How to account for multiple scattering in Photon Doppler Velocimetry spectrograms ?

DE LA RECHERCHE À L'INDUSTRIE

J.A. Don Jayamanne[1,2], J-R. Burie[1], R. Pierrat[2], R. Carminati[2] and O. Durand[1]

[1]CEA DIF, Bruyères-le-Châtel, 91297 Arpajon Cedex, France

[2]Institut Langevin, ESPCI Paris, PSL University, CNRS, 75005 Paris, France

07/02/2023 – PDV Workshop 2023

Outline

Introduction

Light scattering and transport model for PDV

Simulation

Examples

Conclusion and perspectives

<u>Introduction</u>

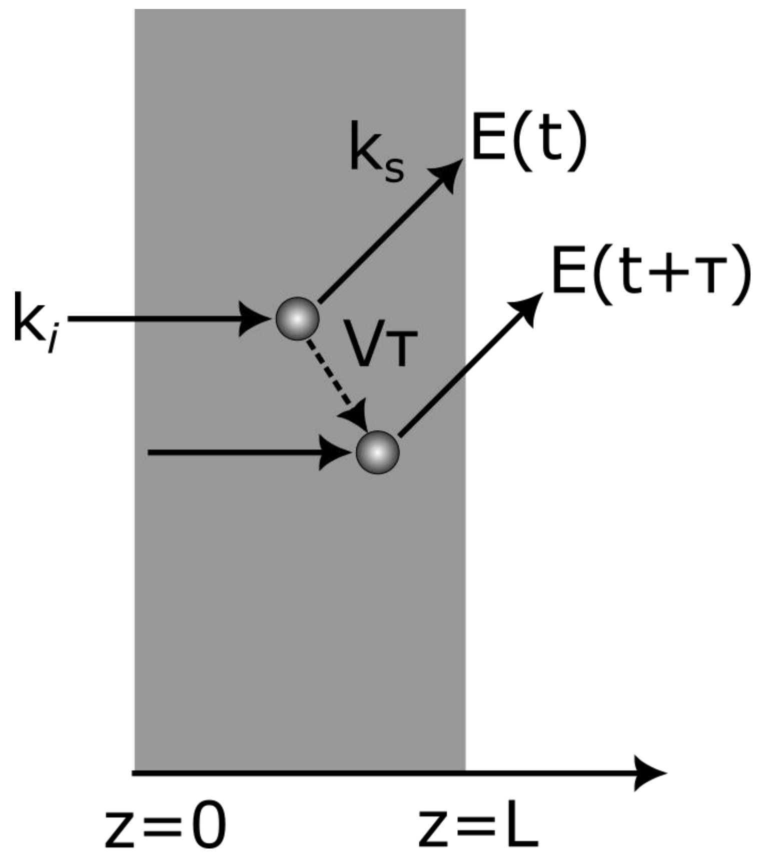
Light scattering and transport model for PDV

Simulation

Examples

Conclusion and perspectives

Principle



Doppler shift

$$\delta\omega = (\boldsymbol{k_s} - \boldsymbol{k_i}) \cdot \boldsymbol{v}$$

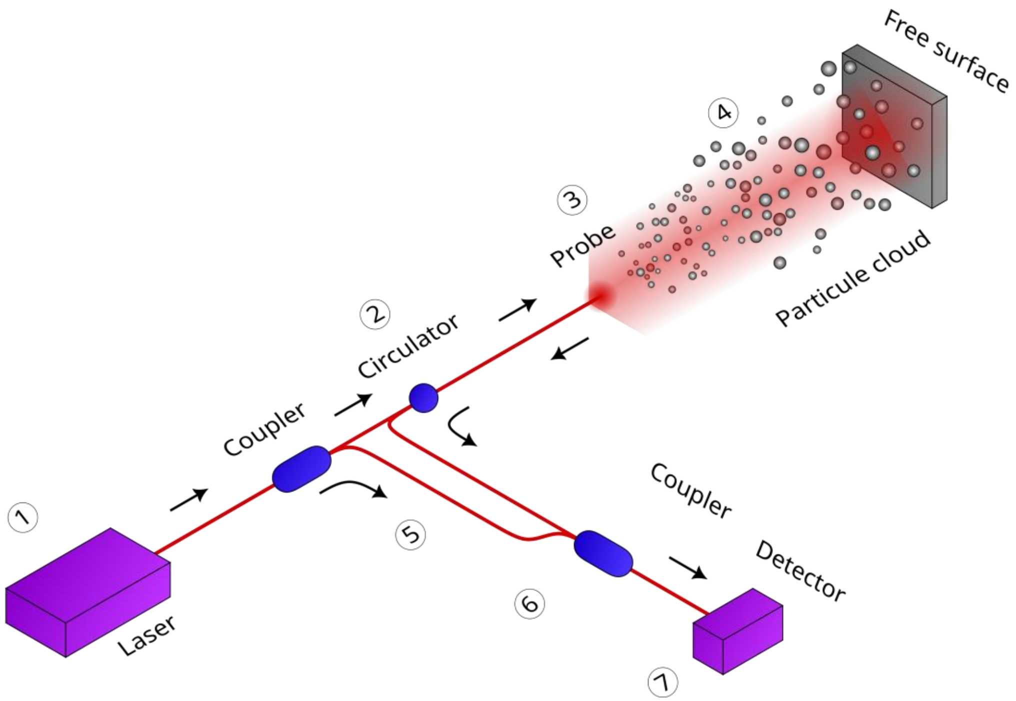
PDV configuration

$$\delta\omega \approx \frac{4\pi}{\lambda} v$$

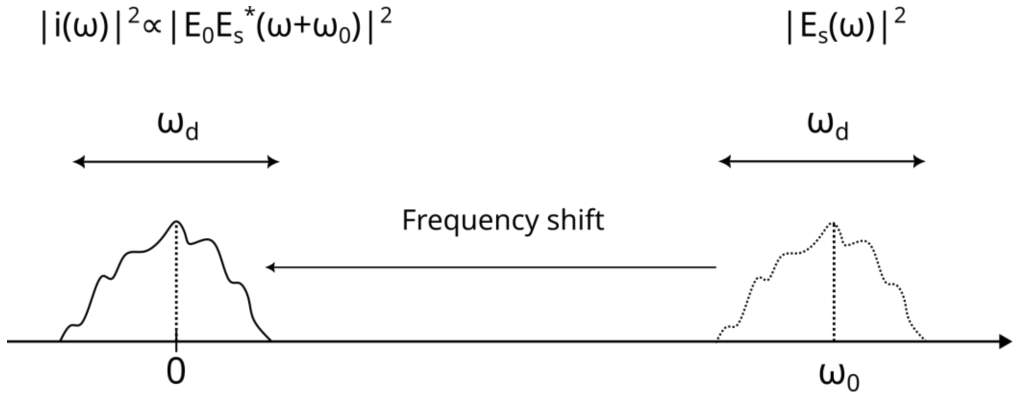
Carminati, Rémi, and John C. Schotland. *Principles of Scattering and Transport of Light*. Cambridge: Cambridge University Press, 2021.

Photon Doppler Velocimetry

Setup



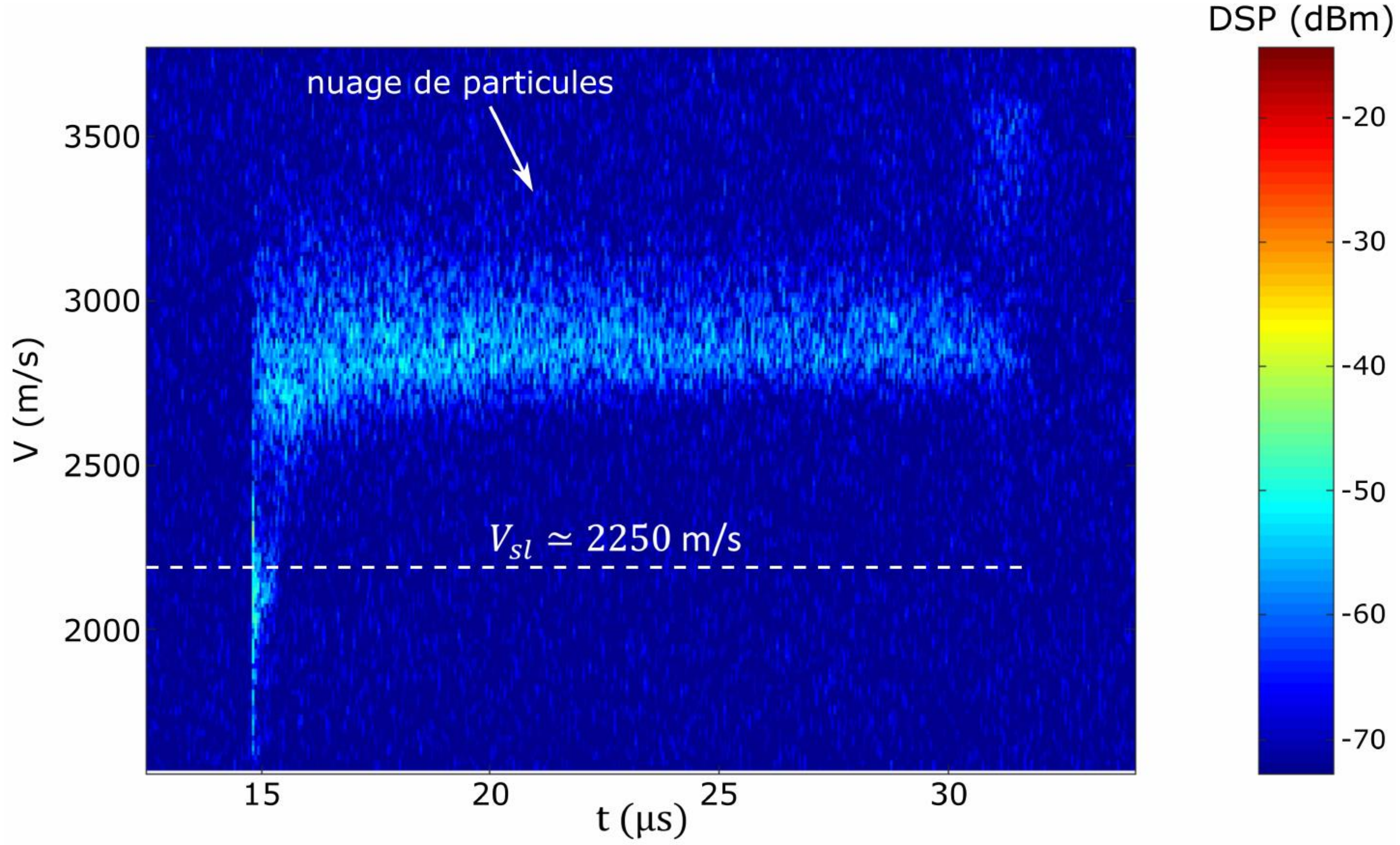
Relevance of heterodyning



Strand, O T, D R Goosman, C Martinez, T L Whitworth, W W Kuhlow, and Bechtel Nevada 2005, 19.

Mercier, P., J. Benier, A. Azzolina, J. M. Lagrange, and D. Partouche, *Journal de Physique IV (Proceedings)* 134, 2006

Spectrogram

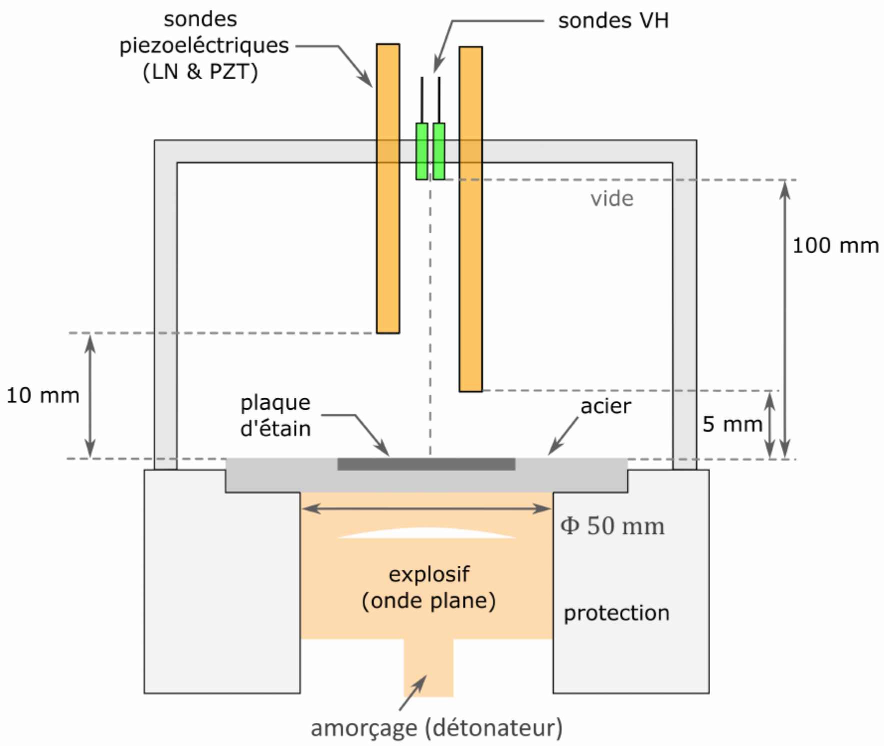


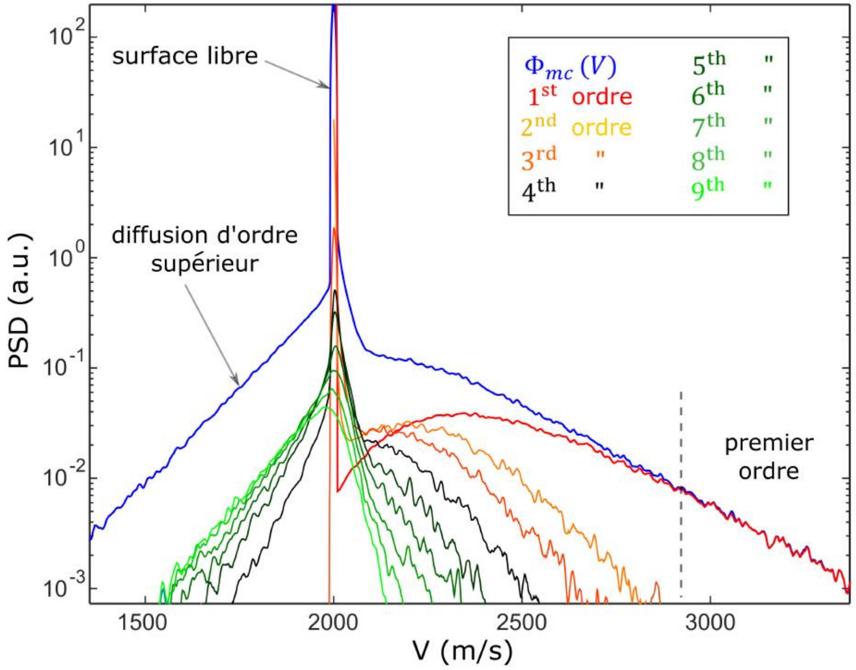
Franzkowiak, Jean-Eloi. « Interaction lumière-nuage de particules micrométriques hautes vitesses: application à la Vélocimétrie Hétérodyne », PhD Thesis, 2018.

The single scattering regime is not enough

Experimental example

The relation $\delta\omega \approx 4\pi/\lambda \times v$ holds as long as $L/\ell_s \leq 1$ but we reach $L/\ell_s \approx 70$.





Franzkowiak, Jean-Eloi. « Interaction lumière-nuage de particules micrométriques hautes vitesses: application à la Vélocimétrie Hétérodyne », PhD Thesis, 2018.

Outline

Outline

Introduction

Light scattering and transport model for PDV

Simulation

Examples

Conclusion and perspectives

State of the art

Recent interest

- Andriyash *et al* . (Ru) : 5 articles (2016-2022)
- Franzkowiak *et al.* (Fr) : 1 article (2018)
- Shi *et al.* (Ch) : 2 articles (2021-2022)
- *Biology community (1990-2000)*

Issues

- Link between the spectrogram and the computed quantities.
- Light scattering theory with multiple scattering and Doppler shifts.

Experimental signal

$$i(t) \approx E(t)^2$$

$$i(t) = [\,E_0(t) + E_s(t)]^2$$

$$i(t) = E_0^2(t) + 2E_0(t)E_s(t) + E_s^2(t)$$

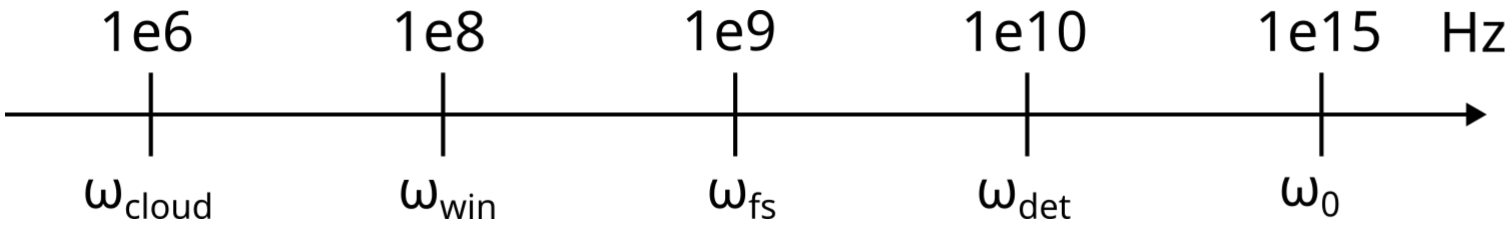
Processing

$$S(t,\omega) = \left|\int i(\tau)w(\tau - r)exp(i\omega\tau)d\tau\right|^2$$

Theoretical quantity

$$\mathrm{I}(t,\omega) = \int \left\langle \bar{E}\left(t + \frac{\tau}{2}\right)\bar{E}^*\left(t - \frac{\tau}{2}\right)\right\rangle exp(i\omega\tau)d\tau$$

Orders of magnitude

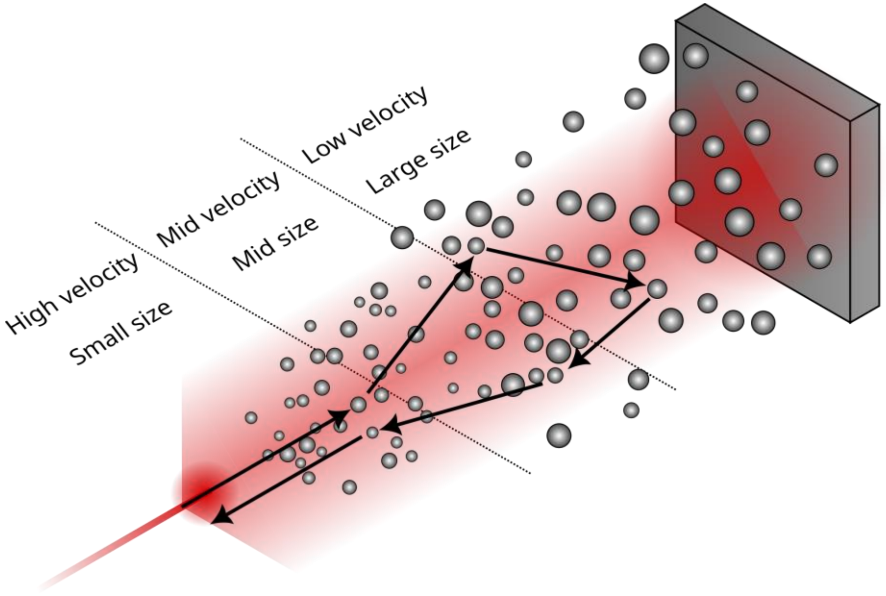


Spectrogram expression

$$S(t,\omega) \propto I_s(t,\omega_0 \pm \omega)$$

Radiative Transport Equation

$$(\boldsymbol{u}.\boldsymbol{\nabla}_{\boldsymbol{r}})I(\boldsymbol{r},\boldsymbol{u},t,\omega) = -\left[\frac{1}{l_s(\boldsymbol{r},t)} + \frac{1}{l_a(\boldsymbol{r},t)}\right] I(\boldsymbol{r},\boldsymbol{u},t,\omega) + \frac{1}{l_s(\boldsymbol{r},t)}\int p(\boldsymbol{r},\boldsymbol{u},\boldsymbol{u}',t,\omega,\omega')I(\boldsymbol{r},\boldsymbol{u}',t,\omega')d\boldsymbol{u}'d\omega'$$



Pierrat, Romain., *Journal of the Optical Society of America A* 25, nº 11, 2008

Introduction

Light scattering and transport model for PDV

Simulation

Examples

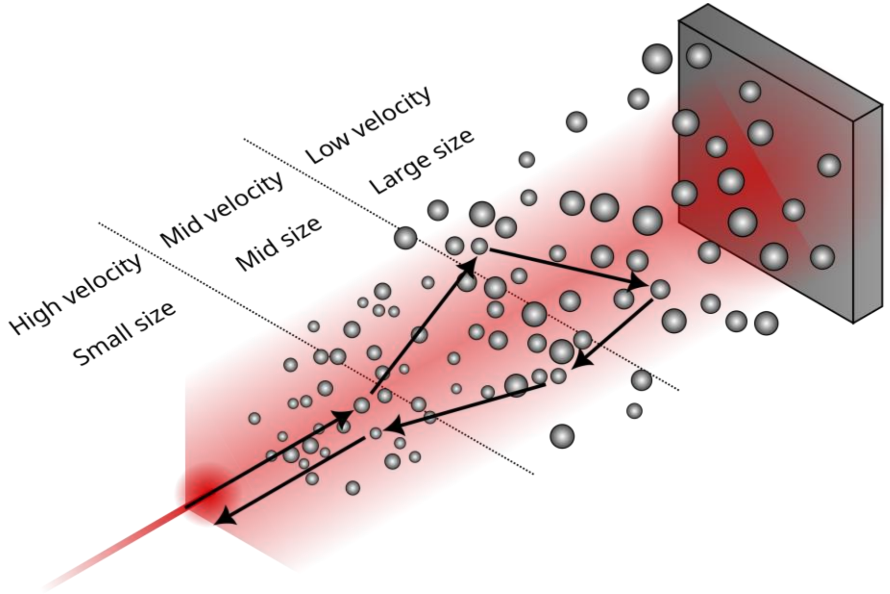
Conclusion and perspectives

Integral form of the RTE

$$I(\boldsymbol{r},\boldsymbol{u},t,\omega) = \int f(s,\boldsymbol{u},\boldsymbol{r},t) p(\boldsymbol{r}+s\boldsymbol{u},\boldsymbol{u},\boldsymbol{u}',t,\omega,\omega')\, I(\boldsymbol{r}+s\boldsymbol{u},\boldsymbol{u}',t,\omega') ds\, d\boldsymbol{u}'\, d\omega'$$

Distance draw

$$f(s,\boldsymbol{u},\boldsymbol{r},t) = \frac{1}{\ell_s(\boldsymbol{r}+s\boldsymbol{u})} \exp\left[-\int_0^s \frac{1}{\ell_s(\boldsymbol{r}+s'\boldsymbol{u})} ds'\right]$$

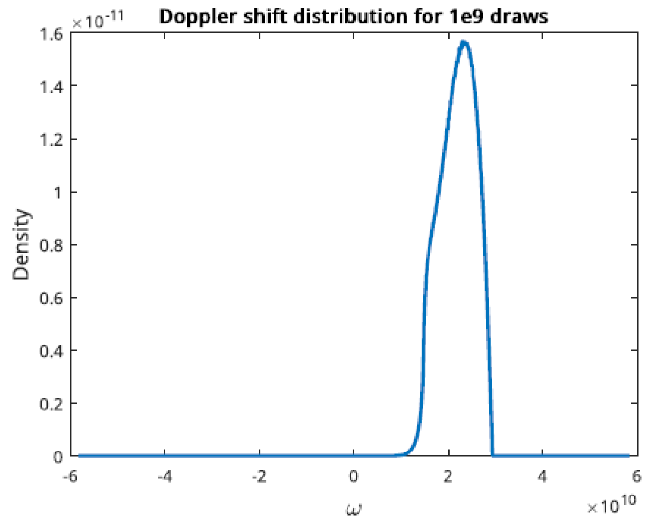


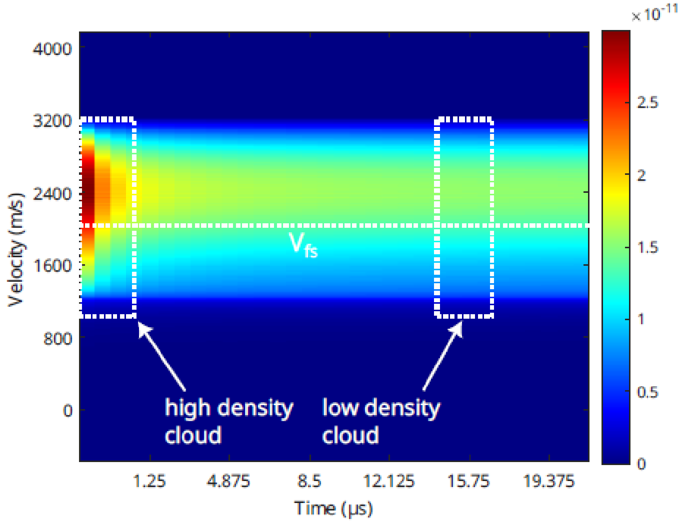
Direction and Doppler shift draw

$$p(\boldsymbol{r},\boldsymbol{u},\boldsymbol{u}',t,\omega,\omega')$$

Method

- Random walk for many quanta
- Doppler shift histogram
- Spectrogram reconstruction

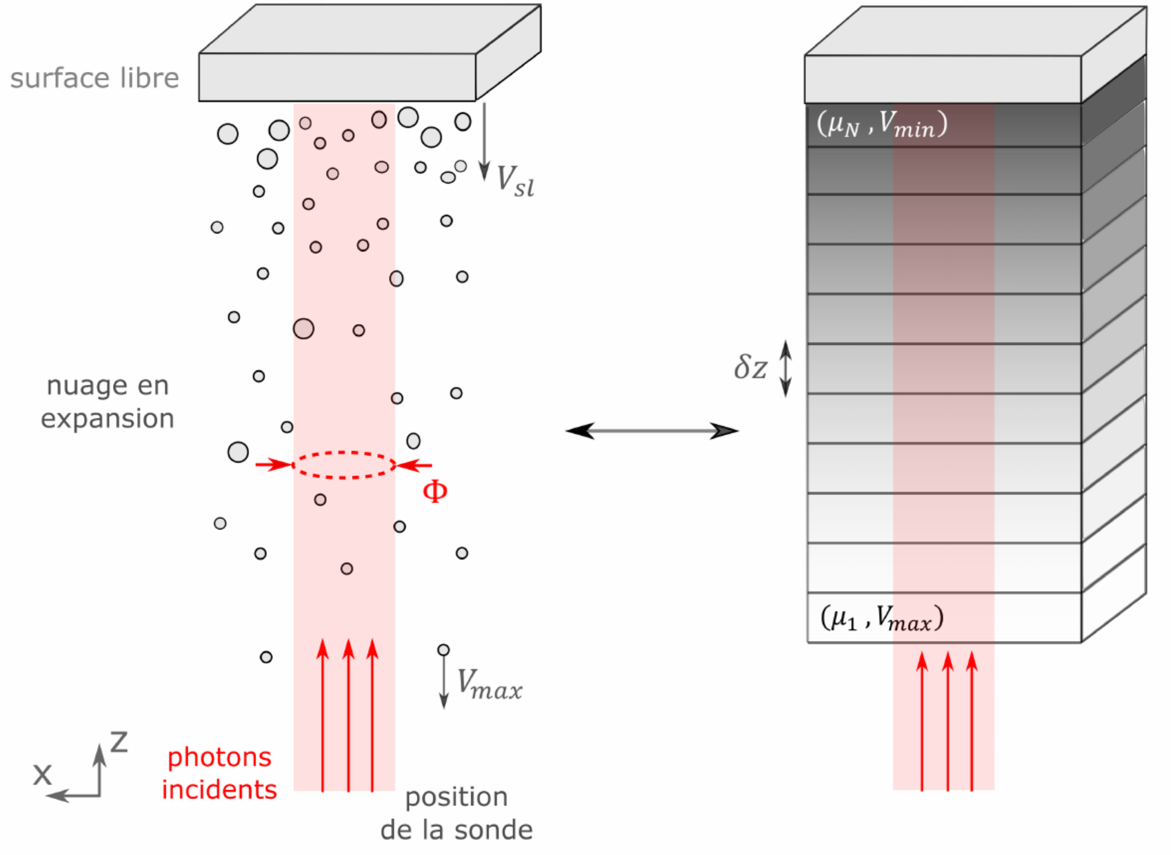




Layered geometry

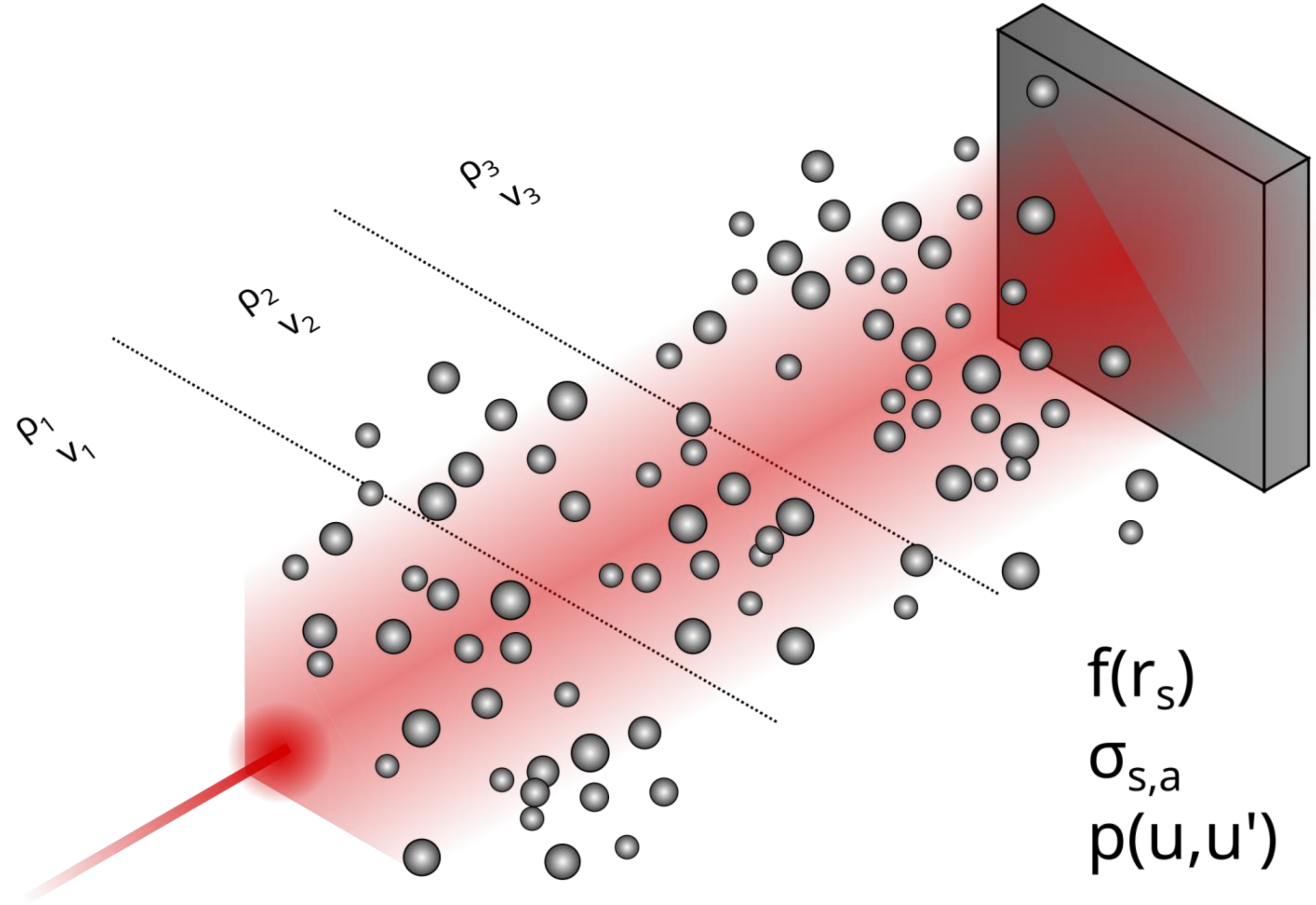
In each layer we compute :

- ℓ_s
- ℓ_a
- $p(\boldsymbol{u}, \boldsymbol{u}', \omega, \omega')$

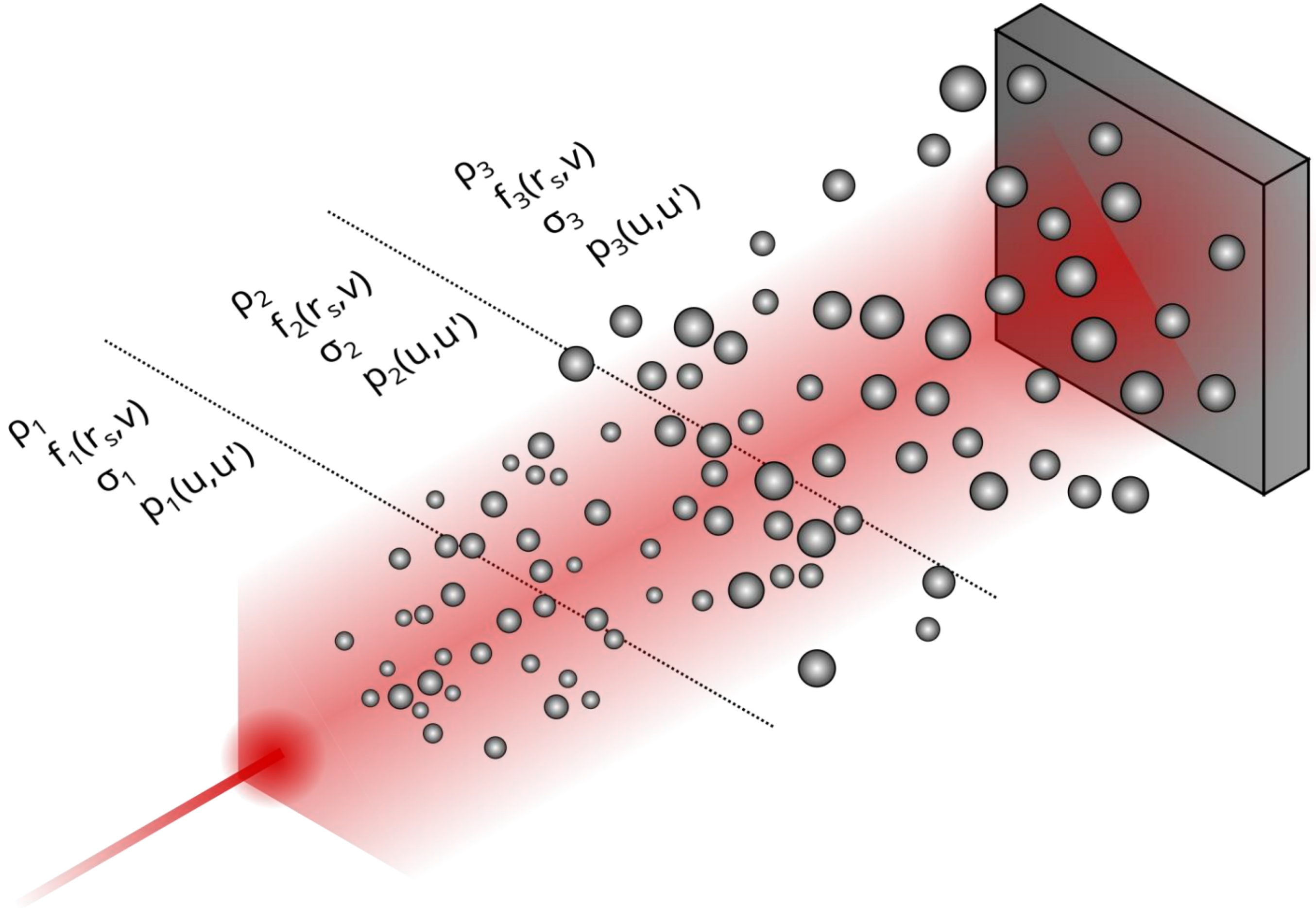


Franzkowiak, Jean-Eloi. « Interaction lumière-nuage de particules micrométriques hautes vitesses: application à la Vélocimétrie Hétérodyne », PhD Thesis, 2018.

Bibliography

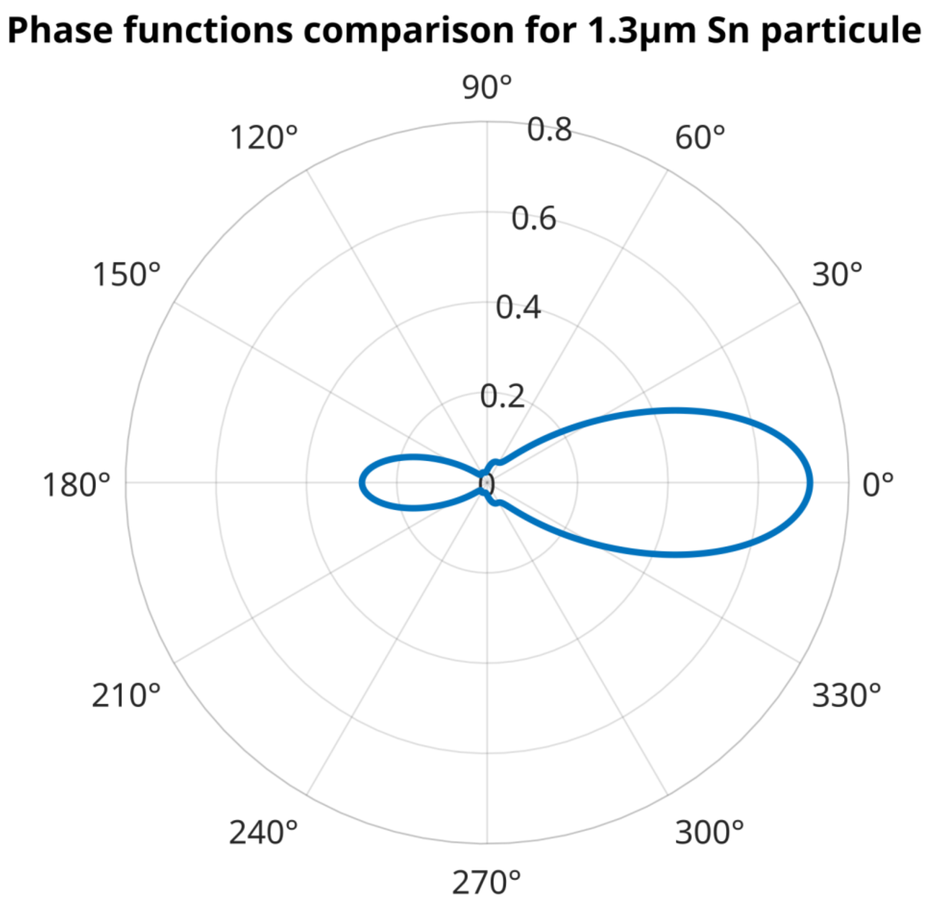


Our work



Mie solution for spherical particles

- Cross sections
- Phase function



Outline

Introduction

Light scattering and transport model for PDV

Simulation

Examples

Conclusion and perspectives

Introduction

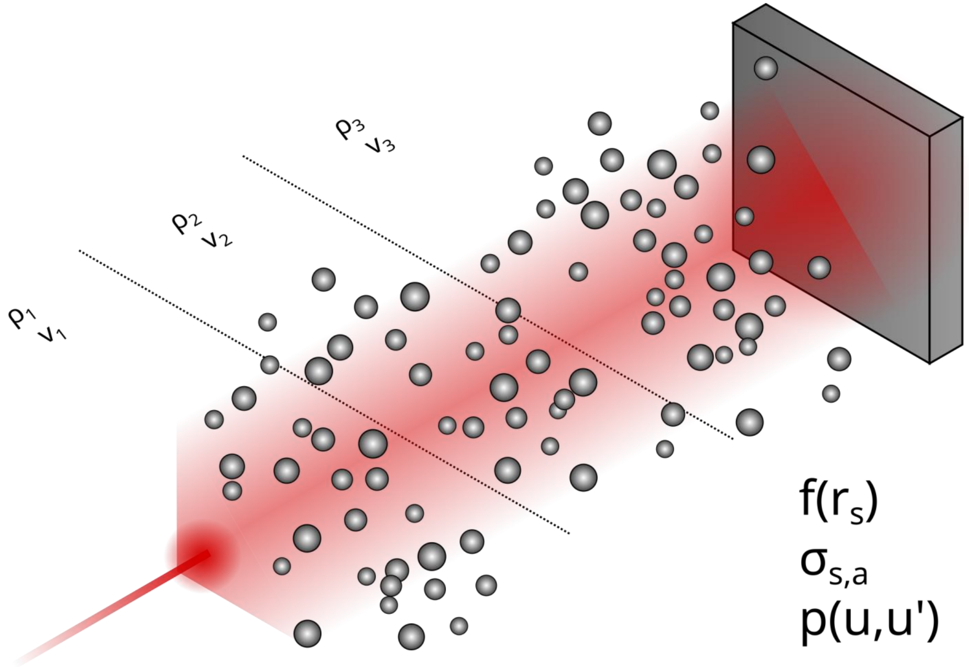
Light scattering and transport model for PDV

Simulation

Examples

Conclusion and perspectives

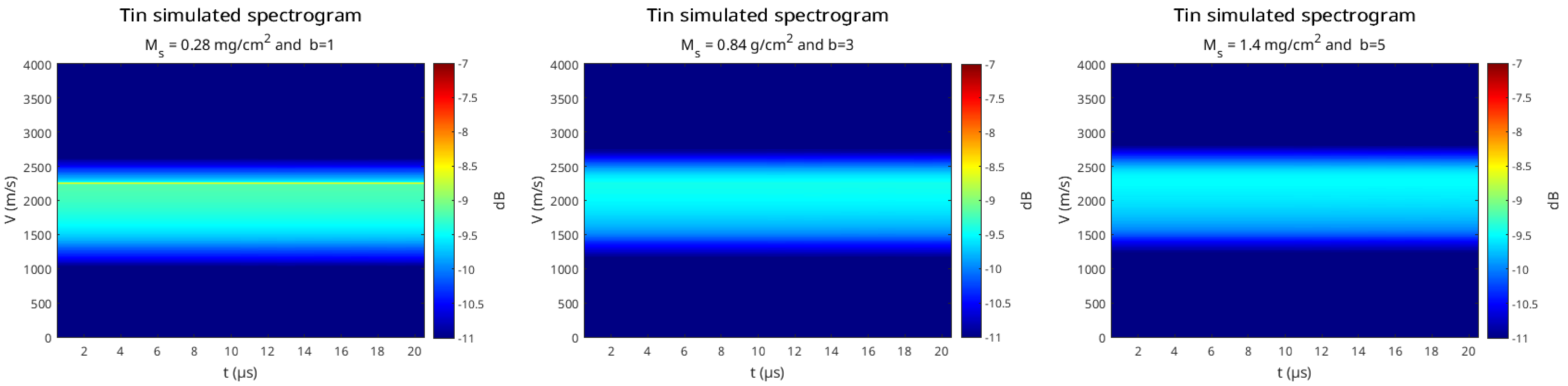
Configuration



Optical thickness

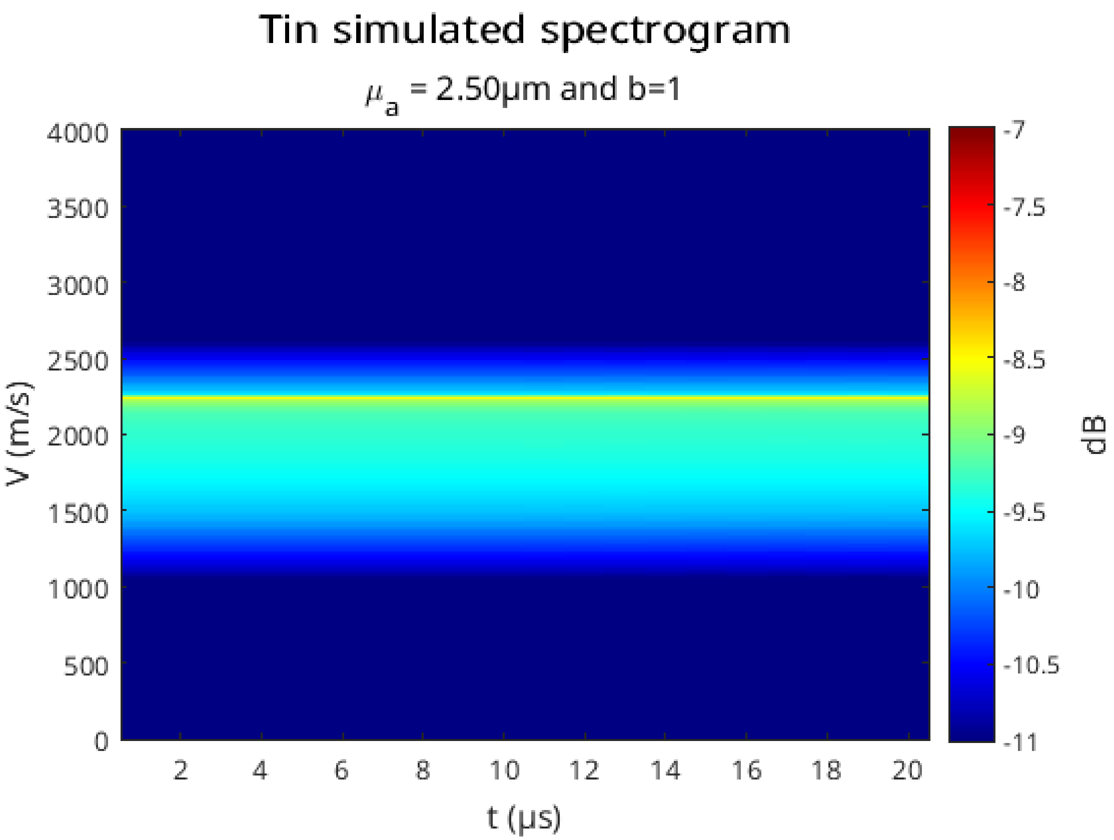
$$b = \frac{M_s \overline{\sigma_s}}{\overline{V} \rho}$$

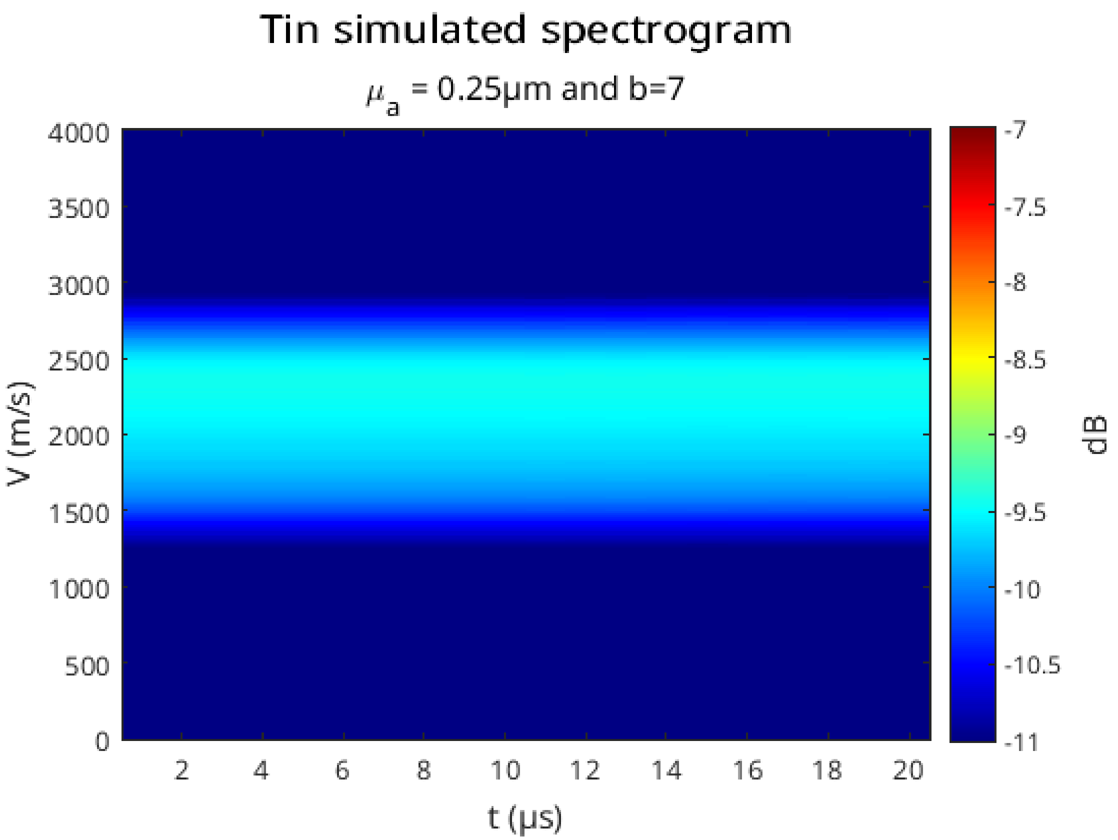
Dependency on surface mass



Uncorrelated velocity and size distribution

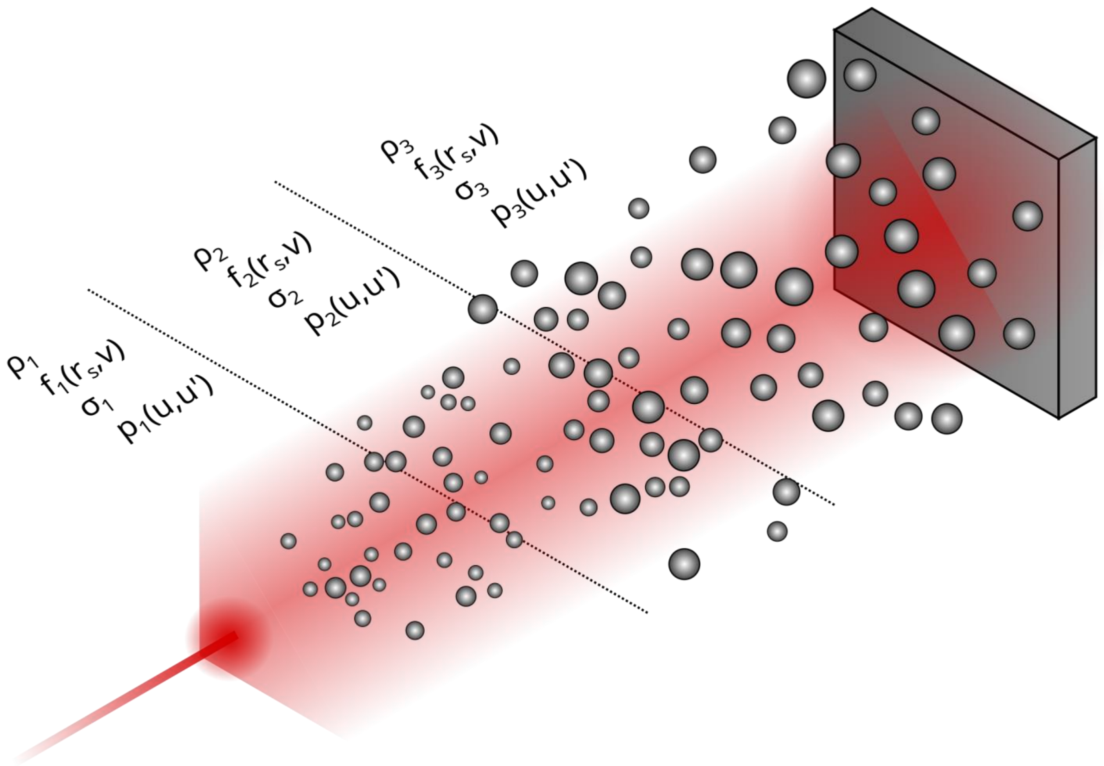
Dependency on mean particle size

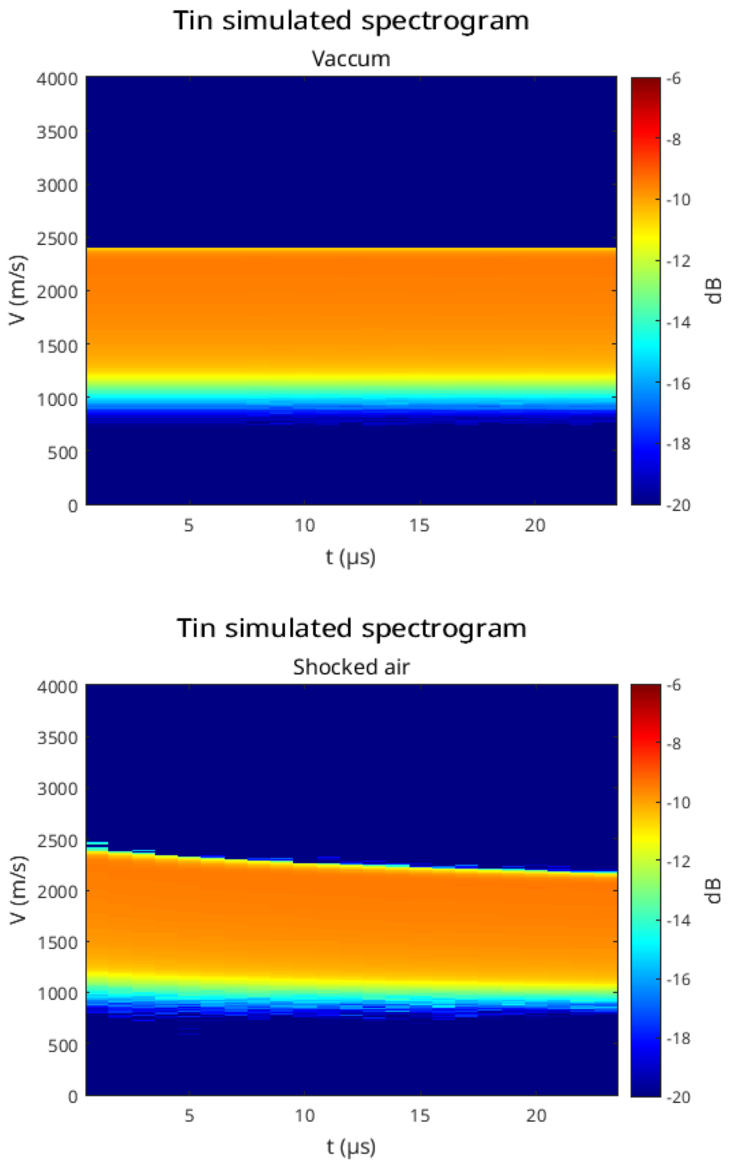




Ejecta shocked air

Configuration





Outline

Introduction

Light scattering and transport model for PDV

Simulation

Examples

Conclusion and perspectives

Outline

Introduction

Light scattering and transport model for PDV

Simulation

Examples

Conclusion and perspectives

Conclusion

- Full theoretical backing for PDV simulation : Relevance and limits.
- Novel RTE valid for any kind of ejecta : Double shock etc.
- Simulation to solve this RTE

Perspectives

- Everything derived here is general : Applies to any wavelength and geometry.
- Useful to anyone using velocimetry techniques : Biological flowmetry etc.

Article in preparation

Take home message

If you can describe your medium with a velocity and size distribution, we can compute its full expected spectrogram.

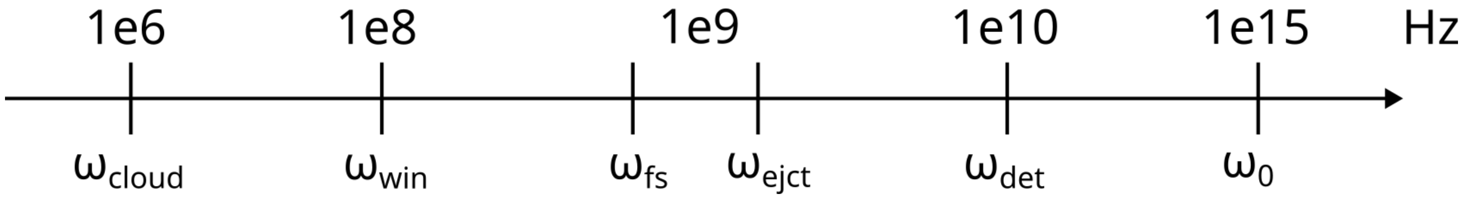
Thank you for your attention

JA Don Jayamanne

DE LA RECHERCHE À L'INDUSTRIE

Appendix

Orders of magnitude



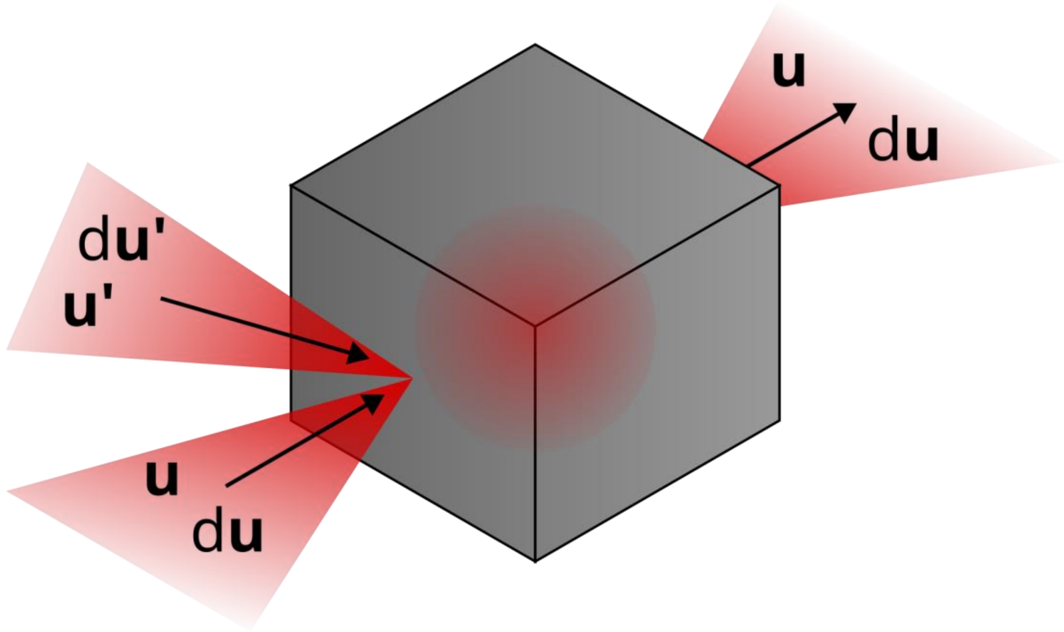
Spectrogram expression

$$S(t,\omega) = \frac{1}{T_0}|E_0|^2[I_s(t,\omega_0-\omega)+I_s(t,\omega_0+\omega)]$$

$$S(t,\omega) = \frac{1}{T_0}|E_0|^2\int_{G_{det}}[I_s(\boldsymbol{r},\boldsymbol{u},t,\omega_0-\omega)+I_s(\boldsymbol{r},\boldsymbol{u},t,\omega_0+\omega)]\boldsymbol{u}\cdot\boldsymbol{n}d\boldsymbol{u}d\boldsymbol{r}$$

Radiative Transfer Equation (RTE)

$$\left[\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{u}.\boldsymbol{\nabla}_r\right] I(\boldsymbol{r},\boldsymbol{u},t,\omega) = -\left(\frac{1}{l_s}+\frac{1}{l_a}\right) I(\boldsymbol{r},\boldsymbol{u},t,\omega) + \frac{1}{l_s}\int \frac{p(\boldsymbol{u},\boldsymbol{u}')}{4\pi} I(\boldsymbol{r},\boldsymbol{u},t,\omega)d\boldsymbol{u}'$$



Integral form of the RTE

$$I(\boldsymbol{r},\boldsymbol{u},t,\omega)=\int\frac{1}{l_s}e^{-\frac{s}{l_s}}\times p(\boldsymbol{u},\boldsymbol{u}')I(\boldsymbol{r}+s\boldsymbol{u},\boldsymbol{u}',t,\omega)ds\,d\boldsymbol{u}'$$

$$f(s)=\frac{1}{l_s}e^{-\frac{s}{\ell_s}}$$

Tweaked step length distribution

$$f(s,\boldsymbol{u},\boldsymbol{r},t)=\frac{1}{\ell_s(\boldsymbol{r}+s\boldsymbol{u})}\exp\left[-\int_0^s\frac{1}{\ell_s(\boldsymbol{r}+s'\boldsymbol{u})}ds'\right]$$