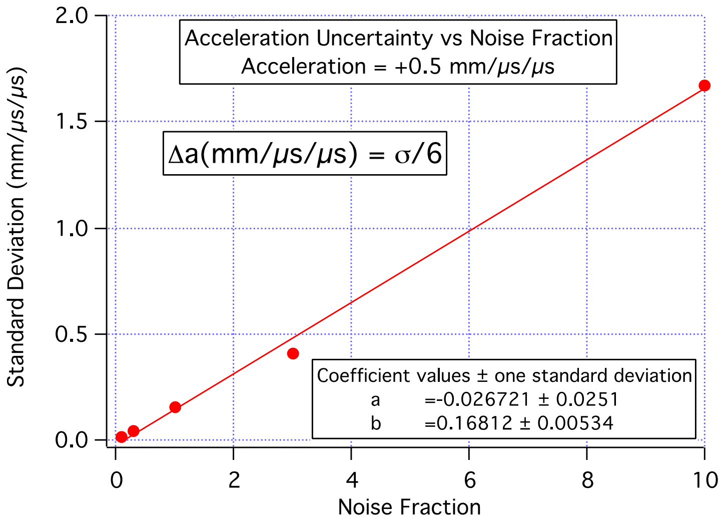
Uncertainties increase weakly with acceleration.

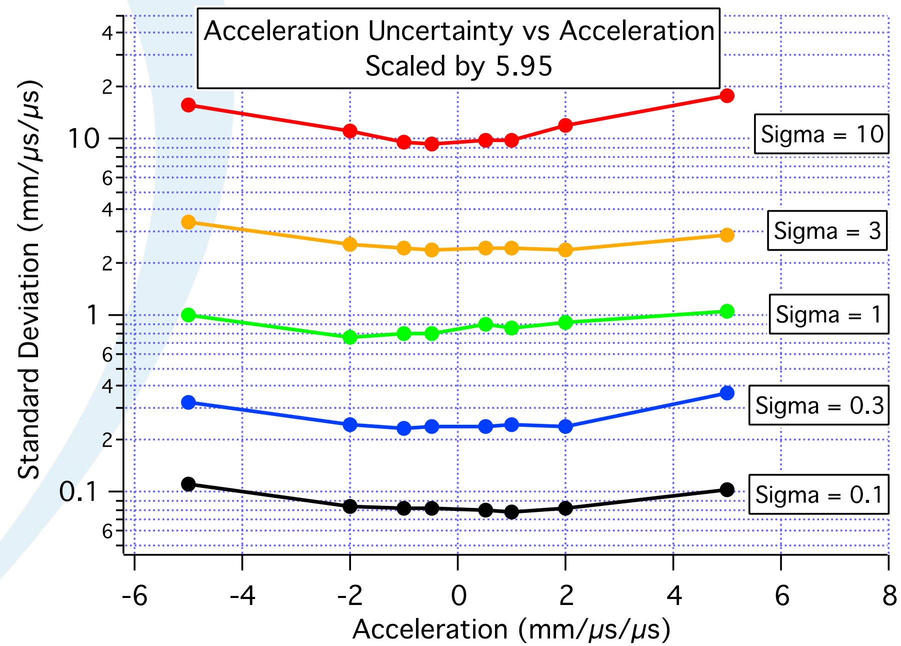
This looks similar to the velocity uncertainties.

Does the acceleration uncertainty scale linearly with noise fraction?

Acceleration uncertainty might have a linear relationship to noise fraction

Use the data at a = +0.5 mm/µs/µs to determine a scale factor.





$$\Delta a(\text{mm}/\mu\text{s}/\mu\text{s}) \approx \sigma/6$$

(For this study with λ = 1554.13 nm, N = 1024, τ = 0.02048 µs)

Reminder: Determining Signal-to-Noise Ratio

Reminder: Determining Signal-to-Noise Ratio

Use the RMS amplitude of the digitizer record to determine SNR(t,dB) for your PDV data

from: Low Noise Receiver/Scope Combinations For PDV, Kirk Miller, et al., PDV Workshop, 6/24/2014

$$SNR_{TIME\text{-}DOMAIN,dB} = 6.02{*}ENOB + 1.76 + 20 \log (2A/V)$$

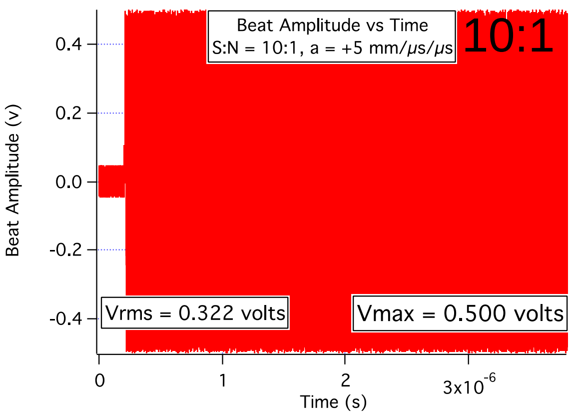
E = effective bits for digitizer, V = full scale range, A = RMS amplitude of applied signal
See Wiley Encyclopedia of Electrical and Electronics Engineering, Vol. 18, J. Blair

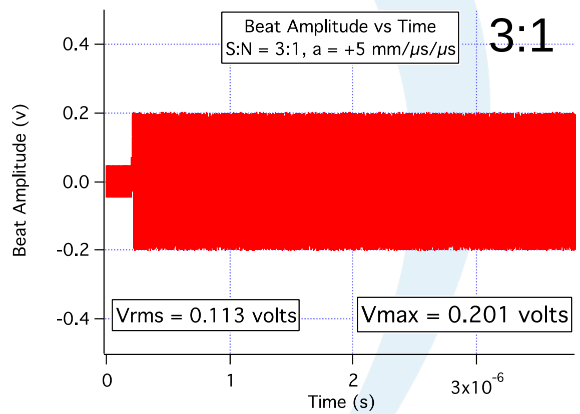
$$SNR_{f,dB} = 6.02{*}ENOB + 1.76 + 20{*}\log(2{*}RMS/V_{FS}) + 10{*}\log(N_{FFT}/2)$$

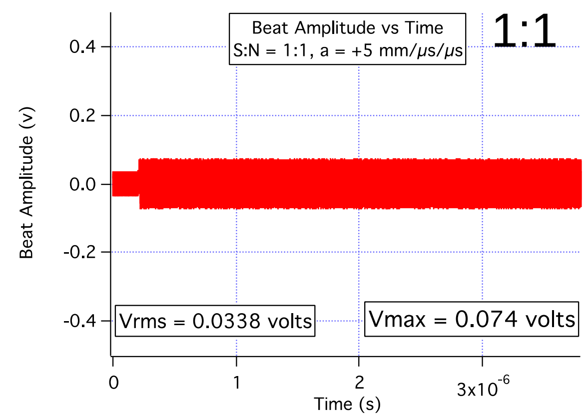
SNR(t,dB) = SNR in the time domain data (digitizer)
SNR(f,dB) = SNR in the frequency domain (spectrogram)
ENOB = effective number of bits for your digitizer
A = RMS = RMS amplitude of the time domain data
V = V_{FS} = full scale voltage range of the digitizer setting
N_{FFT} = number of digitizer points in each FT window

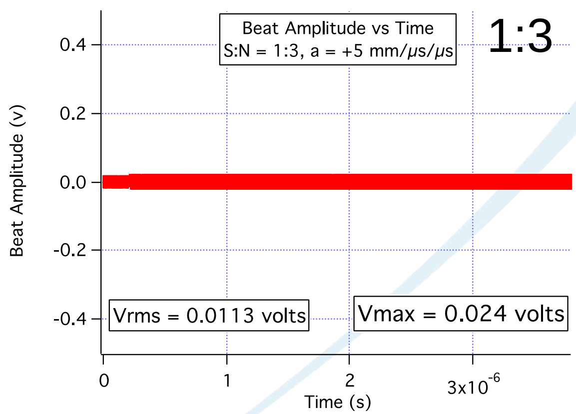
This is what my 'data' would have looked like

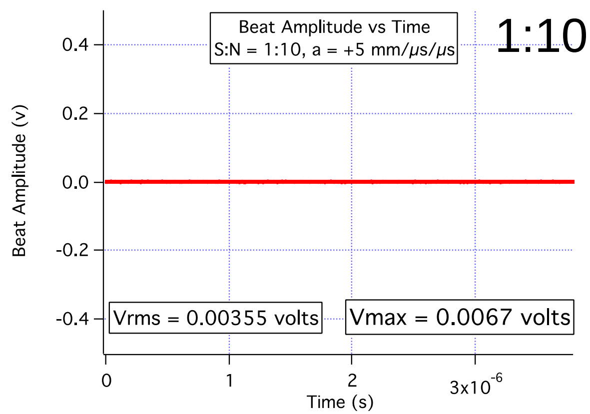
These examples are for acceleration = +5 mm/µs/µs.











(no detector noise included)

Conclusions—PDV errors and uncertainties

Generally good news ($-5 \le a \le +5$ mm/µs/µs, $0.1 \le \sigma \le 10$)

Velocity errors are functions of velocity, acceleration, noise fraction.
- Errors < 0.5 m/s for noise fraction ≤ 3.
- Errors < 4 m/s for noise fraction = 10.

Velocity Uncertainties are constant with velocity.
- Increase weakly with acceleration.
- Δv(m/s) ≈ Noise Fraction (for this study).
- Agrees with analytic expression given by Dolan.

Acceleration errors are constant with velocity.
- Errors are functions of acceleration.
- Increase with increasing noise fraction.
- Errors are negligible (<0.1%).

Acceleration uncertainties constant with velocity.
- Increase weakly with acceleration.
- Δa(mm/µs/µs) ≈ (Noise Fraction)/6 (for this study).

(A final reminder: 50 GS/s, λ = 1554.13 nm, N = 1024, τ = 0.02048 µs)

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