



Modulation Based Ranging (MBR) Analysis of Multi-Surface Data

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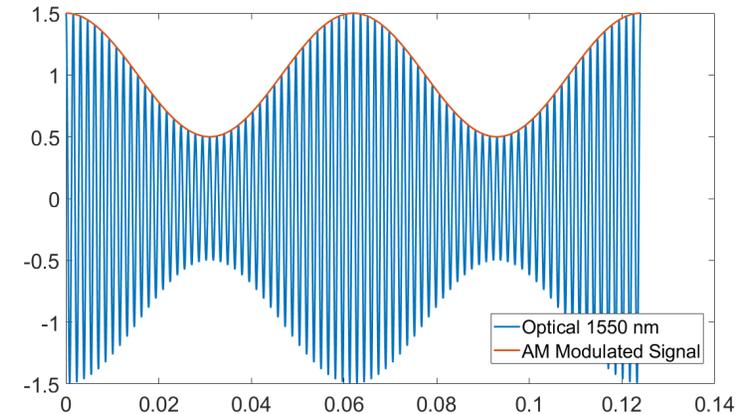
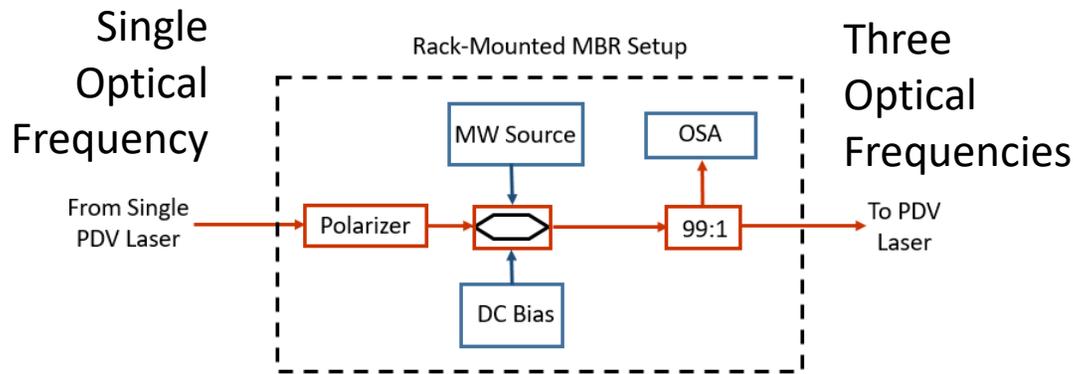
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Outline

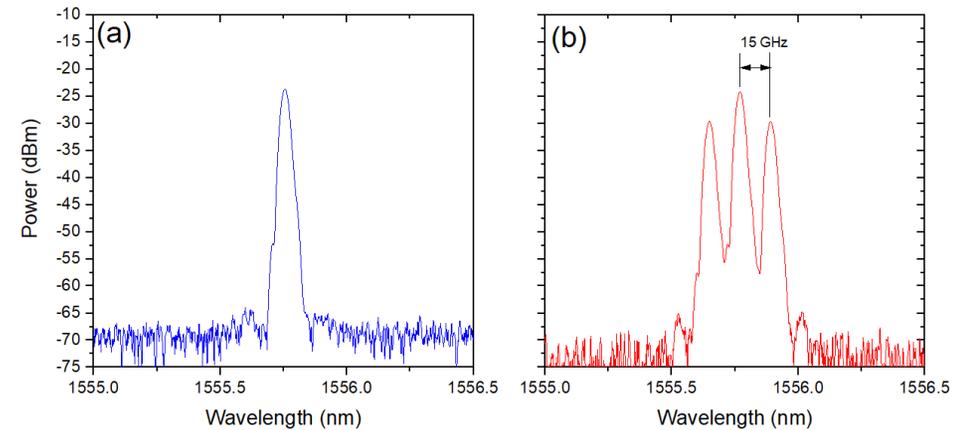
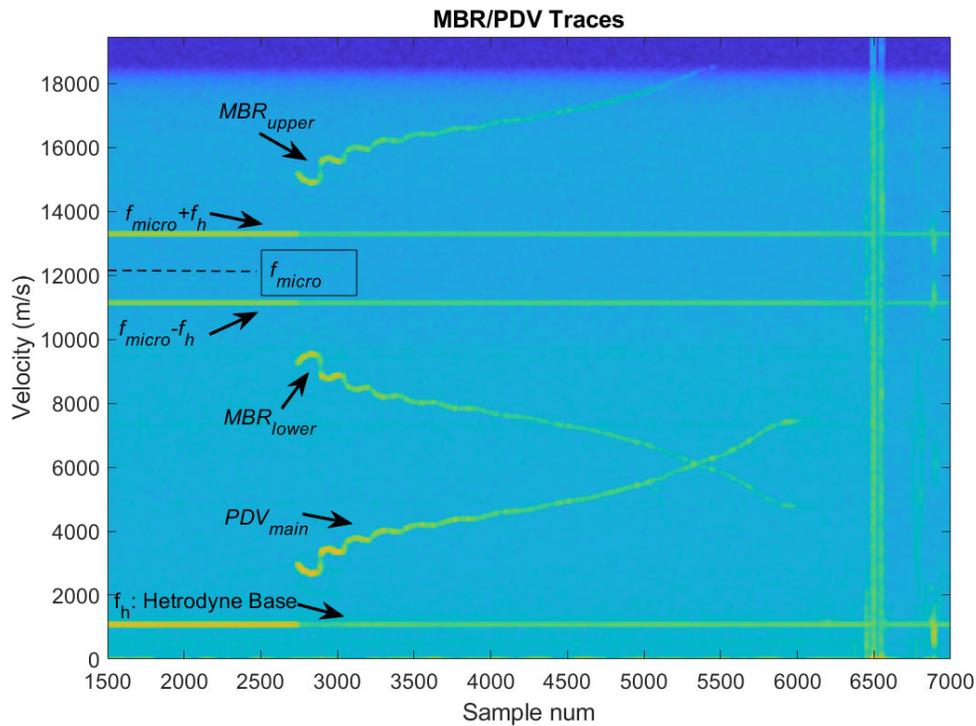
- What is Modulation Based Ranging (MBR)?
- One-surface MBR analysis
- Multisurface and clouds analysis of MBR data
- Example results
- Conclusions and future work

Principle of Displacement Measurements with MBR



- Amplitude modulation at microwave frequencies (~ 15 GHz)
- Displacement extracted from counting the fringes (phase) of this modulated signal
 - Analogous to counting the 1550 nm fringes in a Michelson interferometer expect we are counting the ~ 10 mm half-wavelength fringes from amplitude modulation (6,500 longer than optical wavelength).
 - $\sim 1/100$ wavelength resolution: Greater than 0.1 mm resolution. Dependent on SNR.
- Three distinct frequencies are created by the modulator and sent to the surface.
- After returning, the signals are mixed with the LO – no difference from PDV.

MBR Spectrograms



$$f(t) = c_0 \cos(d_0 + p_1 t) + c_1 \cos(d_1 + p_1 t + f_{microwave}) + c_2 \cos(d_2 - p_1 t + f_{microwave})$$

- After frequency mixing, three dynamic frequencies of interest are created
- PDV signal is conserved

Math and Derivation of MBR

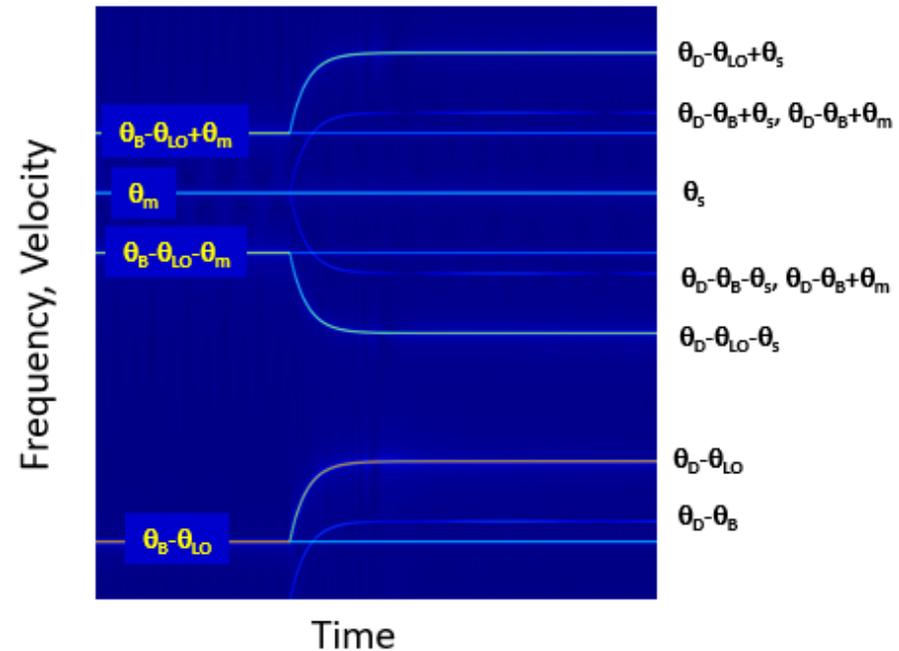
- Heterodyne mixing
 - 21 total frequencies (baselines, out of bandwidth)
 - 3 strong dynamic frequencies of interest
- Dynamic doppler shifted PDV signal (θ_D) present in 'Main', 'Upper', and 'Lower'
- Dynamic microwave signal (θ_S) present in 'Upper' and 'Lower'
 - (θ_S) present at microwave frequency, but very weak (non-heterodyne) signal
- Solve for (θ_S) by subtracting arguments of Upper/Lower relative to the main

$$Displacement = -\frac{Phase * c}{4\pi f_{microwave}}$$

Signals on photodetector

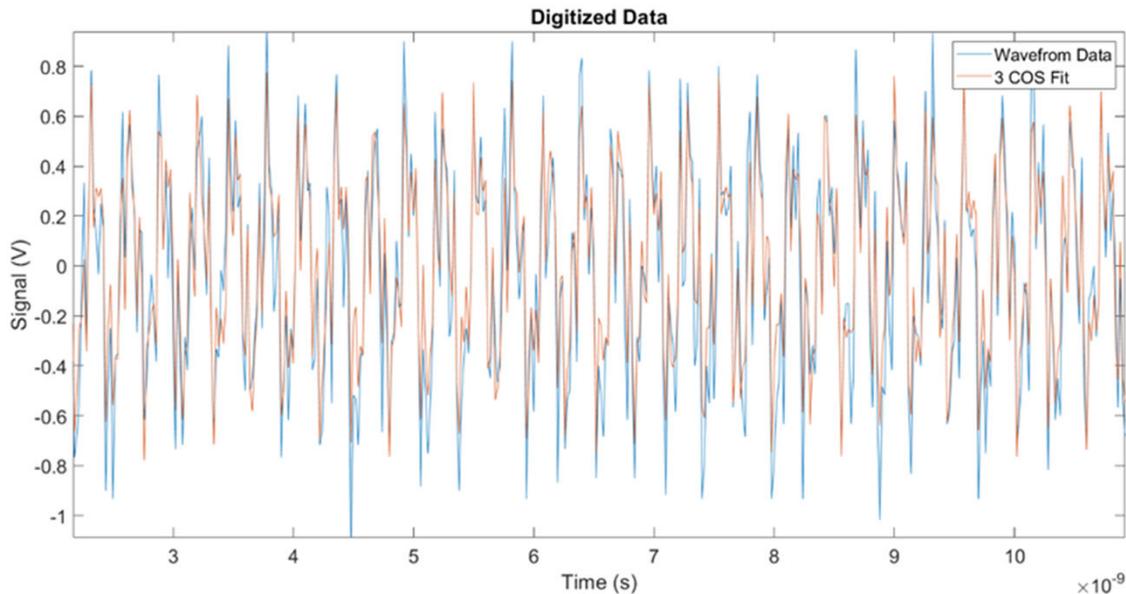
$$[A_{PDV} \cos(\theta_B) + A_{sb} \cos(\theta_B + \theta_m) + A_{sb} \cos(\theta_B - \theta_m) + A_{PDV} \cos(\theta_D) + A_{sb} \cos(\theta_D + \theta_S) + A_{sb} \cos(\theta_D - \theta_S) + E_{LO} \cos(\theta_{LO})]^2$$

Simulated spectrogram



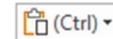
Three $A_{pdv} E_{LO} \cos(\theta_D - \theta_{LO})$
Dynamic $A_{pdv} E_{LO} \cos(\theta_D - \theta_{LO} + \theta_S)$
Terms $A_{pdv} E_{LO} \cos(\theta_D - \theta_{LO} - \theta_S)$

Time-Domain Three Cosine Fitting of MBR Data



- Three-cosine fitting models MBR as a sum-of- sinusoidal waveforms
 - Phases are used to find MBR displacements
 - Frequency is used to find velocity

$$f(t) = c_0 \cos(d_0 + p_1 t + p_2 t^2) + c_1 \cos(d_1 + p_1 t + p_2 t^2 + f_{\text{microwave}}) + c_2 \cos(d_2 + p_1 t + p_2 t^2 - f_{\text{microwave}})$$



Baseline: $v(t) = a_0 \cos(p_0 + p_1 t)$

Dynamic: $v(t) = (a_0) \cos(p_0 + p_1 t + p_2 t^2)$

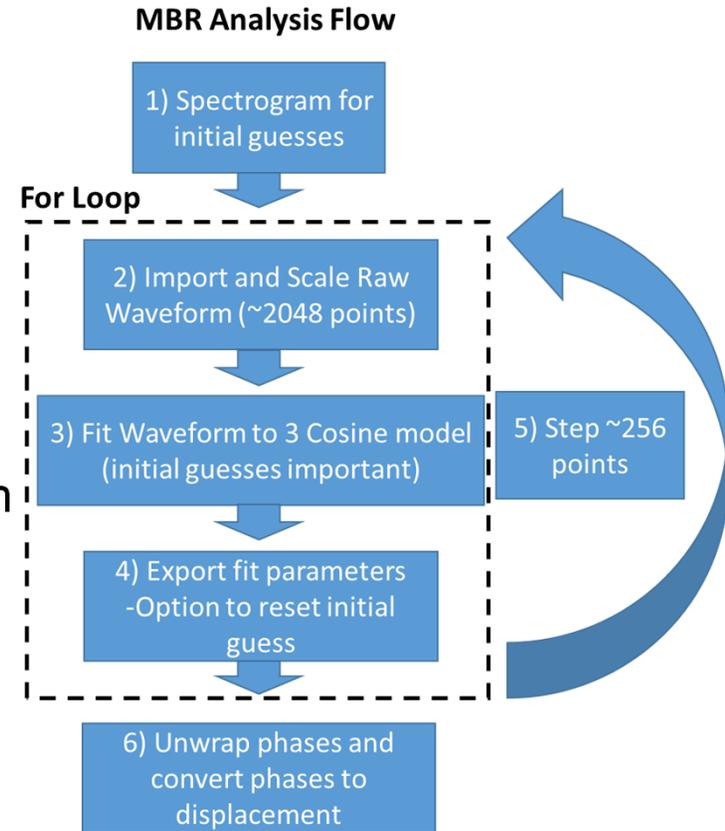
a_0 = amplitude; p_0 = phase; p_1 = velocity; p_2 = acceleration

$$\text{Displacement} = -\frac{(d_0 - d_1) * c}{4\pi f_{\text{microwave}}}$$

Details on Fitting the Data in Time-Domain

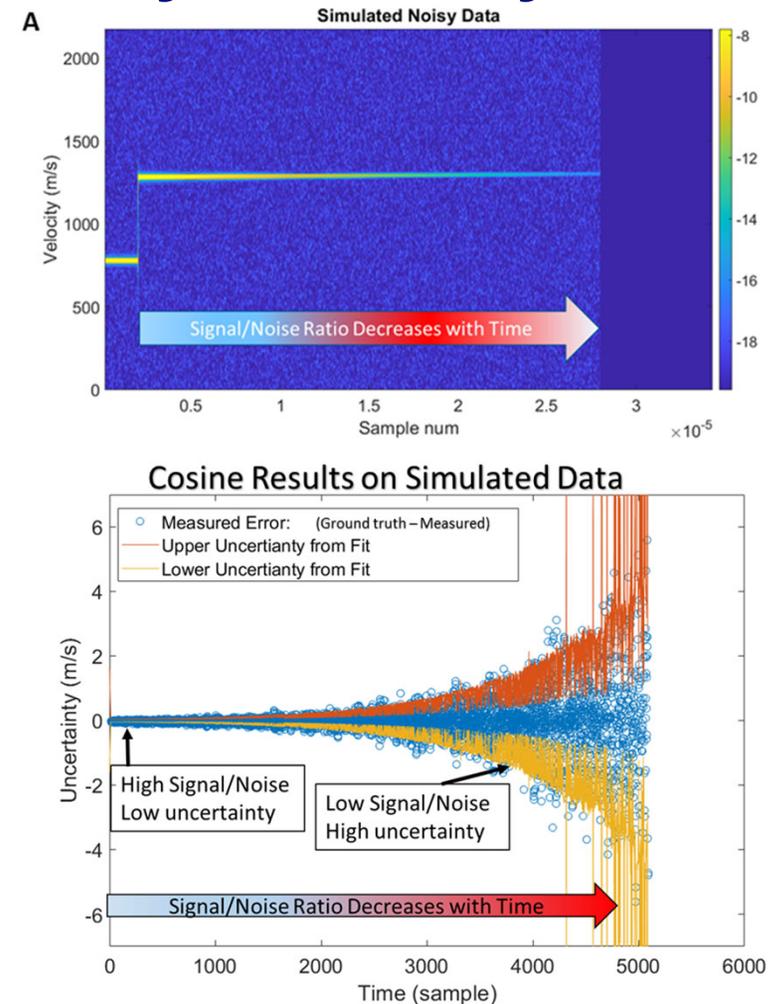
- Non-linear least squares fitting
 - Scaling and initial guesses are important
- Simultaneous velocity and position measurements though frequency (velocity) and phase (displacement) terms
- Eight unknowns
 - Three phases, three amplitudes, frequency, acceleration
- Three knowns (time, microwave frequency, reference time (T_{micro}))

$$\begin{aligned}
 f(t) = & c_0 \cos(d_0 + 2\pi(f_h + f_{main})t + p_2 t^2) \\
 & + c_1 \cos(d_1 - 2\pi(f_h + f_{main})t + 2\pi(f_{microwave})T_{mico} - p_2 t^2) \\
 & + c_2 \cos(d_2 + 2\pi(f_h + f_{main})t + 2\pi(f_{microwave})T_{mico} + p_2 t^2)
 \end{aligned}$$

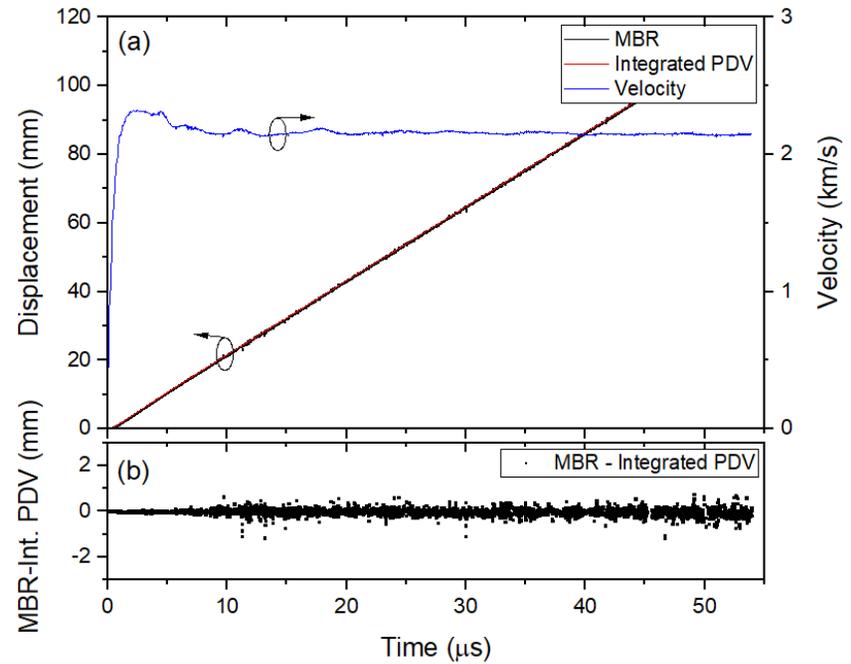
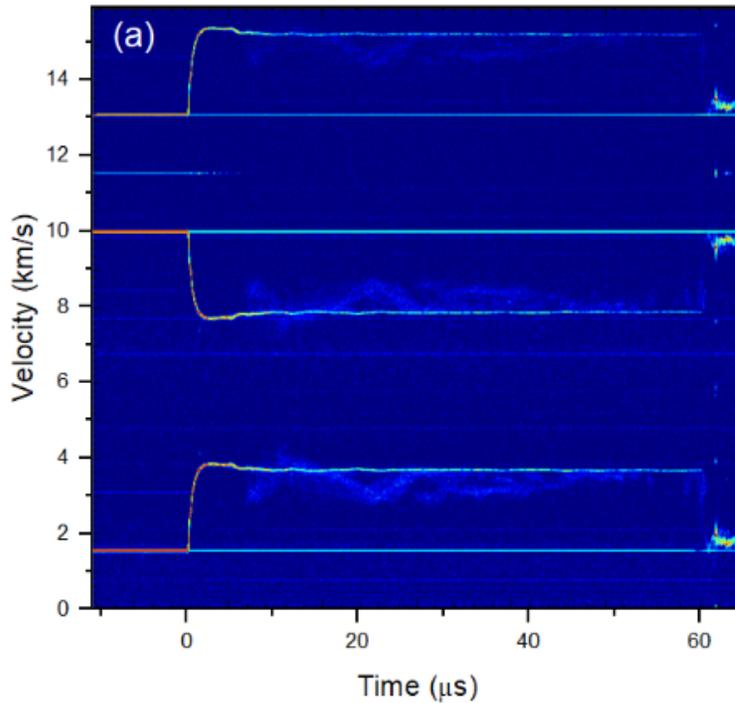


Time-domain fitting predicts uncertainty in velocity and position

- A simulated velocity trace with decreasing signal/noise ratio over time
- Compare values
 - Error = (Measured velocity) – (Known Input)
 - Uncertainty predicted from Cosine Model (Δf)
- The error (defined as the difference between the known velocity input and the extracted velocity from fitting) is shown to be well bounded for ~95% of points by the uncertainty predicted from the cosine-fitted model
- Uncertainty in position extracted from confidence interval of phase terms

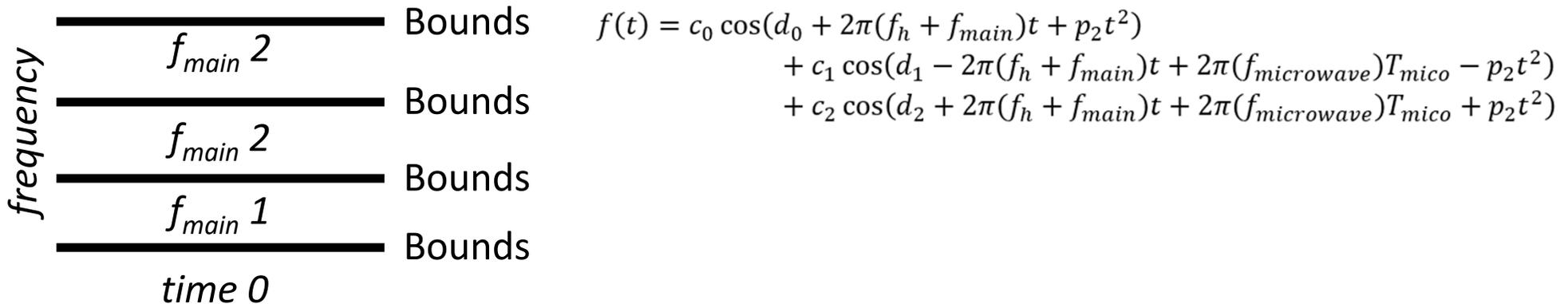


Example of MBR Data on Radial Flyer Plate Test



- Comparison of PDV to MBR
- RMS less than 100 μm

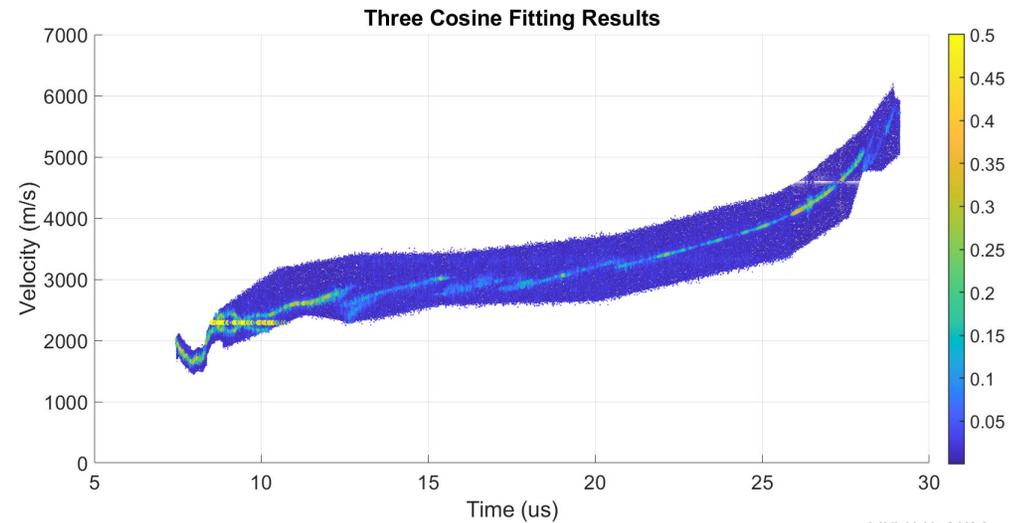
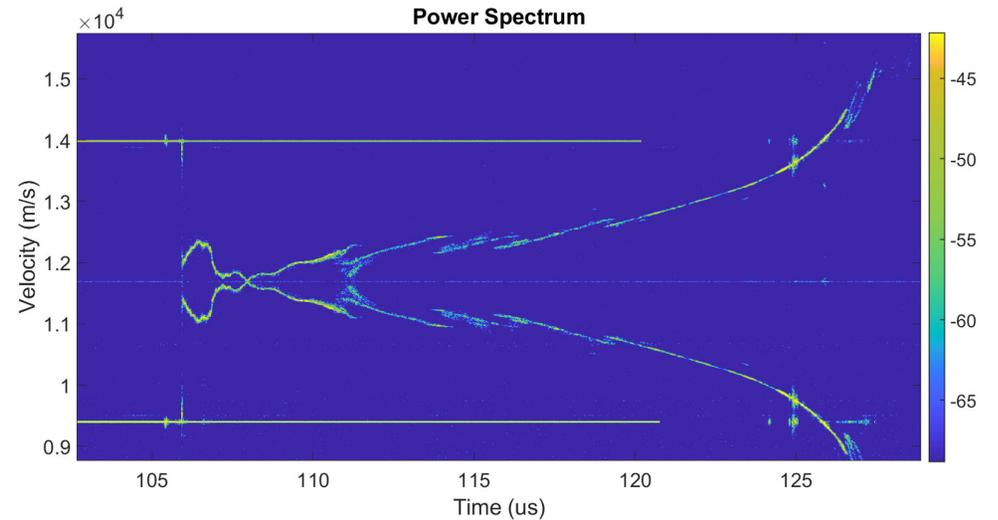
Analysis of Multi-surface MBR data



- Apply three COS fitting over a range of frequencies
 - Analogous to FFT or spectrogram
 - Time-domain fitting using three-cosine model across range of frequencies
- Bin each frequency and force non-linear least squares fitting to converge within each frequency bin
- Threshold and extract MBR displacement data

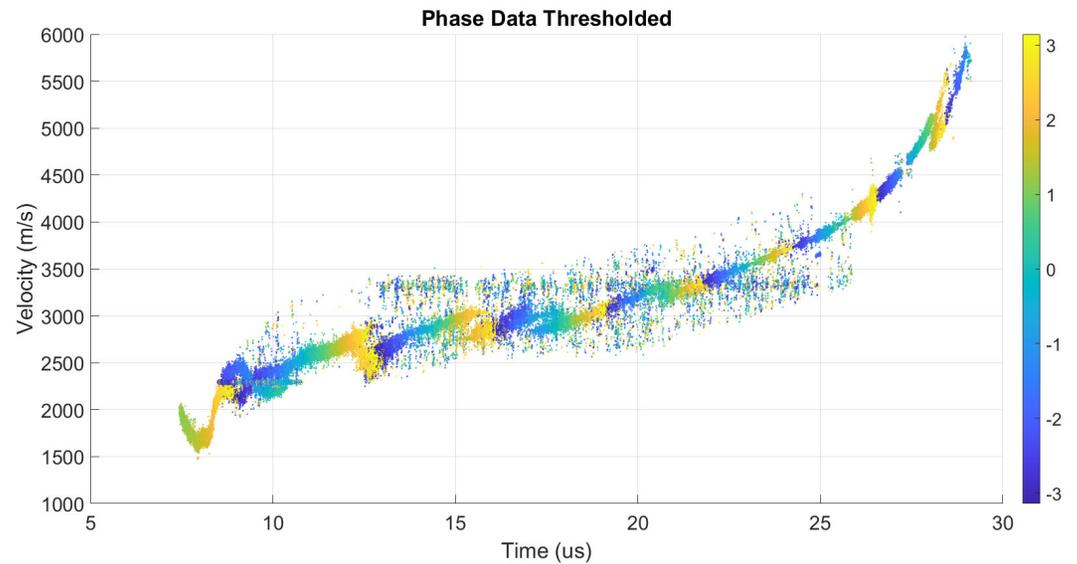
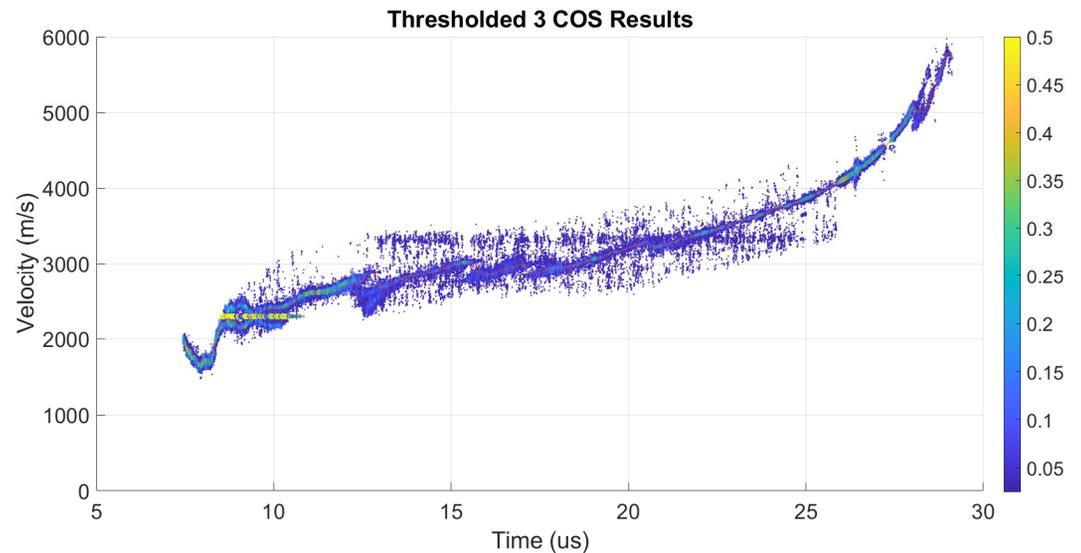
Example of Multi-surface MBR Analysis

- Three-cosine results from time-domain are consistent with spectrogram results
- Each scatter point is amplitude weighted from three-cosine model
- This trace shows regions with multiple velocities

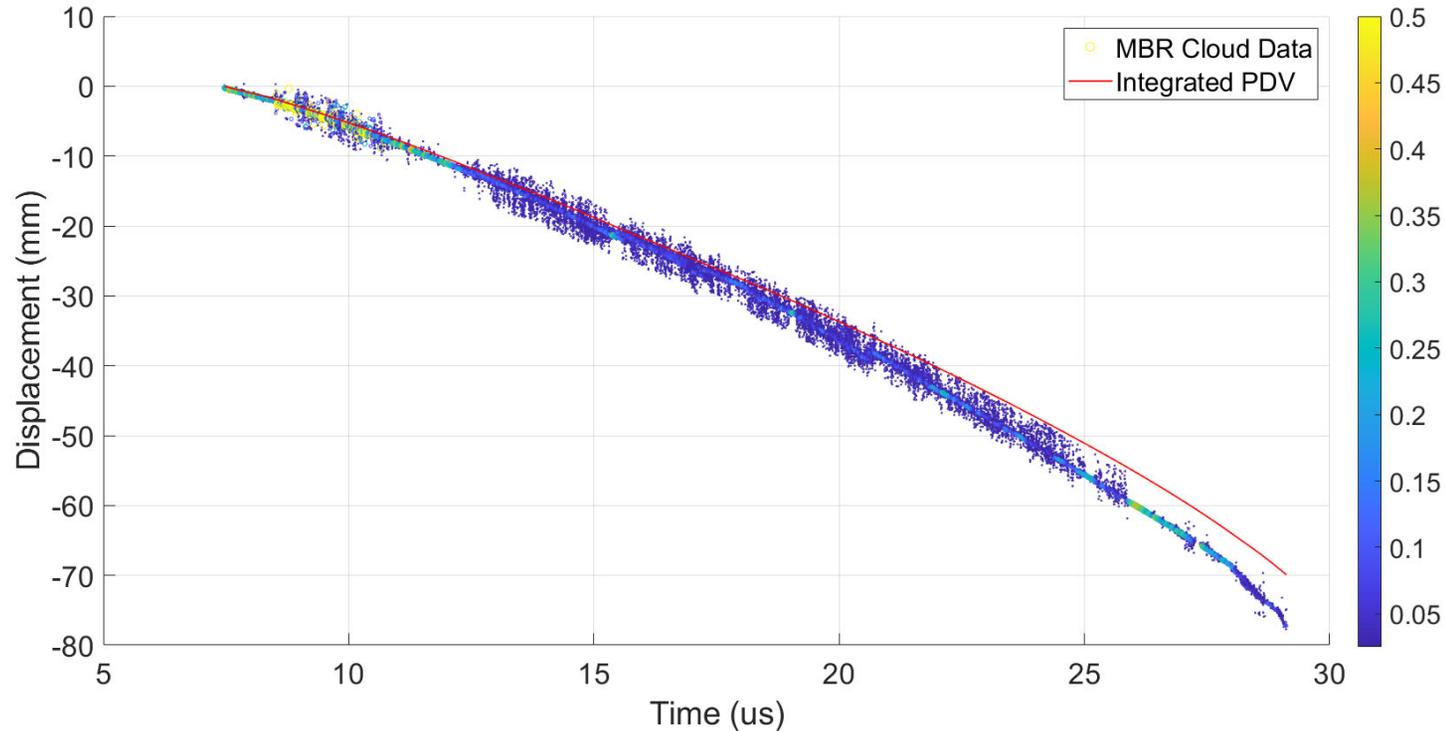


MBR Thresholded Data

- Data is thresholded prior to displacement extraction
 - Similar to PDV cloud data- what is above background noise?
- Extract and unwrap MBR phase data to get displacements

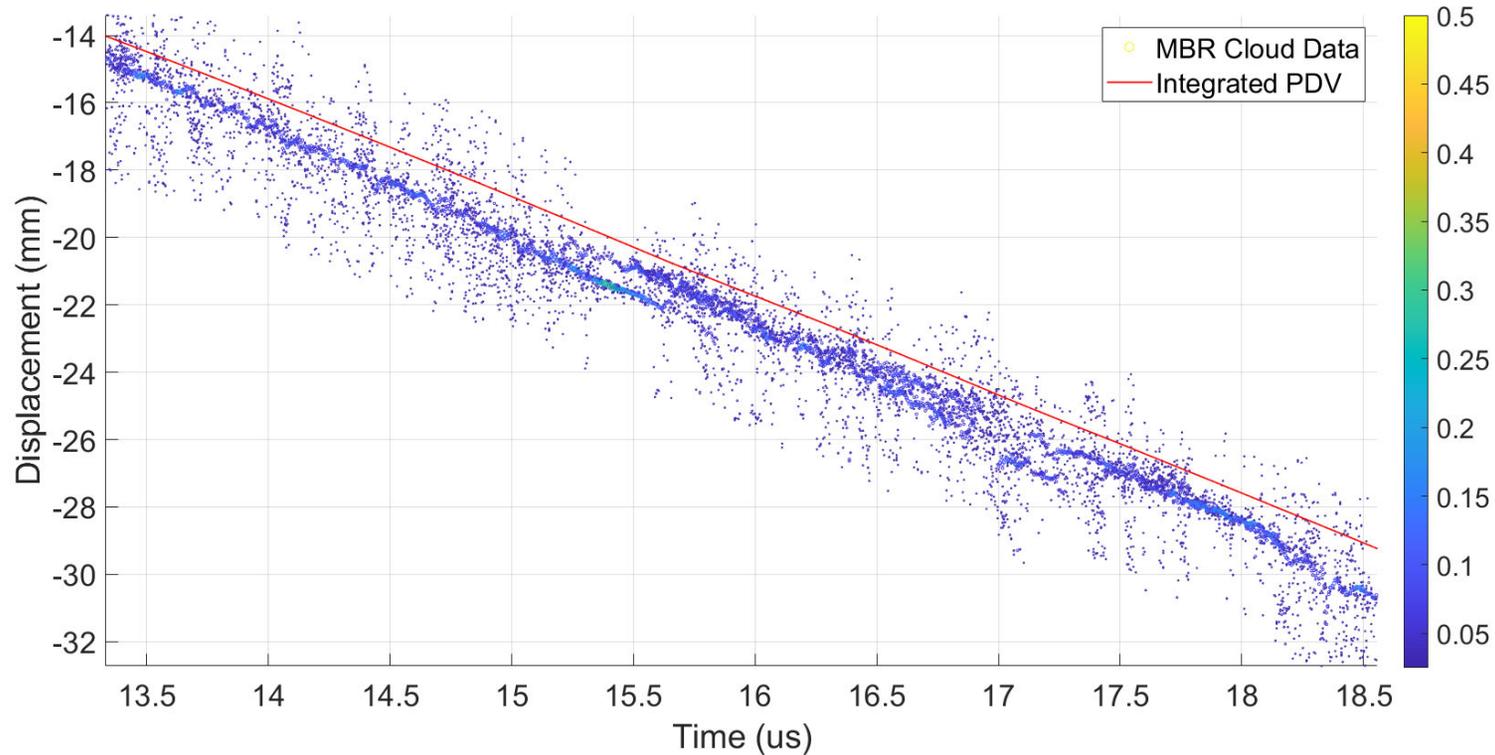


MBR and Cloud Displacements



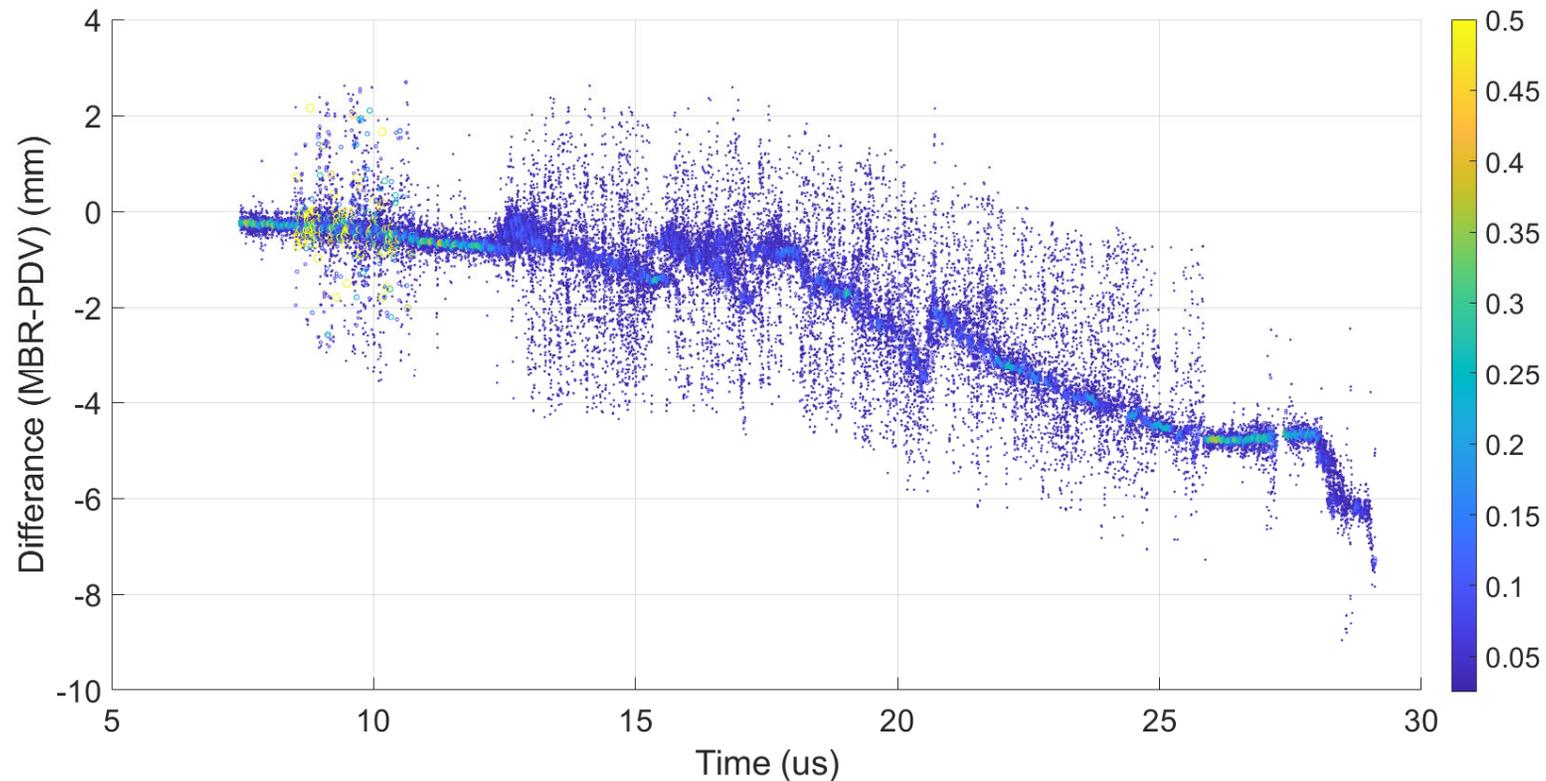
- Discontinuous 'jumps' with surface fragmentation
- Areas of multiple surfaces
- Significant differences between PDV and MBR

Close-up of Multiple Positions



- MBR simultaneously measures multiple surfaces in beam line of sight that are ~ 1 mm apart

Comparing PDV to MBR



- Discontinuous jumps in position (cannot be accurately integrated without prior knowledge)
- Positions with multiple strong velocities

Conclusions

- MBR is a powerful ranging diagnostic
 - Greater than 0.1 mm resolution (50 μm RMS seen over whole experiment on quality trace)
 - PDV data along same beam allows for simultaneous position and velocity measurements
 - Inexpensive and simple! Cost just a few percent of the total cost of a MPDV system
 - Easy to multiplex \rightarrow Huge cost savings. We have 192 points of MBR/PDV (8 slices in time-multiplexed system)
- We have updated our analysis to measure multiple surfaces or distributions of surfaces (clouds) of data
- Clouds, ejecta, and multiple-surfaces can be discontinuous showing 'jump' in position
 - Cannot get their position directly from integrated PDV

Questions?