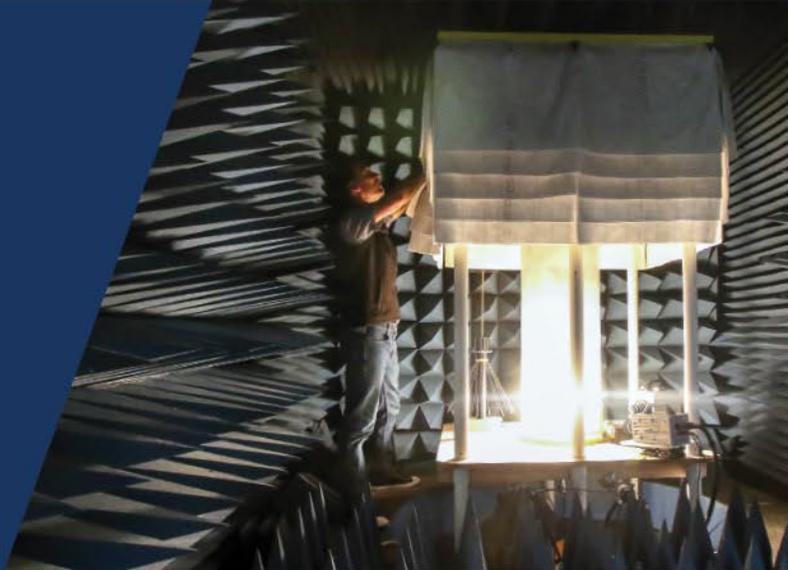




# Non-uniform window phenomena and correction options for PDV



Daniel Champion, NNSS

Dillon Yost, LLNL

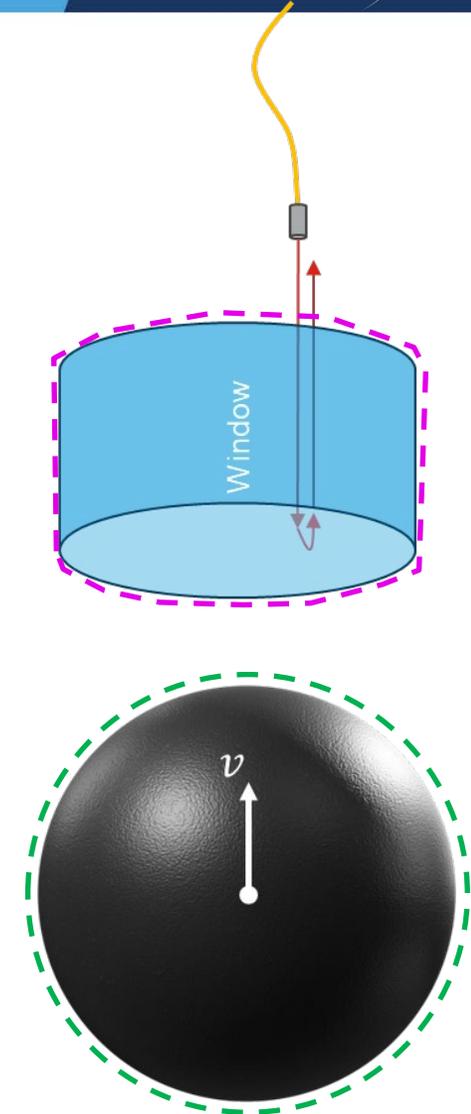
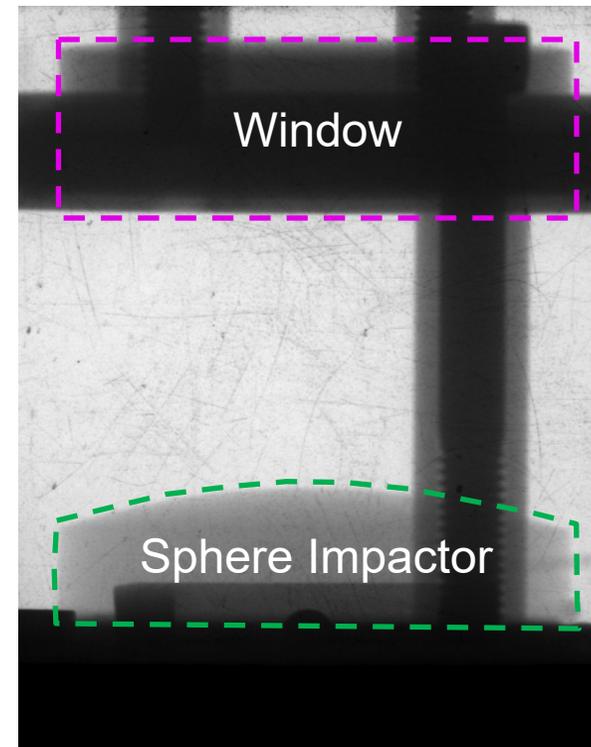


**With acknowledgements-to  
and contributions-from:**

- Shelly Rhodes, LLNL
- Garry Maskaly, LLNL
- Brandon LaLone, STL
- Jerry Stevens, STL
- Boom Box Team, STL

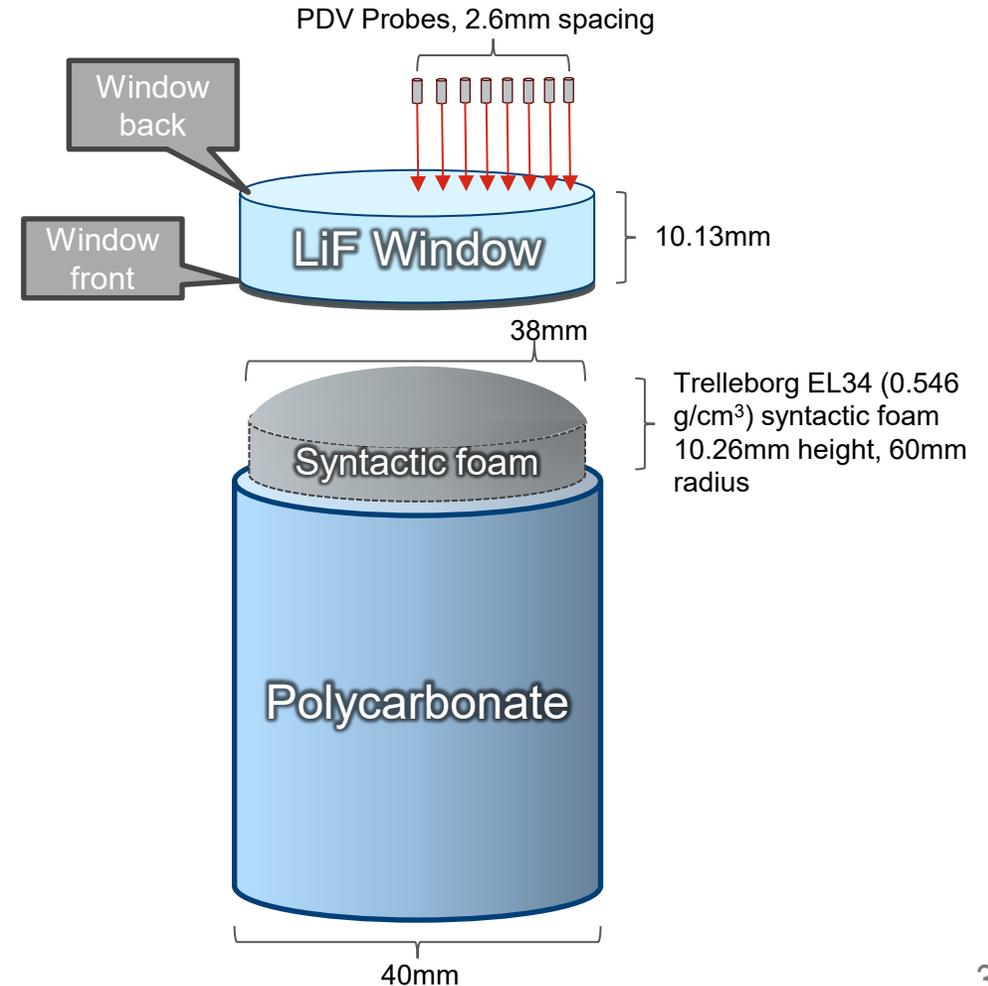
# Introduction, overview, H. catechism

- ▶ *What are we trying to do?*
  - Transform apparent PDV velocity to true velocity in window experiments.
  - Specific goal: accurately interpret and obtain unbiased velocimetry measurements from non-uniform window experiments using PDV.
- ▶ *How is it done today?*
  - Current analysis methods assume 1D behavior in windows and utilize 1D window correction factors (for some materials).
- ▶ *What is new in the approach?*
  - We extend the 1D theory of window corrections to produce a dynamic correction term to account for some 3D effects present in 3D non-uniform experiments, and estimate with forward modeling.
- ▶ *What impact does it make?*
  - Non-uniform corrections can explain counter-intuitive apparent PDV velocities:
    - Negative apparent velocities
    - Mollified/gradual shocks
    - Nonzero velocity prior to shock arrival
  - Non-uniform window corrections can decrease measurement errors (observed experimentally at the 10-20% level).
- ▶ *What are the risks/costs/challenges?*
  - Introduction of forward modelling into velocimetry analysis/sense-making.
- ▶ *How are we verifying success?*
  - Gas gun experiments with known density impactors engineered to produce non-uniform drives on a window



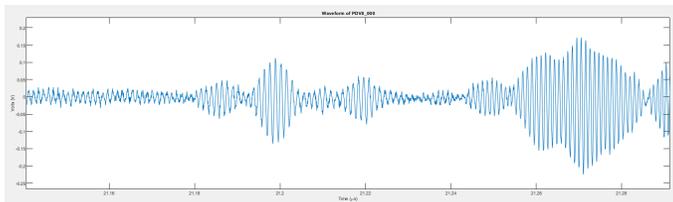
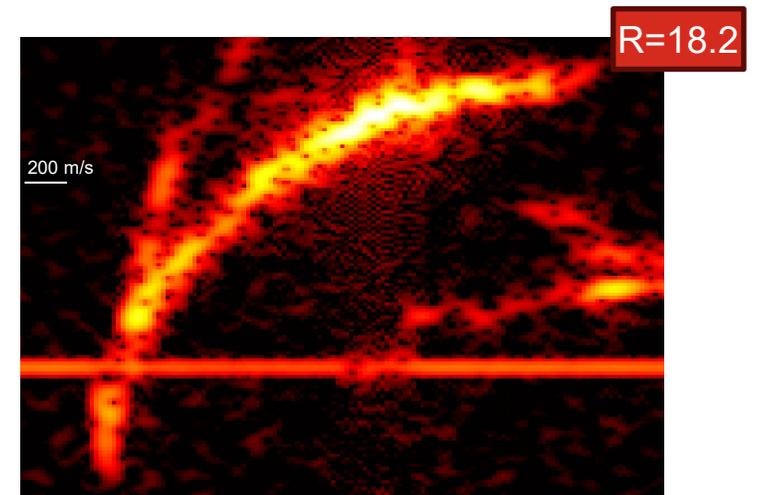
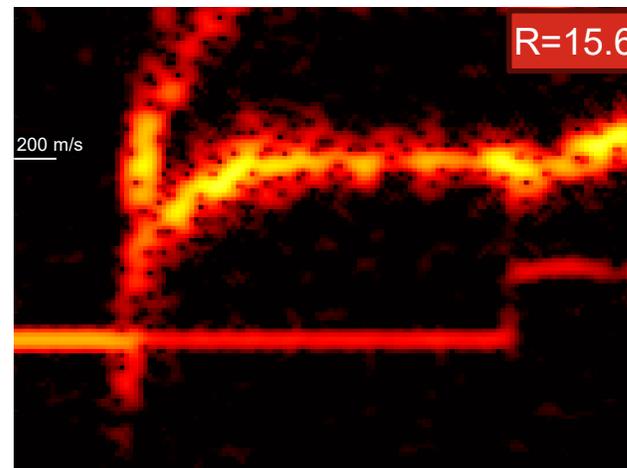
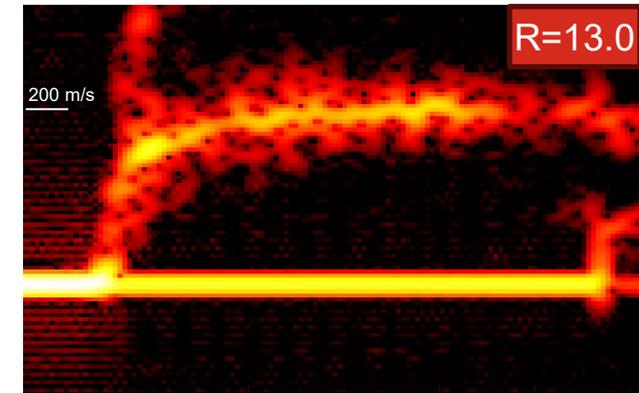
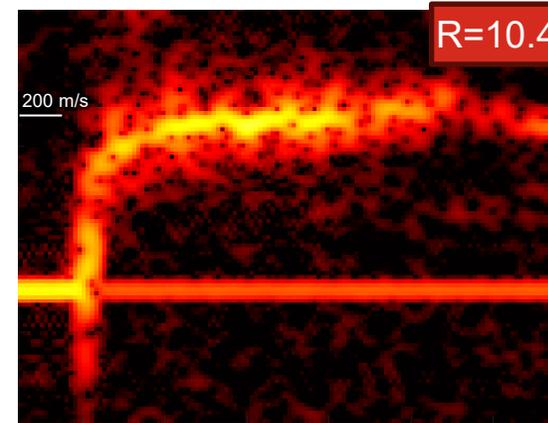
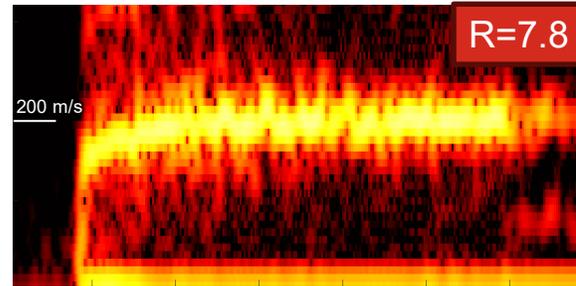
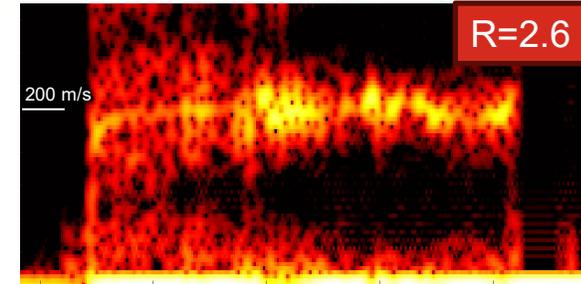
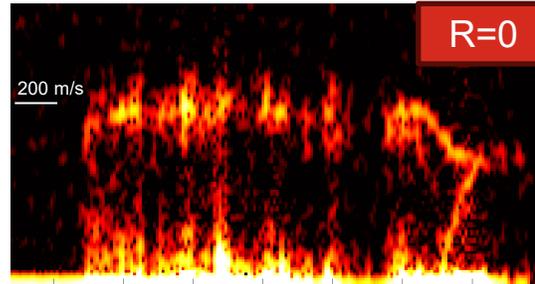
# Foam dome demonstrator and experiment

- ▶ In this talk we will make use of a simple spherical impactor colliding with a cylindrical LiF Asay window as a **demonstrator** and as an **experimental** analysis objective.
  - (Asay, 1978), (Chen et al., 2017)
- ▶ The spherical impactor provides a simple **known-geometry** and **known-answer** non-uniform drive on the window.
- ▶ The “foam dome” experiment was conducted at the NNS Special Technology Laboratory Gas Gun by the STL Boom Box Team (STL PG Exp: 22110):
  - 10.13mm thick LiF Asay window
  - 0.46mm thick Al buffer (PDV reflects off of this surface)
  - Trelleborg EL34 ( $0.546 \text{ g/cm}^3$ ) syntactic foam impactor
  - 1.81 km/sec impactor velocity
  - 8 PDV probes spanning the 0->18.2mm radius range
- ▶ Note the Aluminum (impedance matched) buffer layer, this provides a protected PDV reflection surface and offsets the PDV reflection point into the interior of the window (slightly)
  - Buffered windows introduced: (Chen et al., 2017)



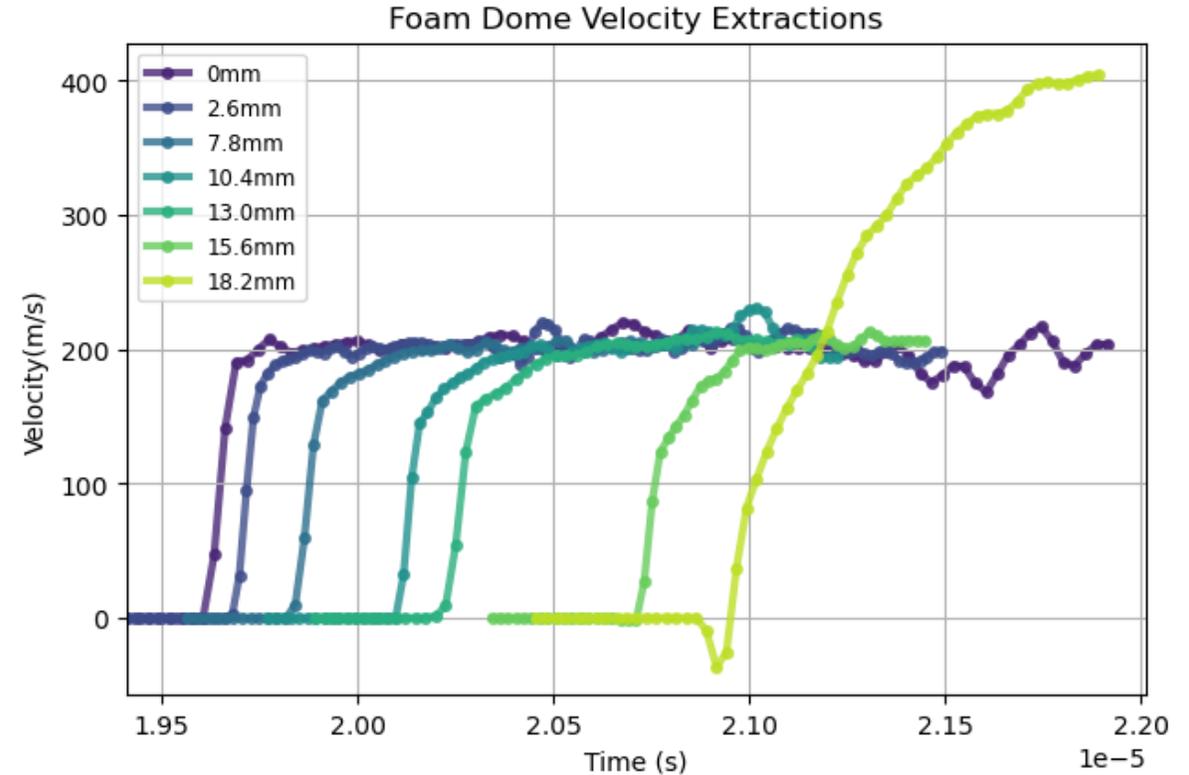
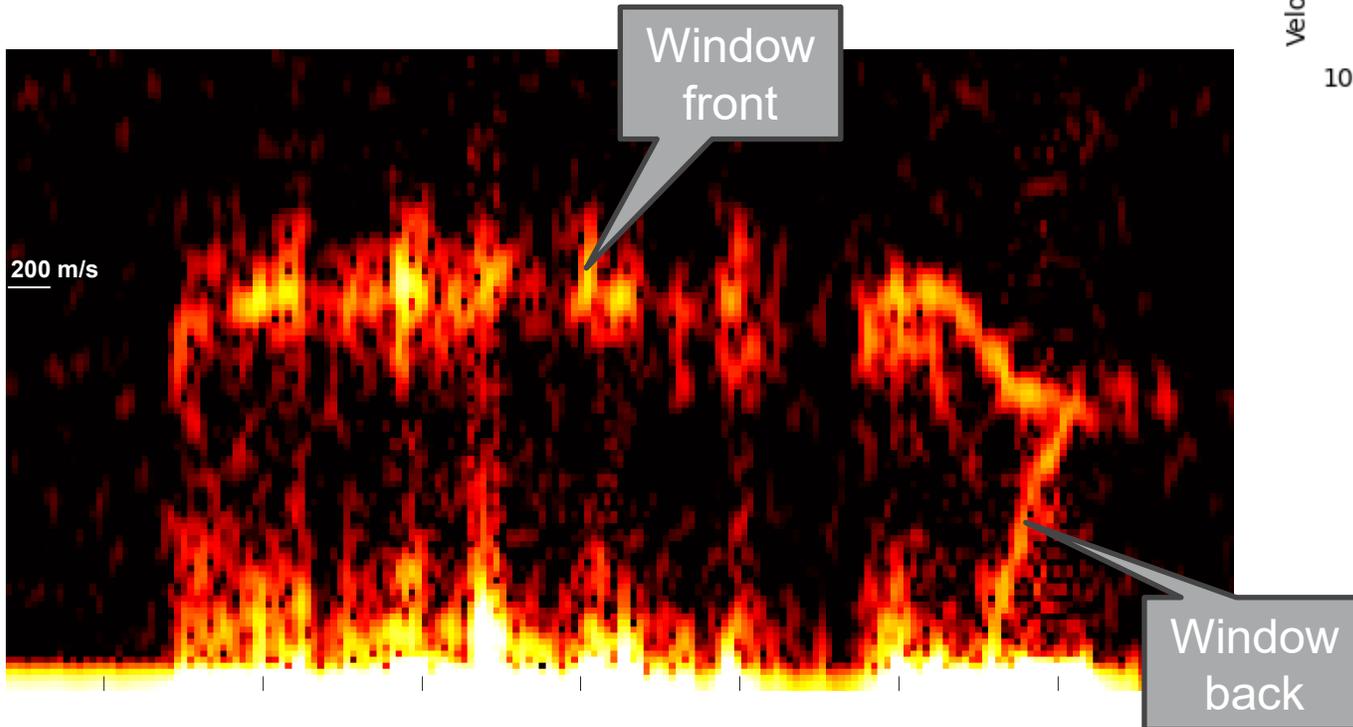
# Foam dome demonstrator and experiment

- ▶ PDV obtained from the experiment shows negative apparent velocities at larger radii
- ▶ The velocity curves at larger radii also show gradually increasing (accelerating) velocity in contrast to the shock-like velocity curves at the center of the window



# Foam dome demonstrator and experiment

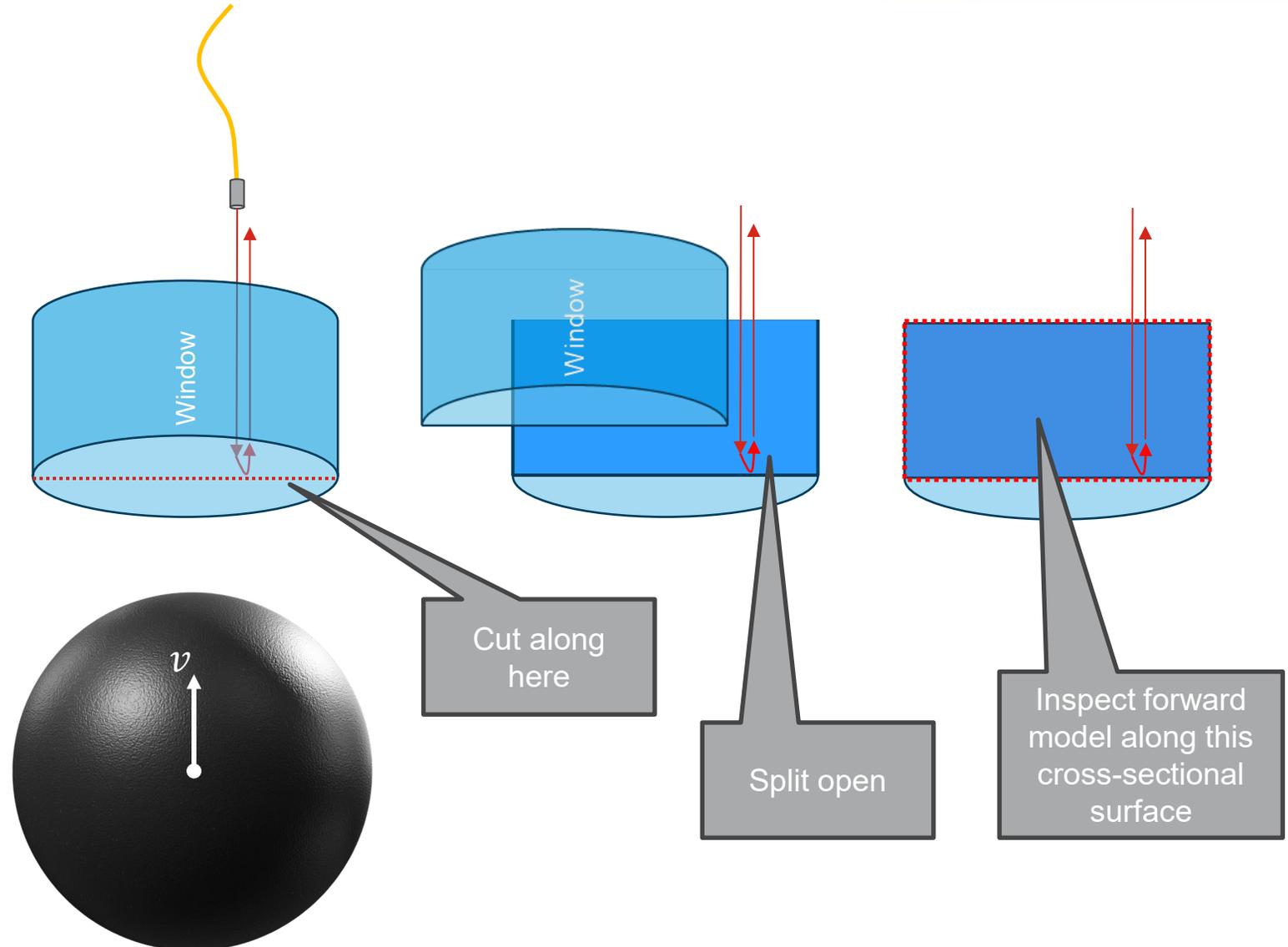
- ▶ Window front and window back PDV signals are present in foam dome experiment data.
- ▶ Our primary interest is in the window front surface signals prior to the rise in velocity caused by shocks from the polycarbonate collisions reach the PDV point.



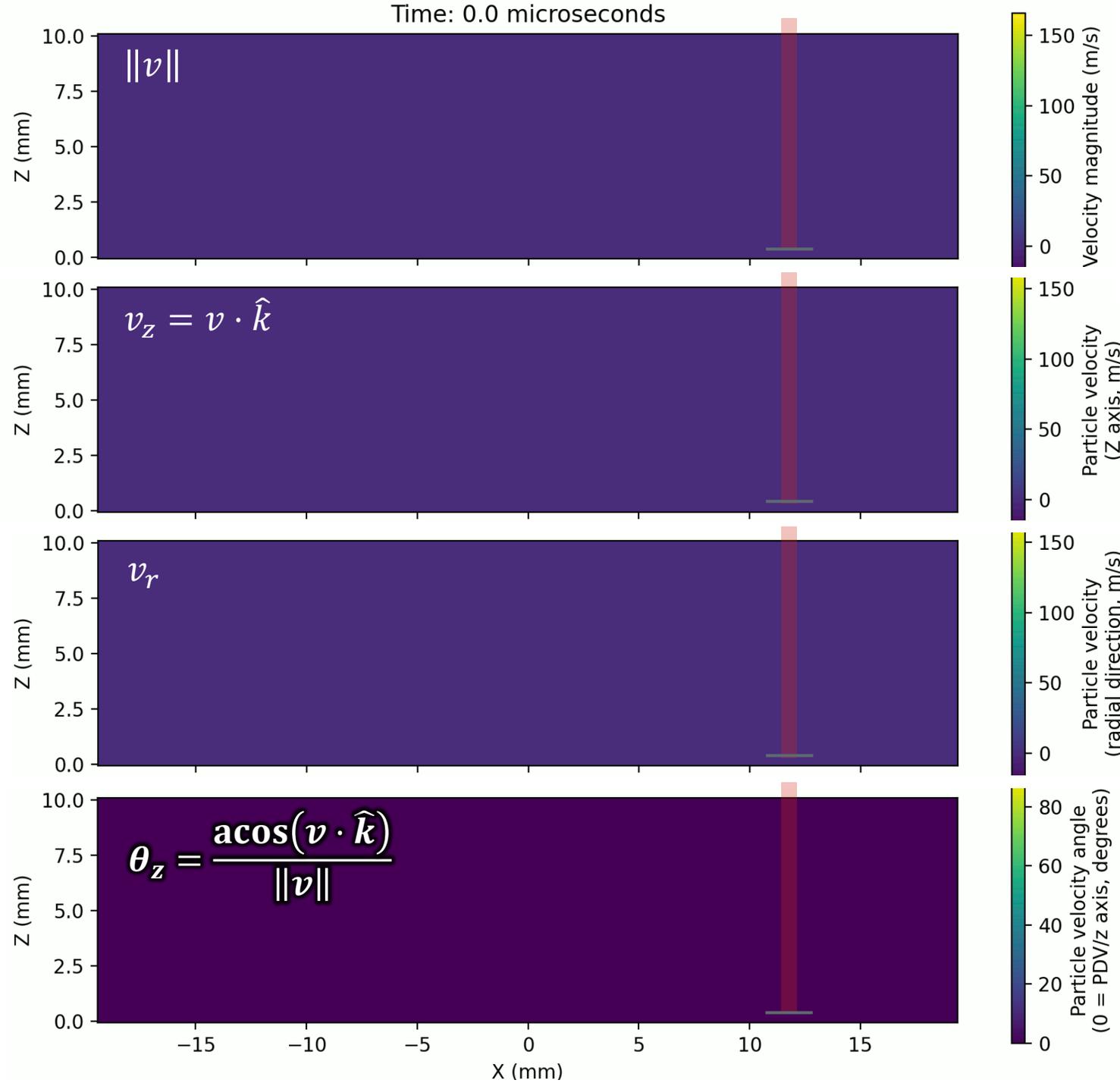
- ▶ At R=18.2mm, we think the  $>200$  m/s velocities correspond to the arrival of edge release at the diagnostic point.

# Lets peek inside the window during the experiment

- ▶ The foam dome has been studied with several forward models to assist with the interpretation and analysis of the PDV data
  - CTH model courtesy Jerry Stevens (STL)
    - (McGlaun, et al., 1989)
  - 3D Lattice Spring Model (LSM),
    - Simple “ball and spring” model with non-Hookean springs.
    - Based on: (Li et al., 2020), (Grady, 2017), (Hockney, 2021)
  - LLNL ARES forward model
    - (Rieben, 2008)
- ▶ Forward modelling allows for inspection of internal particle behavior in the window.
- ▶ In the follow slide we cut open the window to show a side view cross-section.



# Foam Dome Forward Model Cross Section



- As the 3D shock enters into the PDV beam, it does so asymmetrically
- The transverse particle motion (outward radial velocity) results in particles flowing into and out of the beam *unequally*.
- Mass within the beam path is not constant, it dynamically changes based on the varying transverse particle motion throughout the extent of the beam.
- “PDV measures  $\dot{n}$ ” – Ed Daykin
- Dynamic mass  $\rightarrow$  dynamic density  $\rightarrow$  dynamic  $\dot{n}$

# Theory and Math Slide

- ▶ We follow the 1D window correction derivation from (Jensen et al., 2007).
- ▶ Let  $v_{pdv}(t)$  be the apparent PDV velocity, let the position of the window front and back surfaces be given  $x_{front}(t), x_{back}(t)$ , respectively. Index of refraction within the window =  $n(x, t)$

$$v_{pdv}(t) = v_{back}(t) - \frac{d}{dt} \left[ \int_{x_{front}(t)}^{x_{back}(t)} n(x', t) dx' \right]$$

- ▶ As in (Jensen et al., 2007), (LaLone et al., 2008) we use a linear relationship between  $n$  and density  $\rho$ . For a relevant alternative: (Rigg, 2014)

$$n(x, t) \approx a_{1550} + b_{1550} \cdot \rho(x, t)$$

$$n(x, t) \approx 1.2669 + 4.438 \cdot 10^{-8} \cdot \rho(x, t)$$

- ▶ Substituting into Equation 1, we obtain:

$$\begin{aligned} v_{pdv}(t) &= v_{back}(t) - a_{1550} \cdot \frac{d}{dt} [x_{back}(t) - x_{front}(t)] - b_{1550} \cdot \frac{d}{dt} \int_{x_{front}(t)}^{x_{back}(t)} \rho(x', t) dx' \\ &= v_{back}(t) \cdot (1 - a_{1550}) + a_{1550} \cdot v_{front}(t) - b_{1550} \cdot \frac{d}{dt} \int_{x_{front}(t)}^{x_{back}(t)} \rho(x', t) dx' \end{aligned}$$

- ▶ Let  $m_{beam}(t) = \int_{x(t)}^{x_s(t)} \rho(x', t) dx'$ , the areal mass within the PDV window beam path.

- ▶ If we further assume the window back is at rest (which holds until we observe release at the window a probe), we have:

$$v_{pdv}(t) = a_{1550} \cdot v_{front}(t) - b_{1550} \cdot \frac{d}{dt} m_{beam}(t)$$

This is the "normal"  
1D scalar correction

This term vanishes  
in 1D

# Non-uniform window correction

$$v_{pdv}(t) = -0.2669 \cdot v_{back}(t) + 1.2669 \cdot v_{front}(t) - 4.438 \cdot 10^{-8} \cdot \frac{d}{dt} m_{beam}(t)$$

or prior to window back motion...

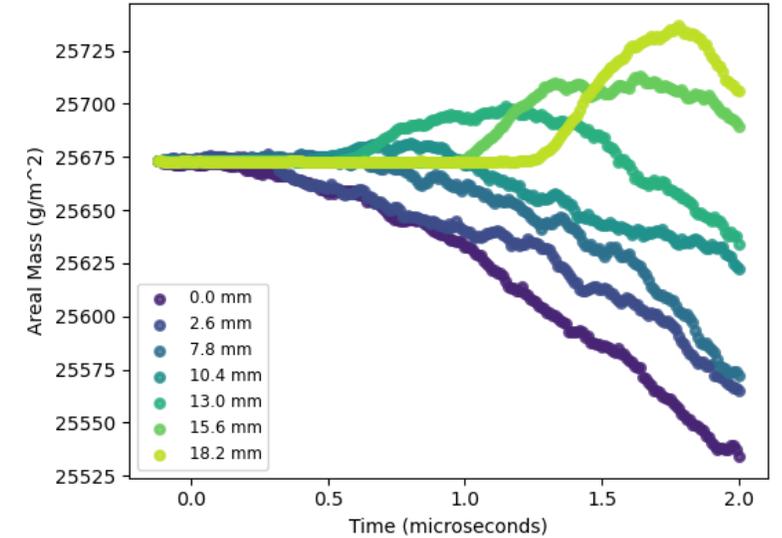
$$v_{pdv}(t) = 1.2669 \cdot v_{front}(t) - 4.438 \cdot 10^{-8} \cdot \frac{d}{dt} m_{beam}(t)$$

- ▶ If areal mass increases, as it would when the transverse 3D shock first enters into the PDV beam, then the  $\frac{d}{dt} m_{beam}(t)$  term becomes negative and can influence PDV observed velocity.
- ▶ These dynamic changes in PDV beam areal mass can be very small, but if they occur during a sufficiently small time interval (1-100 nanoseconds) they can result in large valuations of  $\frac{d}{dt} m_{beam}(t)$
- ▶ Primary non-uniform correction questions:
  1. How significant is the non-uniform correction term (red/right)?
  2. How can we estimate  $m_{beam}(t)$  and  $\frac{d}{dt} m_{beam}(t)$  with experiment data?
  3. Can this non-uniform correction be validated?

\* The CTH dynamic areal masses were calculated in post from output of a foam dome CTH simulation that was kindly provided/created/run by Jerry Stevens (STL)

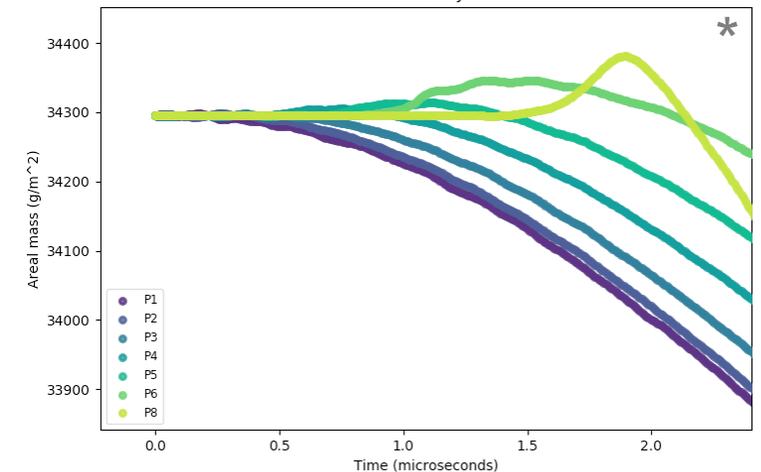
## LSM beam path areal density measurement

Dynamic PDV beam path areal mass, DSR driven forward model



## CTH beam path areal density measurement

Foam Dome: CTH tracer dynamic areal mass

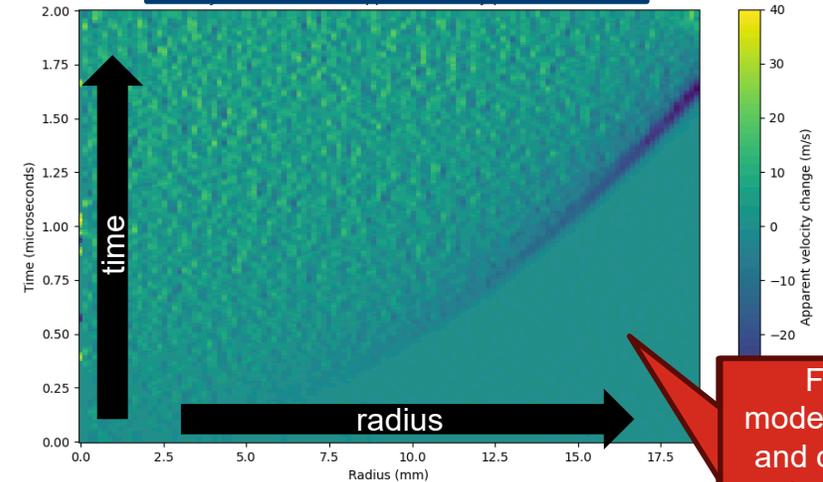


# Non-uniform window correction

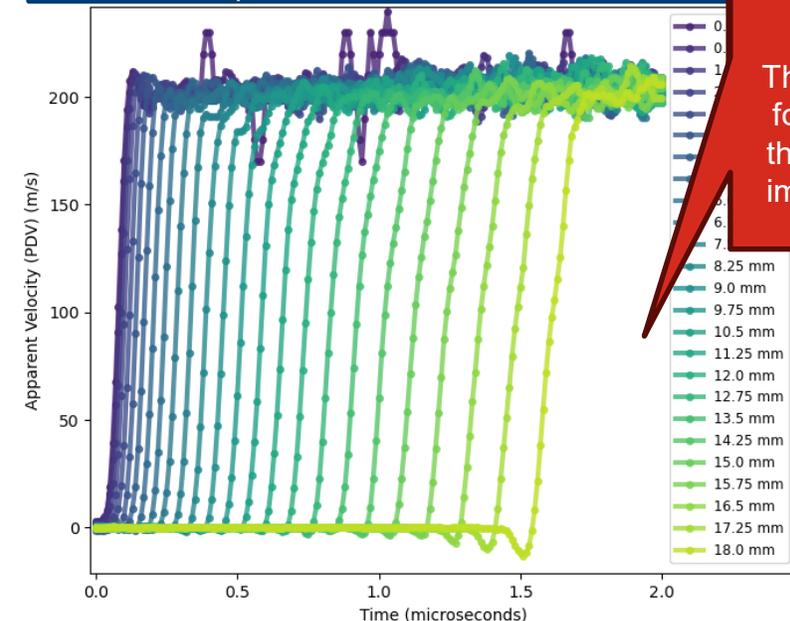
$$v_{pdv}(t) = 1.2669 \cdot v_{front}(t) - 4.438 \cdot 10^{-8} \cdot \frac{d}{dt} m_{beam}(t)$$

- How significant is the non-uniform correction term (red/right)?
  - On foam dome, non-uniform correction term can contribute up to -40m/s to the apparent velocity at extreme radii (18mm), -10m/s or less for radii below 12.5mm
  - Higher density projectiles, higher collision velocity, or increased non-uniformity can increase the contribution to 100's m/s scale
- How can we estimate  $m_{beam}(t)$  and  $\frac{d}{dt} m_{beam}(t)$  with experiment data?
  - ARES, CTH, LSM forward simulations can produce sufficient estimates for these quantities and show good agreement.
  - Helpful to have a forward model that can be driven by a boundary condition (observed PDV). More on this next...
- Can this non-uniform correction be validated?
  - Applying non-uniform correction computed from CTH/LSM/ARES models to the tracer velocity histories produces similar ramp behavior and negative velocity behavior to the foam dome experiment data: figure at lower right

Velocity correction (LSM): time and radius



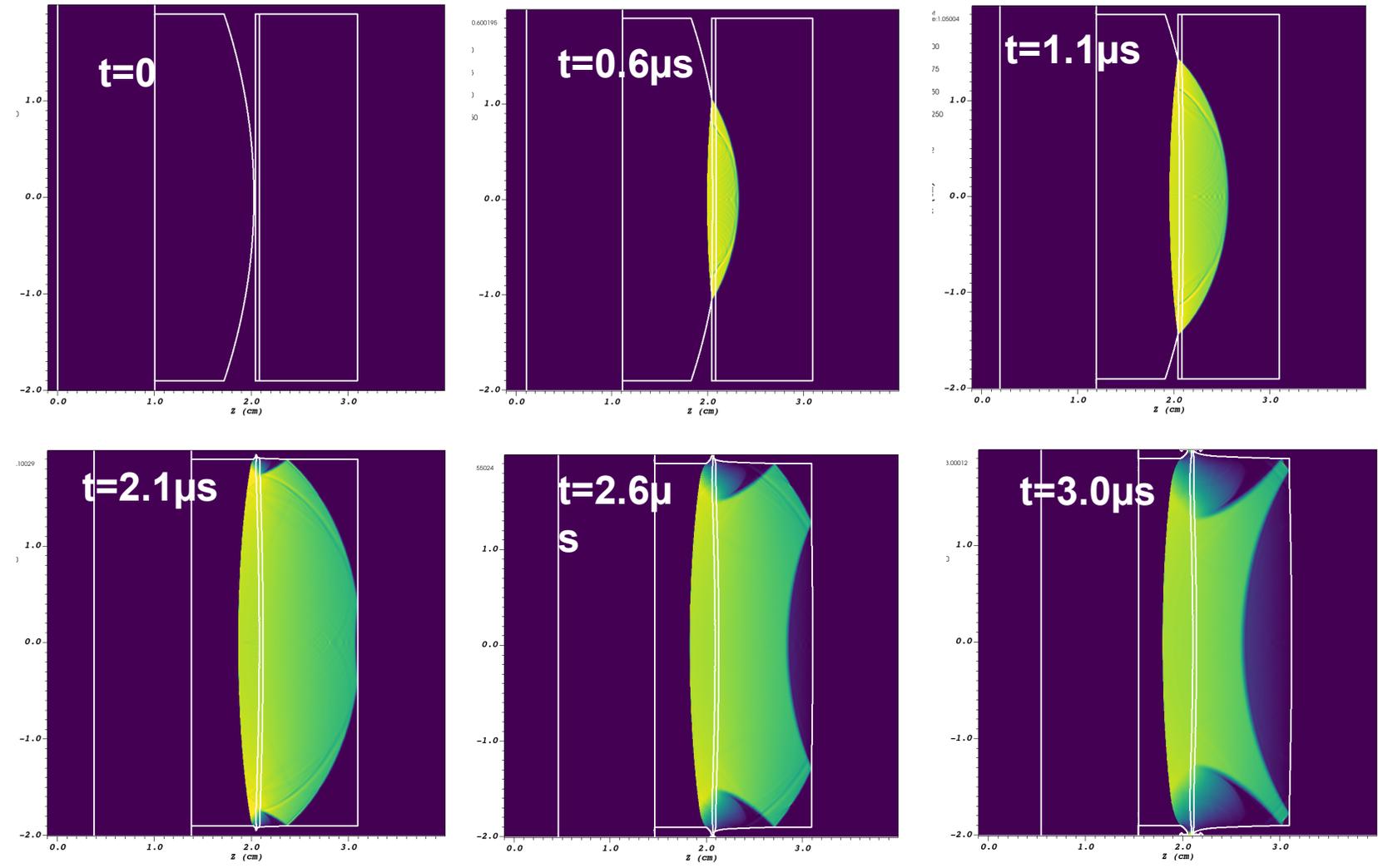
Simulated apparent velocity compared to Foam experiment data: time and radius



Forward modelling results and corrections calculated from simulation in post.

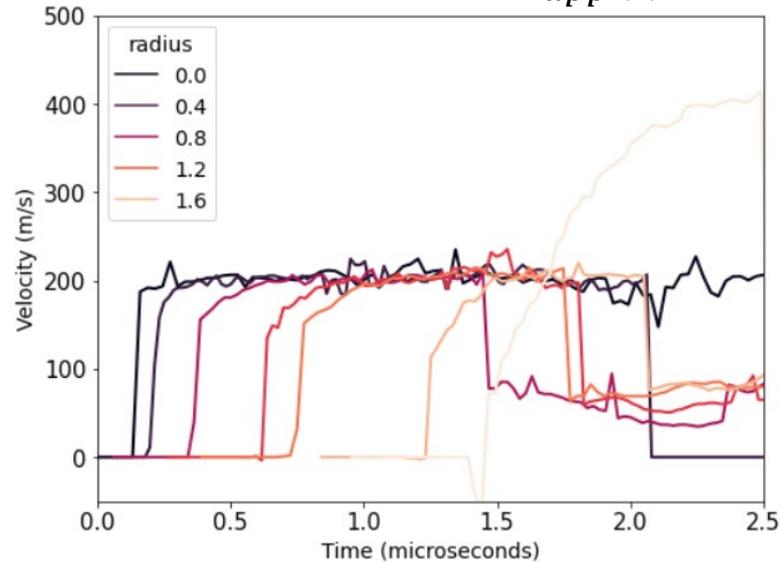
This was a "full" forward model that collided an impactor with a window.

# Foam Dome ARES Simulation

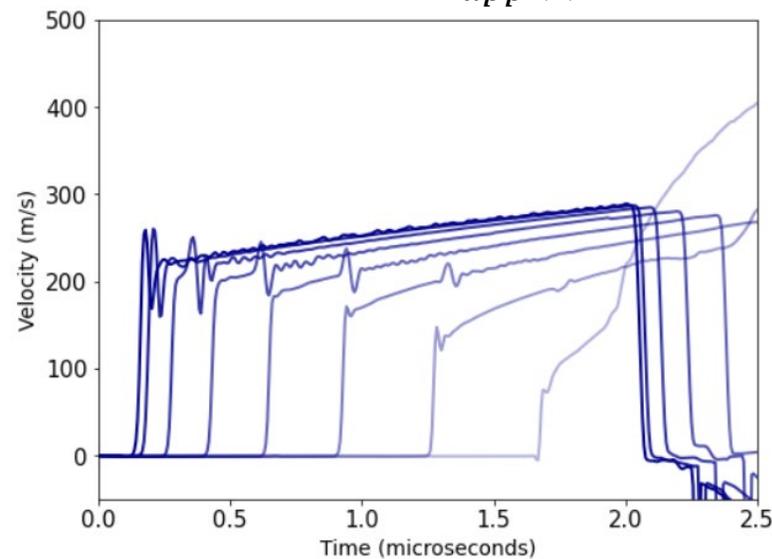


# ARES simulated PDV apparent velocities

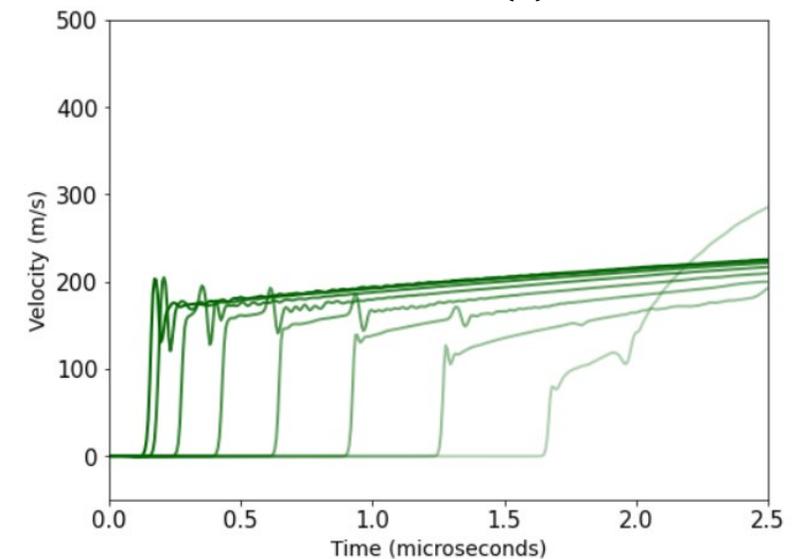
### Experimental $v_{app}(t)$



### Simulated $v_{app}(t)$



### Simulated $v(t)$



- With the ARES hydrocode simulation of the foam dome experiment, we can calculate apparent velocity on-the-fly via
  - $v_{app}(t) = \frac{-dZ(t)}{dt}$ ,  $Z(t) = \int_{beam\ path} n(x, t) dx$
- Agreement with experiment shows room for improvement: our material model for LiF overestimates upward drift due to release waves
- Beam wandering may account for window-back timing discrepancies

# How can we estimate $m_{beam}(t)$ and $\frac{d}{dt}m_{beam}(t)$ with experiment data?

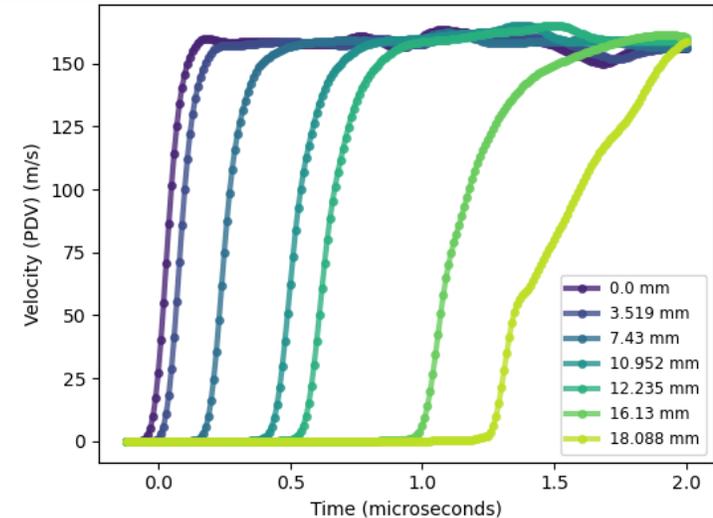
We can attempt to find a self-consistent solution given what we know about non-uniform effects on apparent velocity.

- ▶ Let  $\mathcal{F}$  be a forward model of a window that can be driven by a window front surface velocity function  $v(r, t)$ .
  - Forward model driven by a boundary condition/constraint
- ▶ Denote by  $\mathcal{F}[v(r, t)]$  the estimated apparent velocities produced by the forward model when driven by  $v(r, t)$ 
  - That is, with the dynamic areal mass apparent velocity modifications propagated to the results:  $\mathcal{F}[v(r, t)] = \mathbf{a}_{1550} \cdot v(r, t) - \mathbf{b}_{1550} \cdot \frac{d}{dt}m_{beam}(t)$
- ▶ We seek a window front surface velocity estimate  $v^*(r, t)$  satisfying:

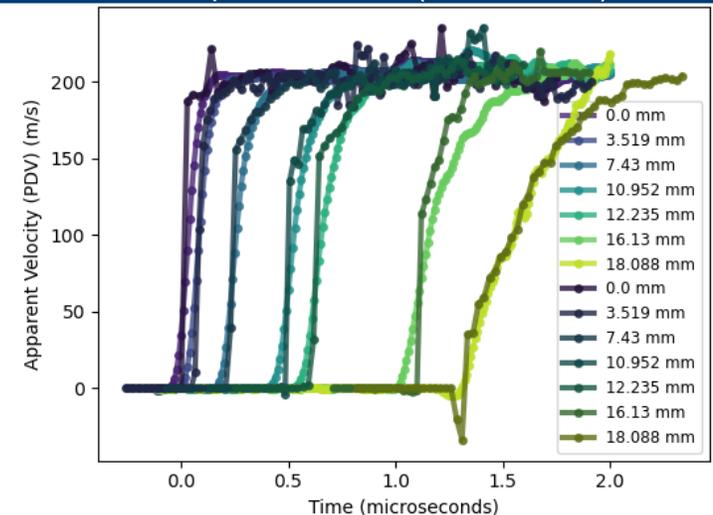
$$\mathcal{F}[v^*(r, t)] = v_{pdv}(r, t)$$

This is a different kind of forward model than what was shown earlier and mentioned on the previous slide. The physics is the same, but here there is no impactor, its driven by the boundary condition (velocity)

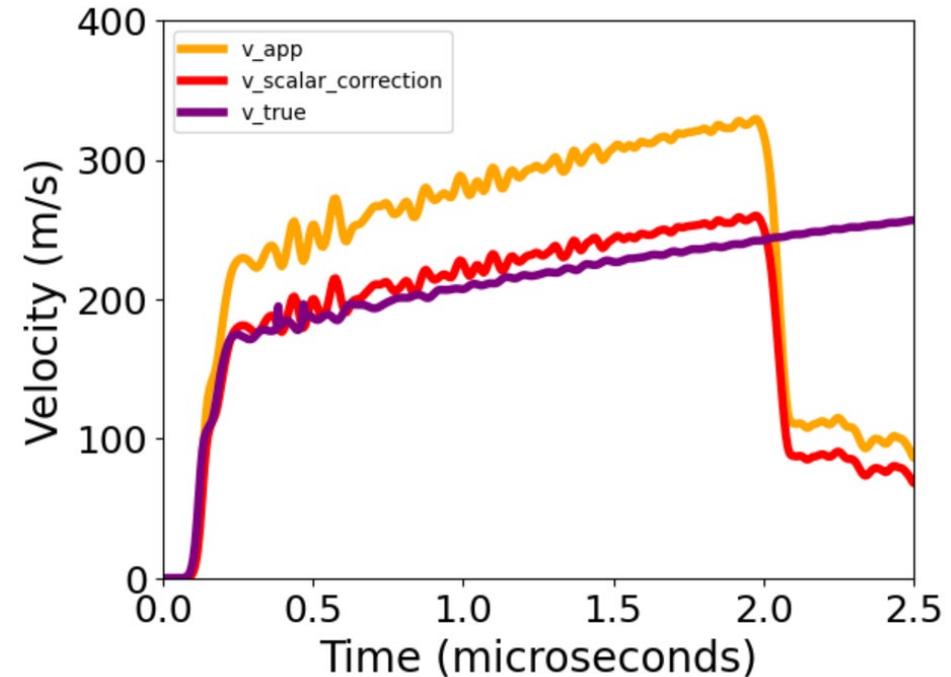
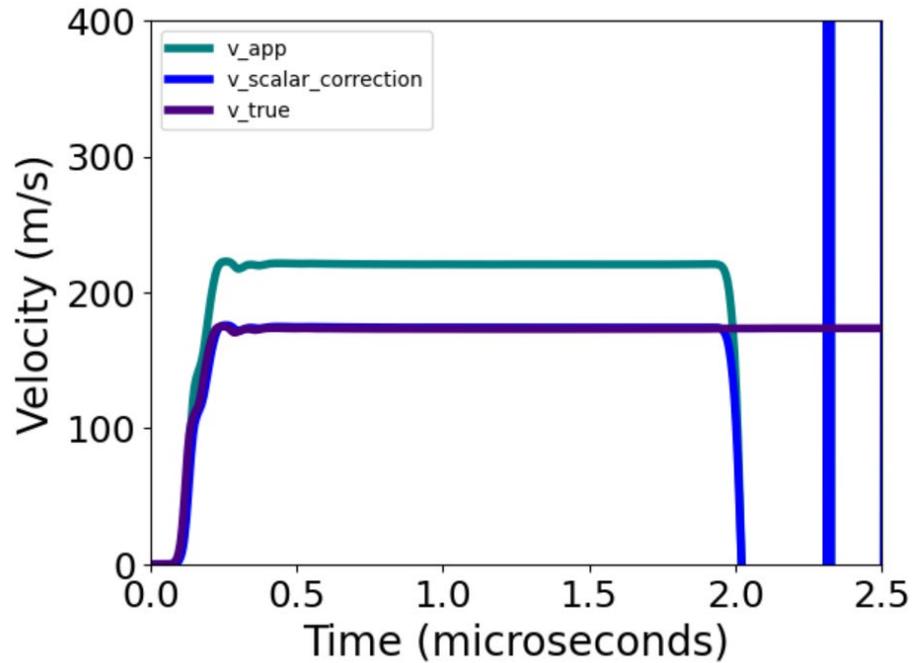
Iterative recovery of embedded reflector velocity (18 iterations)



Iterative recovery of apparent velocities compared to experiment data (18 iterations)



# Scalar correction is inaccurate even for simple cases



- ▶ We cannot expect to improve Asay window inferences without forward modeling of colliding material with window and simulated PDV

$$v_{app}(t) = \frac{-dZ(t)}{dt}, \quad Z(t) = \int_{beam\ path} n(x, t) dx$$

← We need to calculate  $\frac{-dZ(t)}{dt}$  directly in simulations and match to experiment

# How can we estimate $m_{beam}(t)$ and $\frac{d}{dt}m_{beam}(t)$ with experiment data?

- ▶ We seek a window front surface velocity estimate  $v^*(r, t)$  satisfying:

$$\mathcal{F}[v^*(r, t)] = v_{pdv}(r, t)$$

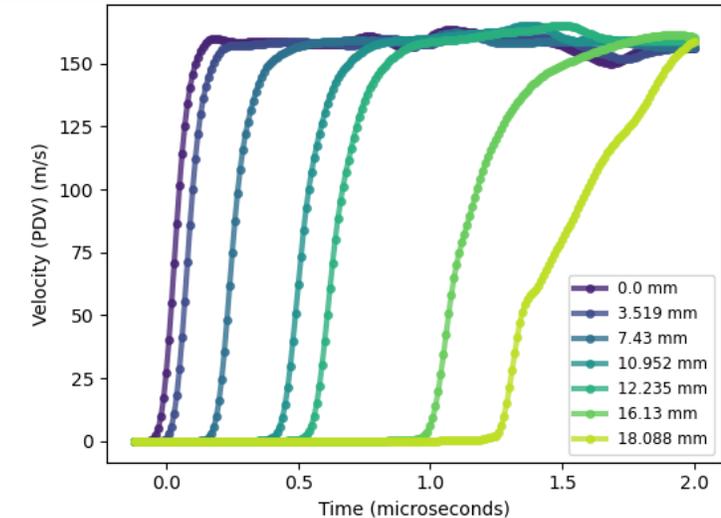
- ▶ An iterative process can be used to recover incremental improvements to  $v(r, t)$  resulting in satisfaction of the self-consistent condition described above.

## ▶ Algorithm overview:

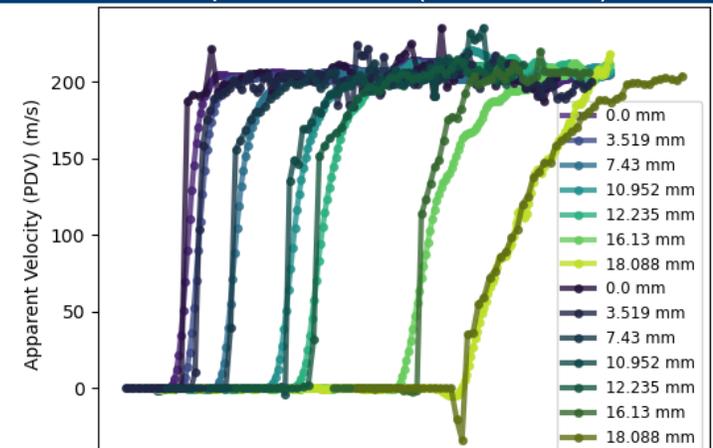
1. For a presumed radial-temporal velocity correction  $C'(r, t)$ , apply the correction to obtain a putative window front velocity function:  $v'(r, t) = \frac{1}{1.2669} \left( C'(r, t) + v_{pdv}(r, t) \right)$ .
2. Forward simulate using a window front simulator (3D) with boundary condition  $v'(r, t)$ .
3. Use the mass distribution from the forward simulation to compute  $\frac{d}{dt}m'_{beam}(r, t)$  for each PDV beam radius.
4. Update the velocity correction:  $C'(r, t) = \text{Interp} \left( \left\{ \left( r, \frac{d}{dt}m'_{beam}(r, t) \right) \right\} \right)$
5. Repeat steps 1-4 until the correction function converges.

Time permitting/appendix: Application to foam dome

## Iterative recovery of embedded reflector velocity (18 iterations)

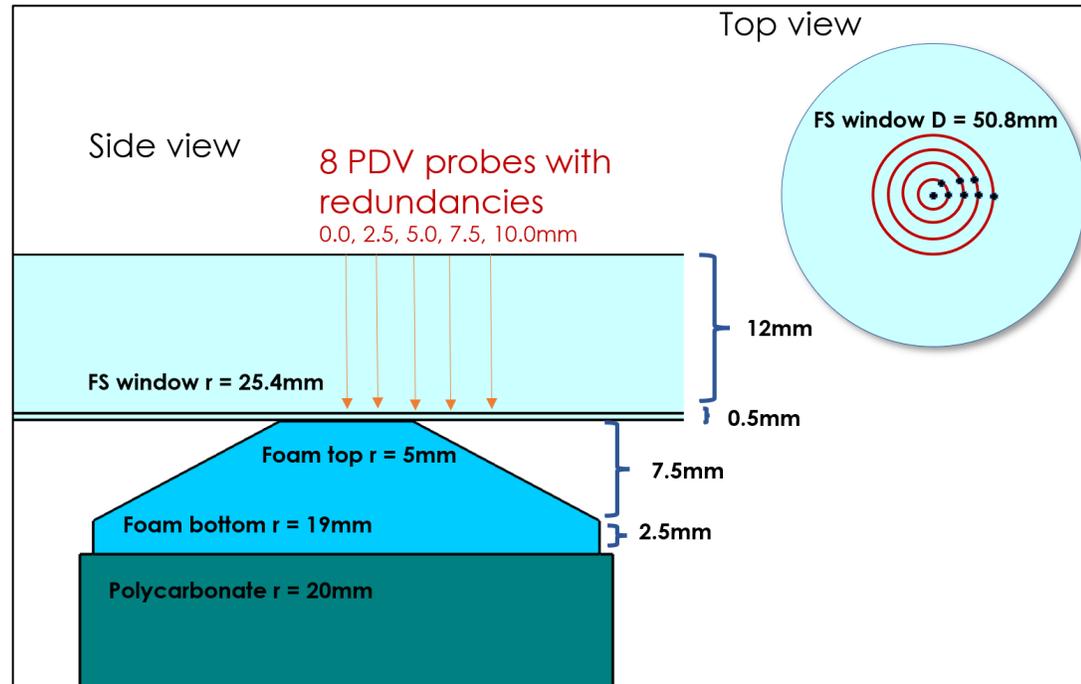


## Iterative recovery of apparent velocities compared to experiment data (18 iterations)

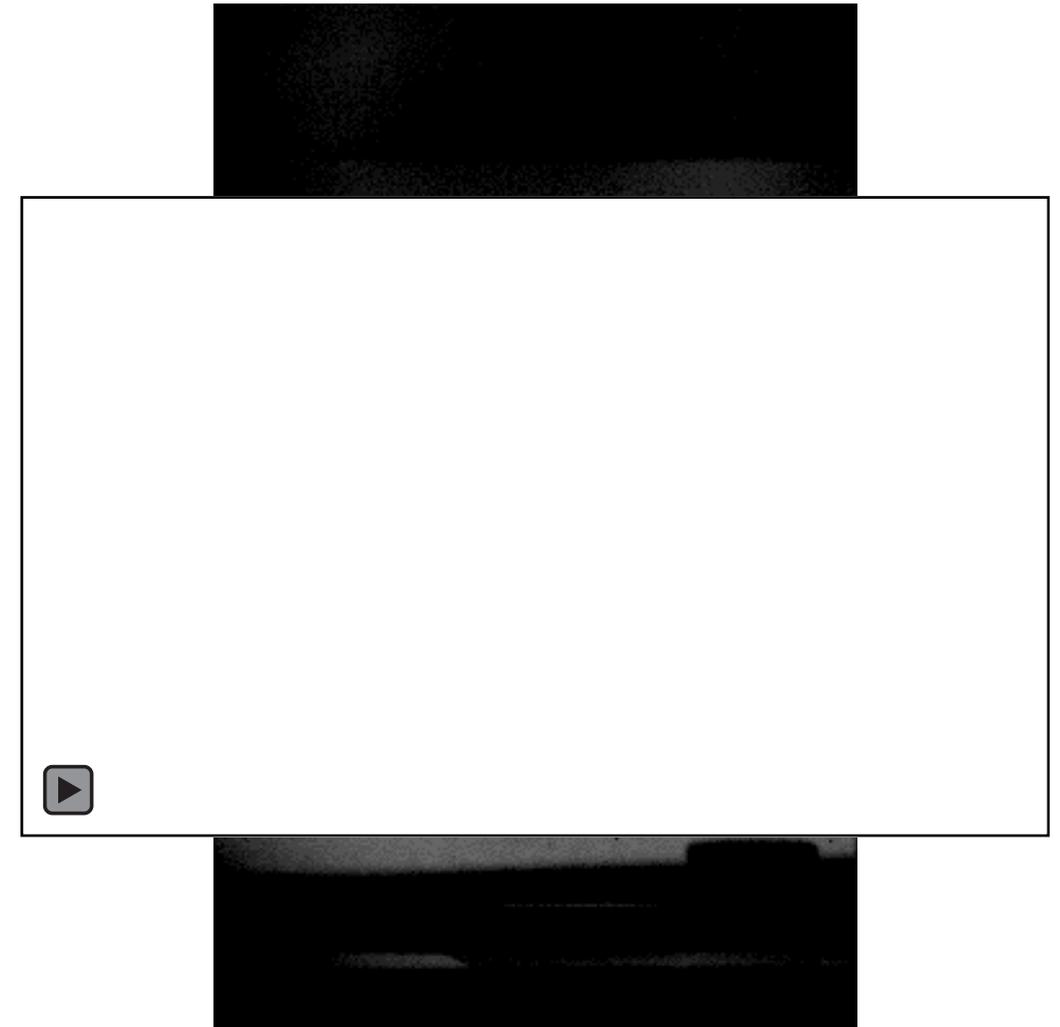


Neat! Problem solved right?

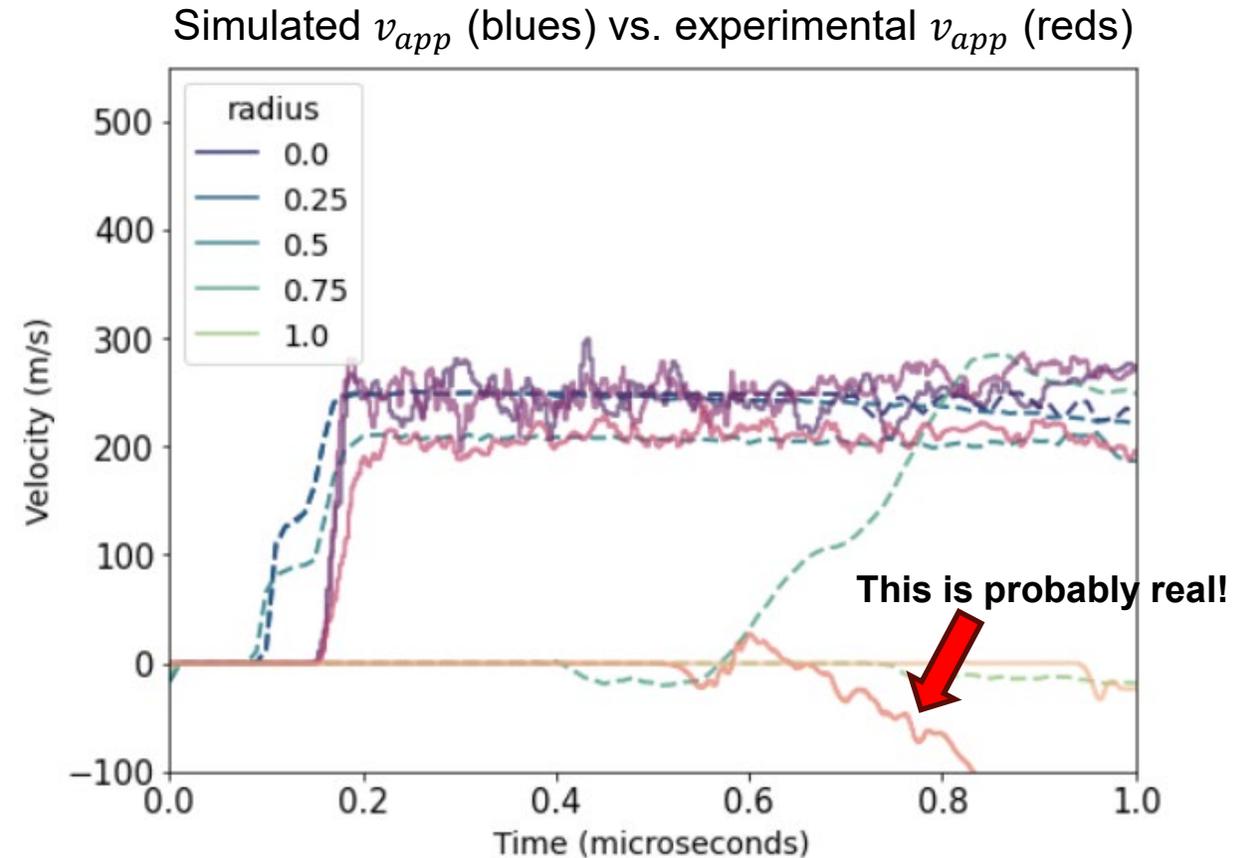
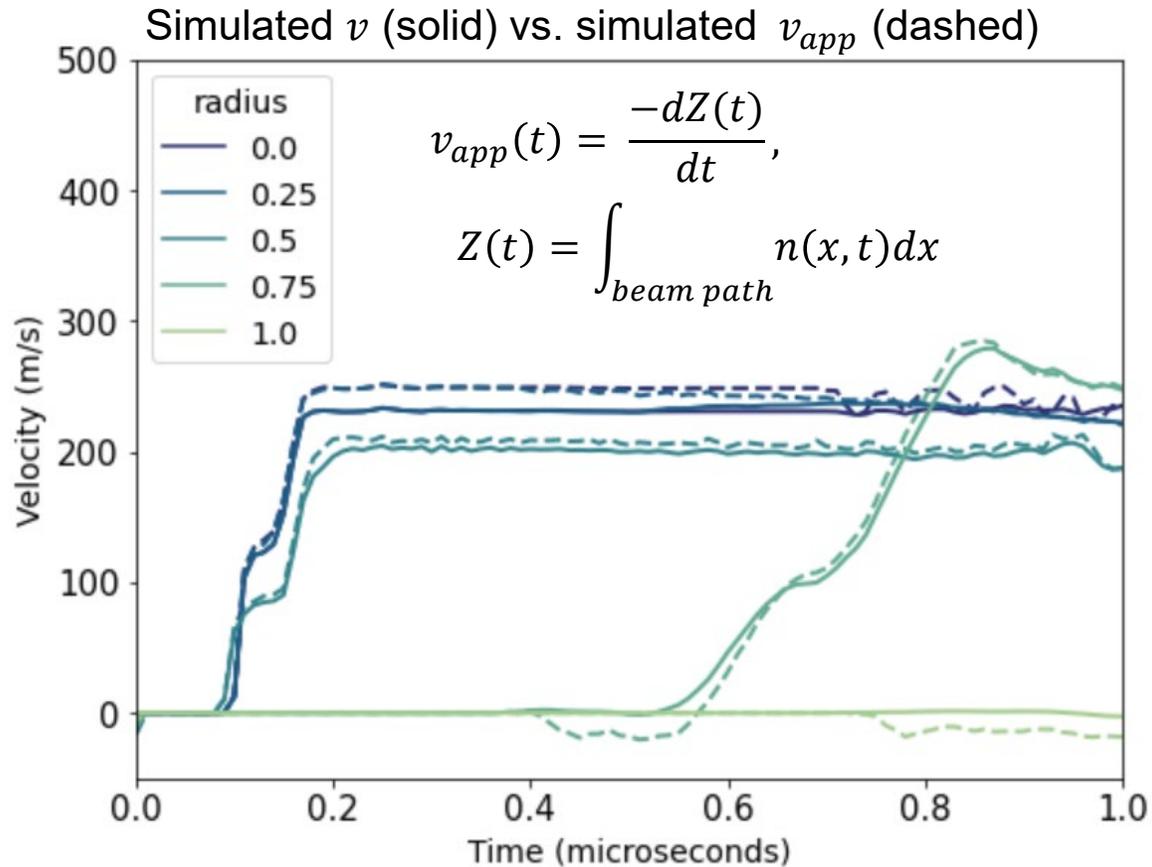
# A foam cone experiment attempted to investigate non-uniform impact effects in a fused silica window



- A **fused silica (FS) window** was impacted by a foam with known density, velocity, and geometry
- Flat-top cone used so that we have a quasi-1D impact at the center at early times.
- ARES hydrocode was used to forward model the experiment (FS strength model matters a lot)

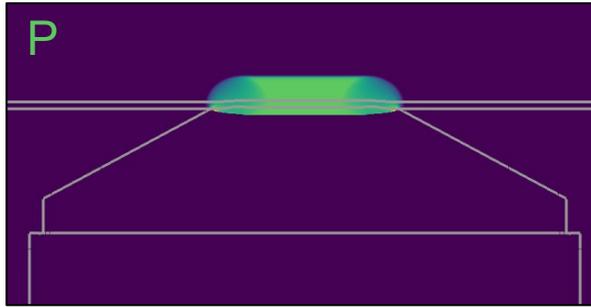


# Simulated PDV compares fairly well to experiment, but there is still much room for improvement

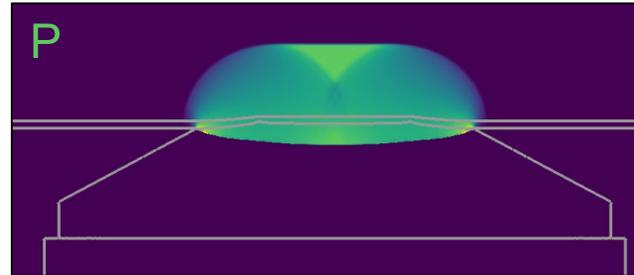


- Difference between real and apparent velocity is relatively small
- We are not using a damage model for FS (probably important)

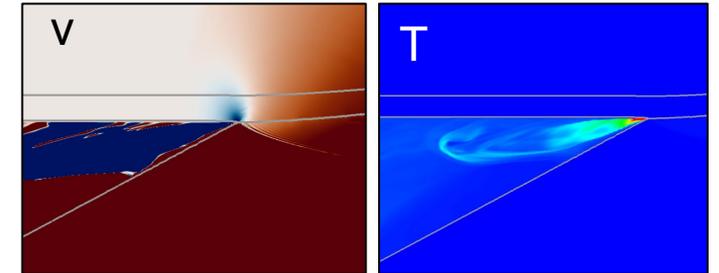
# Negative velocities due to non-uniform impacts can be real



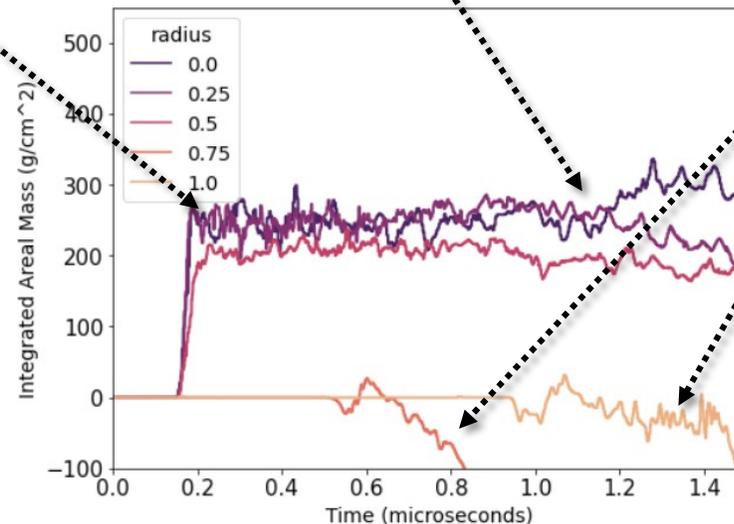
Early times: quasi-1D behavior at center of window



Later times: release wave interaction at center effects apparent velocity (we still don't model this very well)

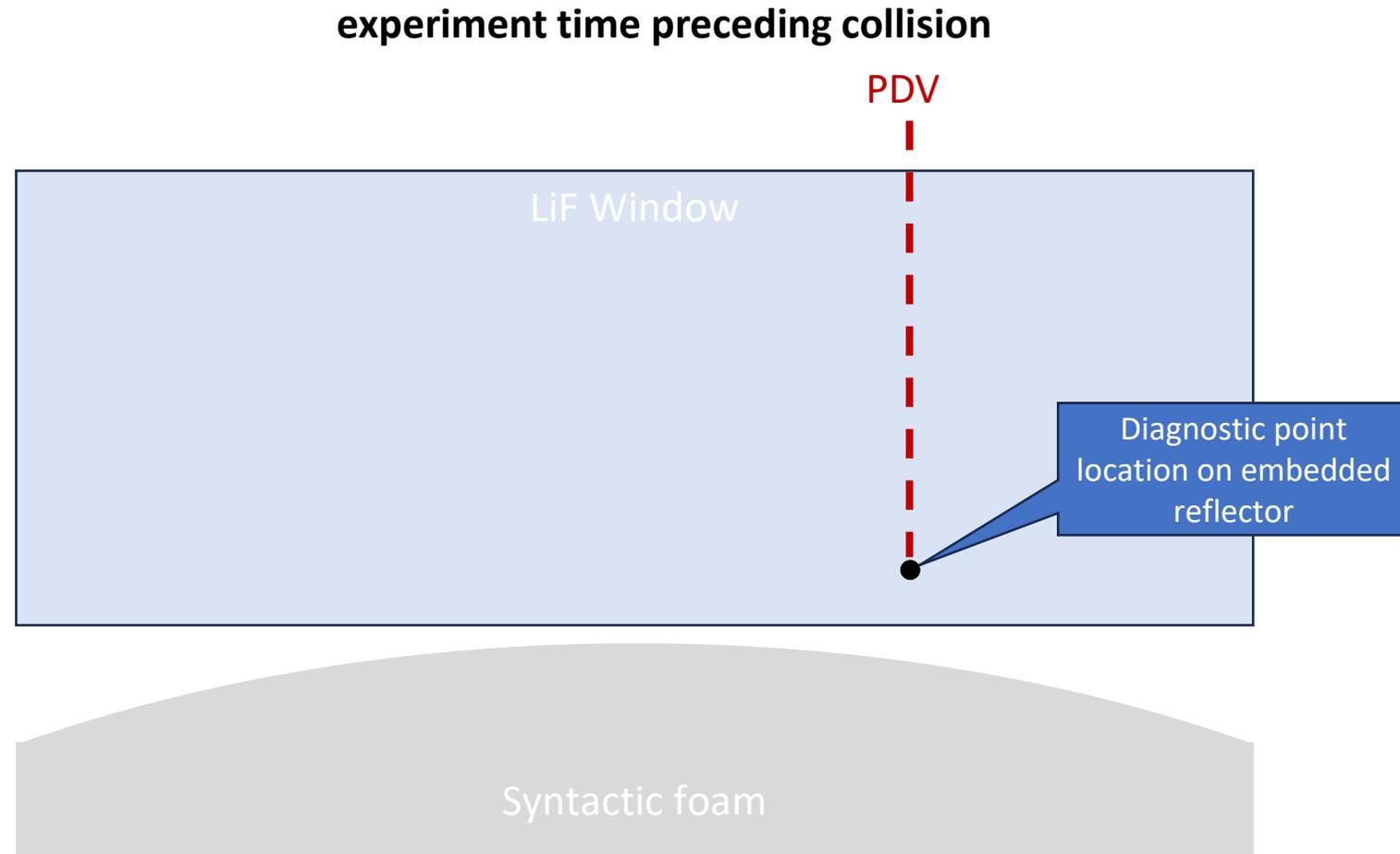


If you see negative velocity, do not assume it is an artifact of the optical path change. Jetting at material-window interface may pull window surface apart.



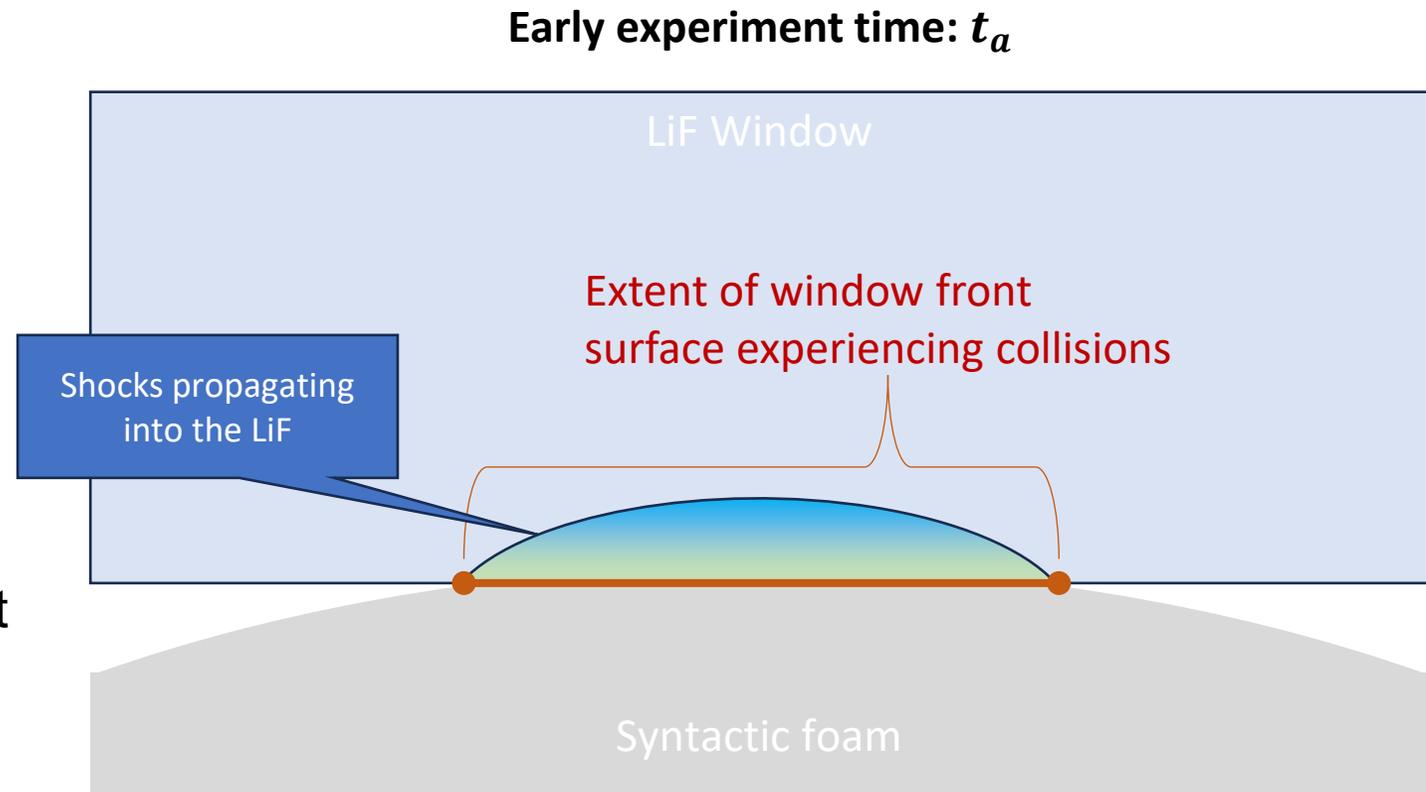
# EVEN MORE NUANCE: angles and unequal regions of influence

- ▶ Consider again the foam dome experiment at right
- ▶ We will be considering diagnostic points that reflect off of an internal reflection point within the window
  - due to a buffer layer or an embedded reflector as in (Chen, et al. 2017)
  - This greatly improves PDV signal quality



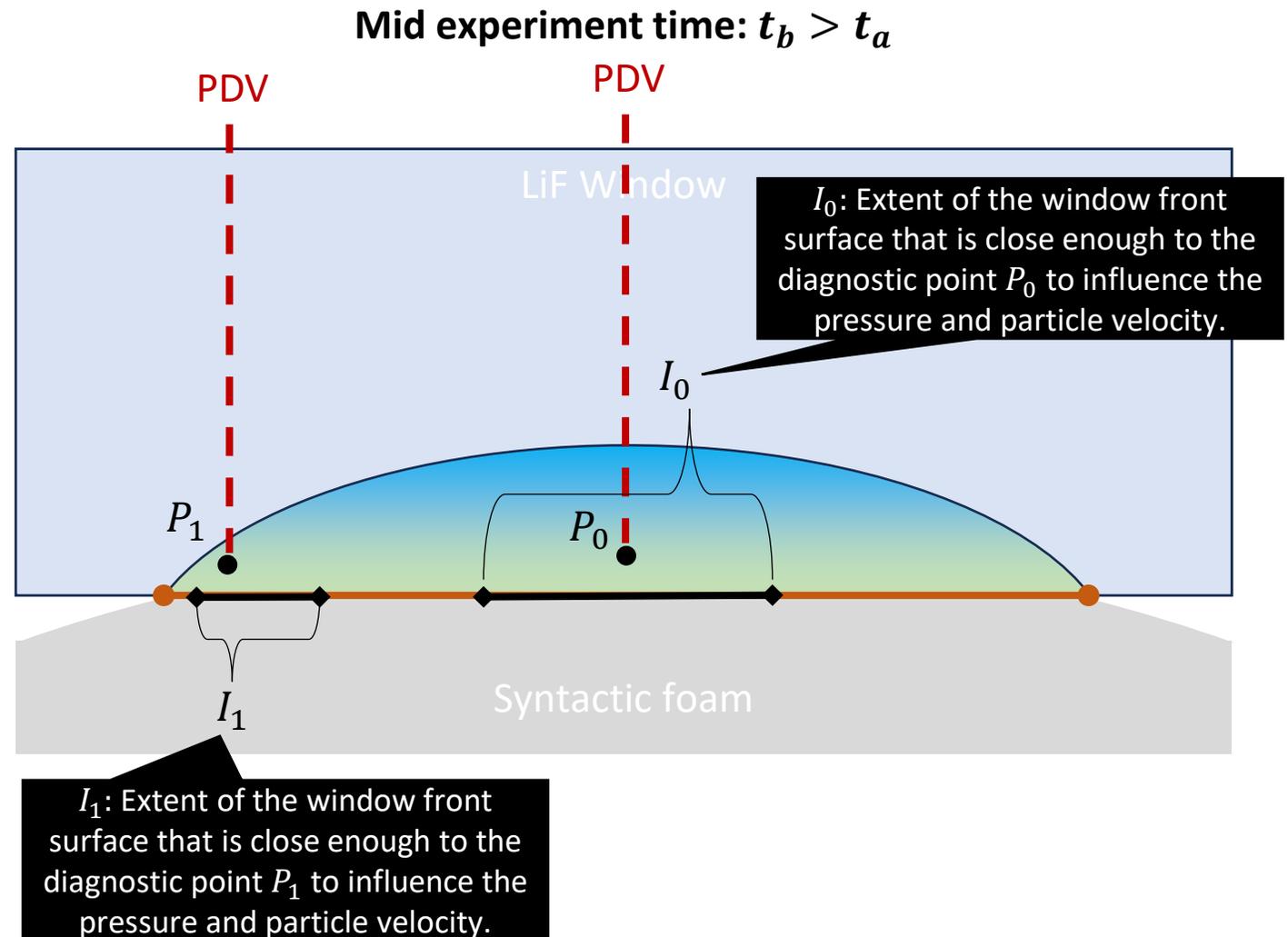
# More nuance to window corrections: angles and unequal regions of influence

- ▶ In this side view illustration I have shown an artist rendition of the shock propagation into the LiF window.
- ▶ The colored region shows shocked LiF.
- ▶ The orange line segment shows the spatial extent of the collision on the window front surface.
- ▶ The rate that this spatial extent increases is between  $\infty$  and  $10 \frac{mm}{\mu s}$



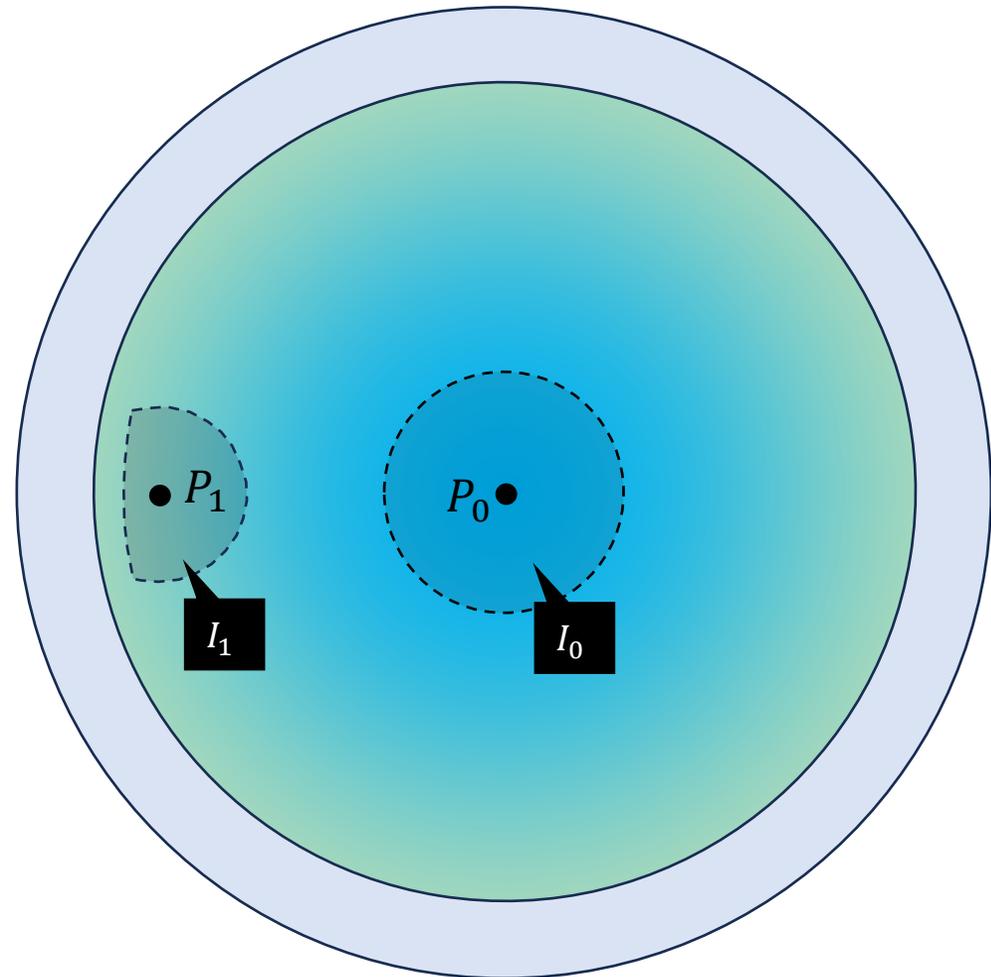
# More nuance to window corrections: angles and unequal regions of influence

- ▶ Consider two hypothetical diagnostic points, one near the center ( $P_0$ ), and one at a higher radius ( $P_1$ )
- ▶ These two points, at the embedded reflector, have different collision regions for which shocks have had sufficient time to influence the velocity we record at the embedded reflector.

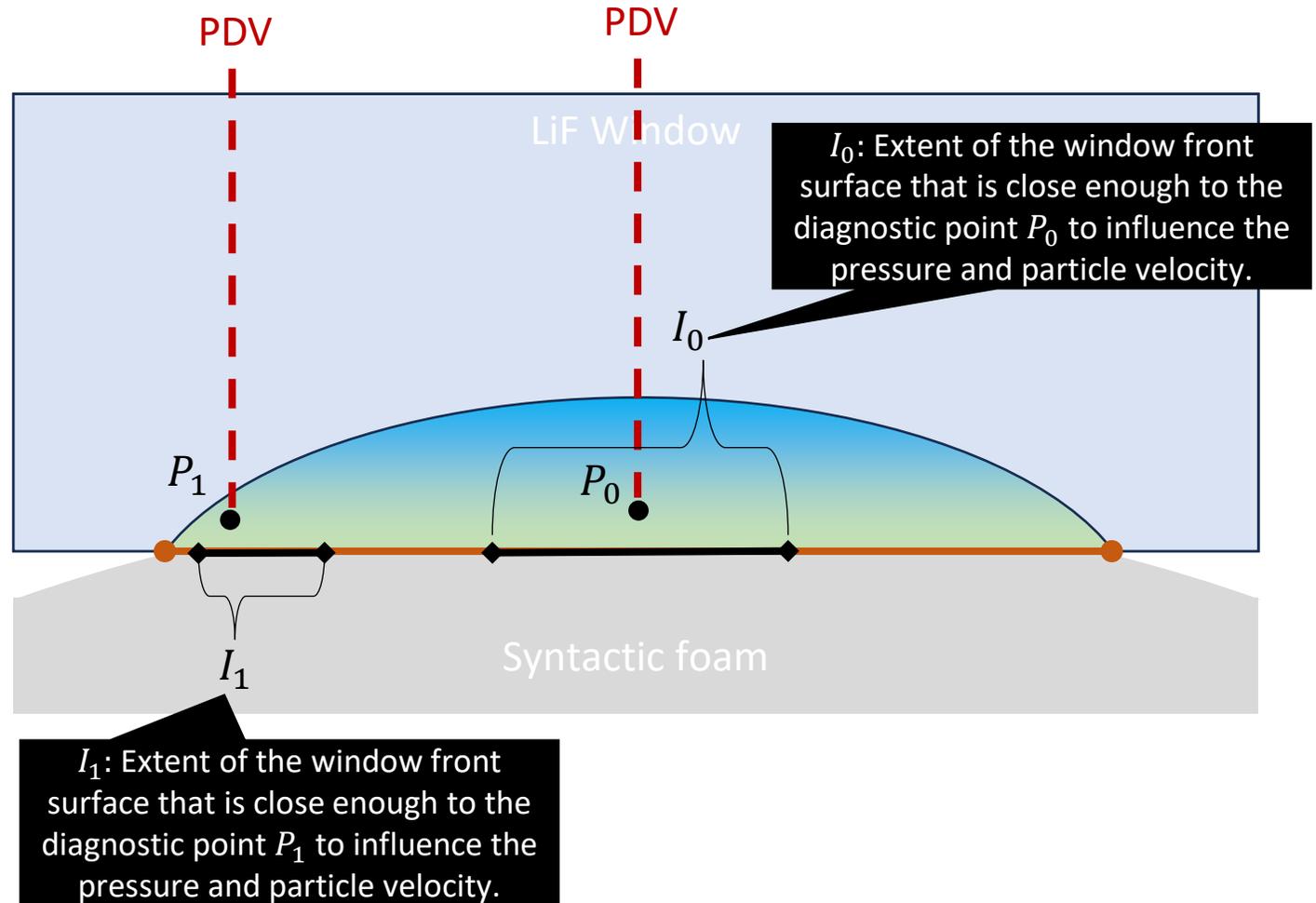


# More nuance to window corrections: angles and unequal regions of influence

Mid experiment time:  $t_b$   
Top-down view  
(PDV probe perspective)



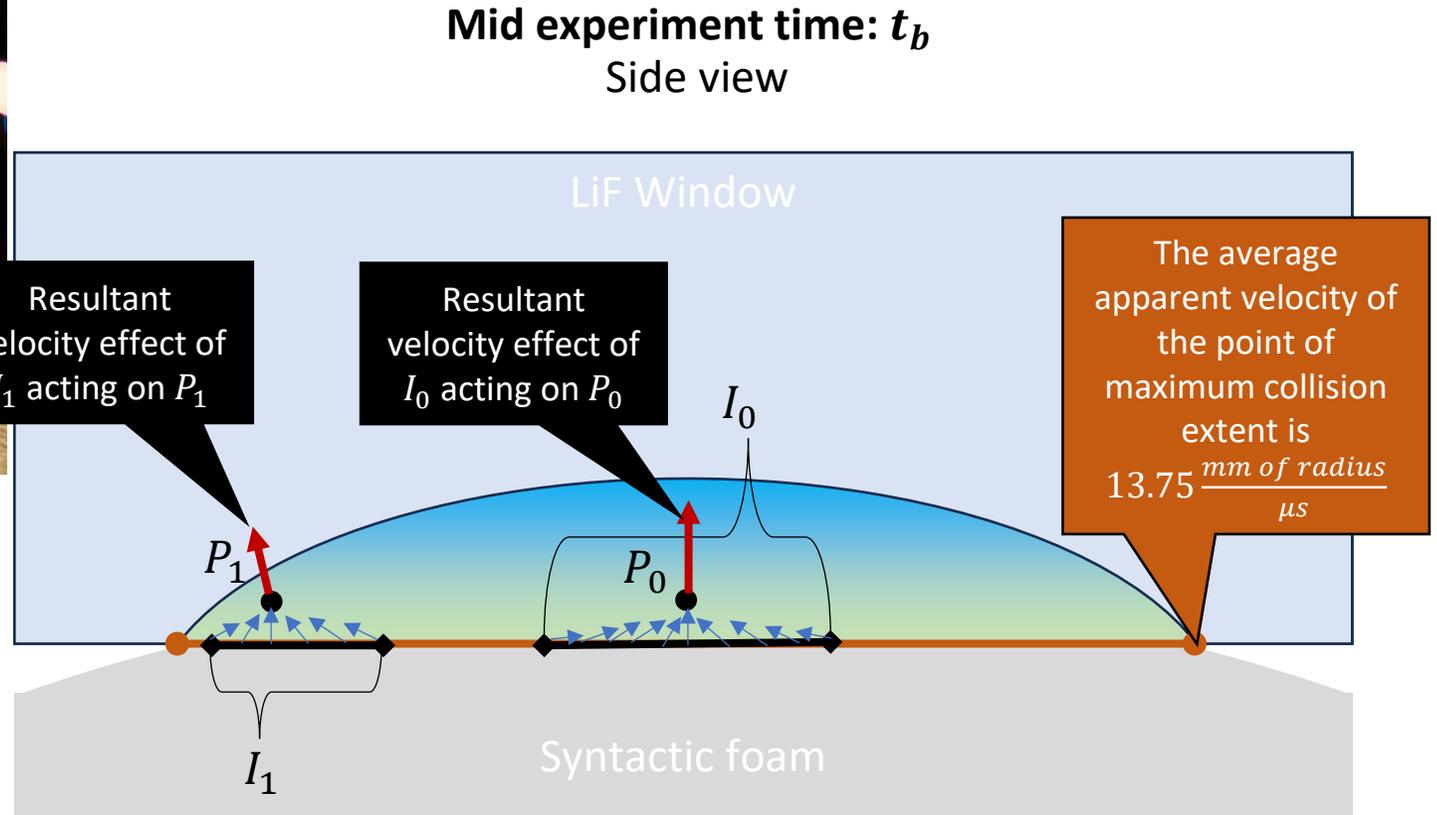
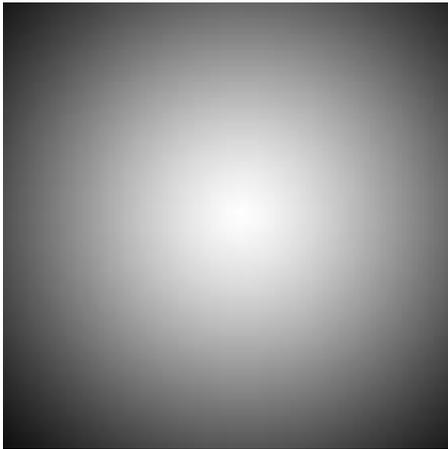
Mid experiment time:  $t_b > t_a$



# More nuance to window corrections: angles and unequal regions of influence



► “Cat-eye Bokeh”, and resulting vignetting is a similar mechanism for attenuating received off-axis light in photography

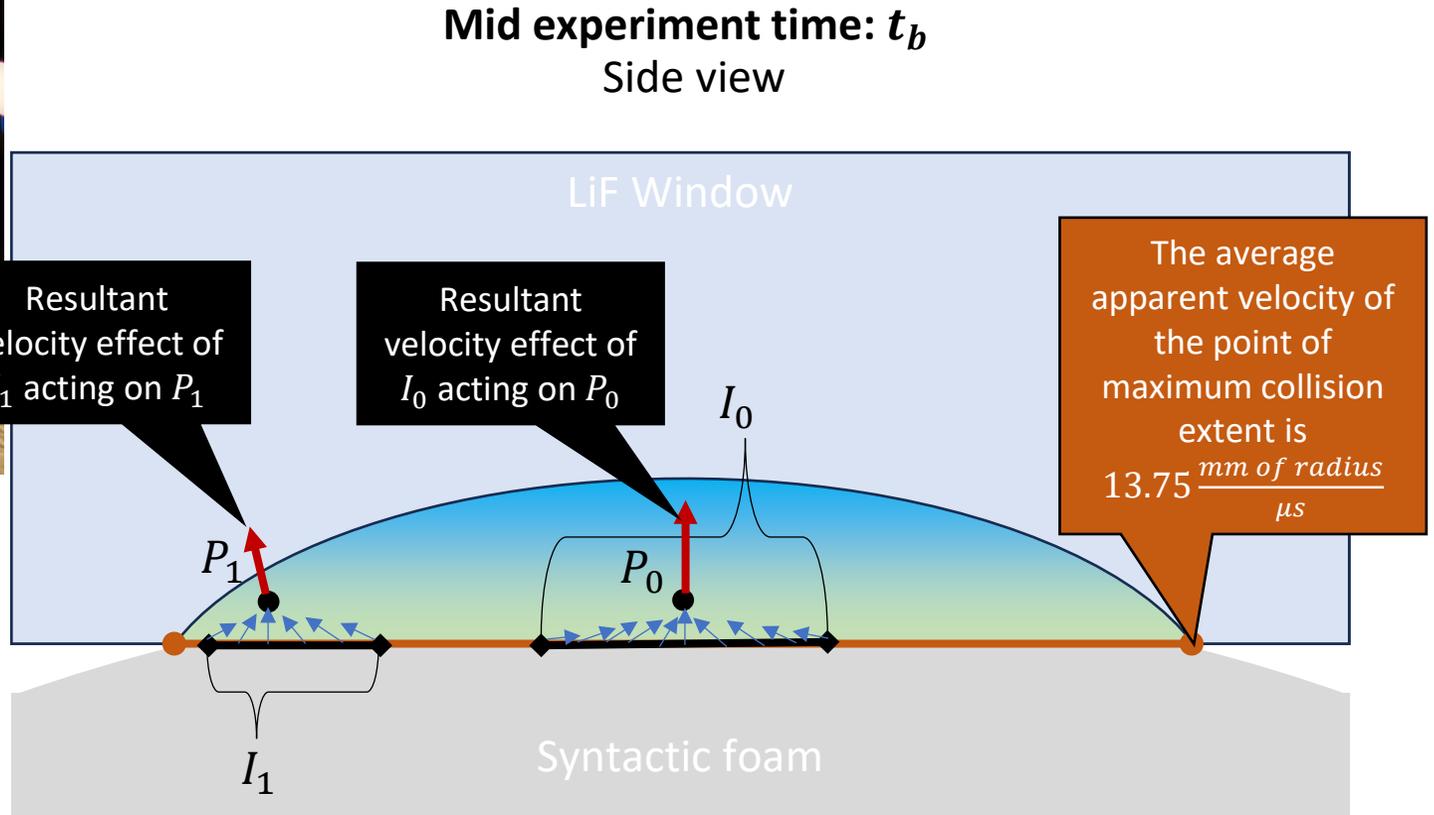
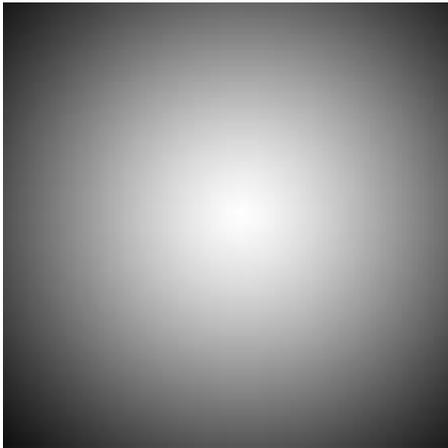


- The unequal regions influencing the velocity behavior at  $P_0$ ,  $P_1$  can result in lower magnitude and transverse velocities at  $P_1$ .
- This can have a “vignetting” attenuation effect on velocities at high radii

# More nuance to window corrections: angles and unequal regions of influence



► “Cat-eye Bokeh”, and resulting vignetting is a similar mechanism for attenuating received off-axis light in photography



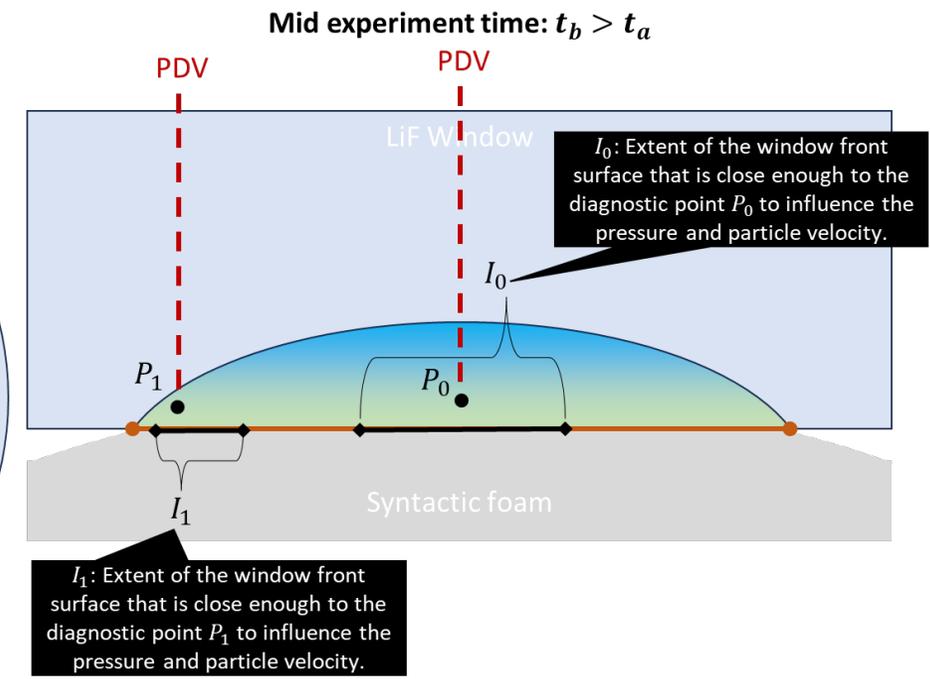
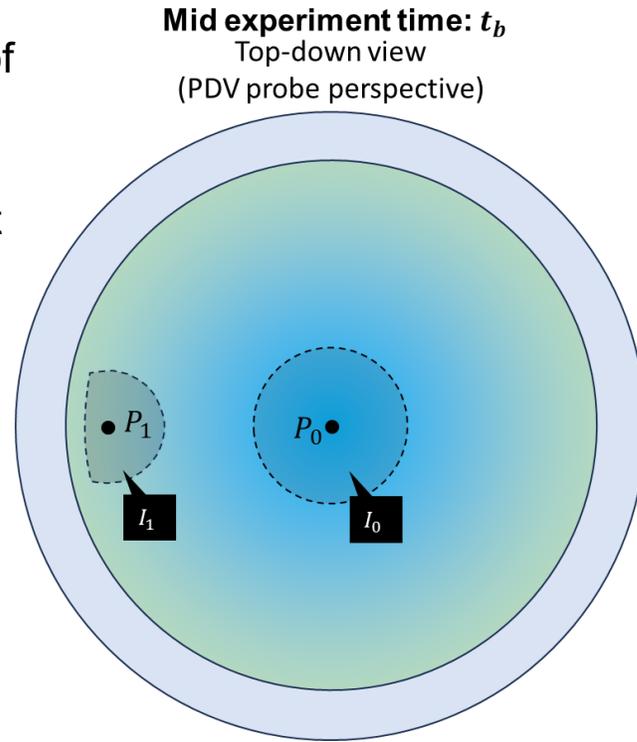
- This means that even if identical material is arrive beneath  $P_0$  and  $P_1$ , the spatial-temporal non-uniformity implies we should have no expectation that dynamic areal mass corrected velocities would produce a match between these two probes.
- **Do we need some kind of flat-field correction for non-uniform experiments??**

# More nuance to window corrections: angles and unequal regions of influence

- ▶ Perhaps we need a non-uniform region of influence correction factor.
- ▶ A scaling factor based on the geometry and spatial-temporal arrival of material at the window front surface.
- ▶  $I(x, y, t)$  = indicator function representing the presence of colliding material  $t(x, y)$  at time  $t$ .
- ▶  $P = (P_x, P_y, P_z)$  = diagnostic point
- ▶ A correction factor would summarize the angular and spatial influence upon the diagnostic point, maybe something resembling this:

$$C(P, t) = \iint_S \frac{I(x, y, t) \cdot \cos(\angle(\langle P_x, P_y, 0 \rangle, P, \langle x, y, 0 \rangle))}{d(P, \langle x, y, 0 \rangle)^2} dA$$

$$= \iint_S \frac{I(x, y, t) \cdot \cos\left(\frac{P_z}{d(P, \langle x, y, 0 \rangle)}\right)}{d(P, \langle x, y, 0 \rangle)^2} dA$$



- ▶ **Why would we want a correction to an actual velocity:** because we want to use the velocity at our diagnostic points to estimate material properties of the colliding matter (momentum diagnostics)
- ▶ **Future work needed:** experiments, 3D shock theory, more experiments, ...

# Conclusions and final thoughts

- ▶ In window experiments, analysis may need to consider and account for the window behavior in the entire extent of the window PDV beam path.
- ▶ Dynamic beam-path areal mass corrections can be applied to window experiments.
- ▶ Identical colliding material at two locations on a window can produce different velocity records when non-uniformity is present.
- ▶ Forward modelling can provide quantification of beam path effects within their operating capability. Simulation edge release and 3D shock propagation would benefit from more verification for common window materials.

## Non-uniform phenomena that can impact apparent velocimetry (that we know of):

1. 1D index of refraction change correction (scales velocity by factor of 1.2669 in LiF)
2. Dynamic beam path areal mass ( $-4.438 \cdot 10^{-8} \cdot \frac{d}{dt} m_{beam}(t)$  in LiF)
3. Window back motion ( $-0.2669 \cdot v_{back}(t)$  in LiF)
4. Dynamic changes in the optical beam path (unknown impact, optical transport modeling needed)
5. Edge release (can be investigated with forward modeling)
6. Unequal collision regions of influence (relevant for momentum diag., need a flat field capability)

# Acknowledgements

The authors thank **Jerry Stevens** and **Brandon LaLone** and the **boom-box team at STL** for their ongoing assistance with experimental validation and theoretical physics discussions on Asay windows and the foam dome experiment.

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Thank you! Questions?

# Extra Content

# Optical beam path wandering during experiment

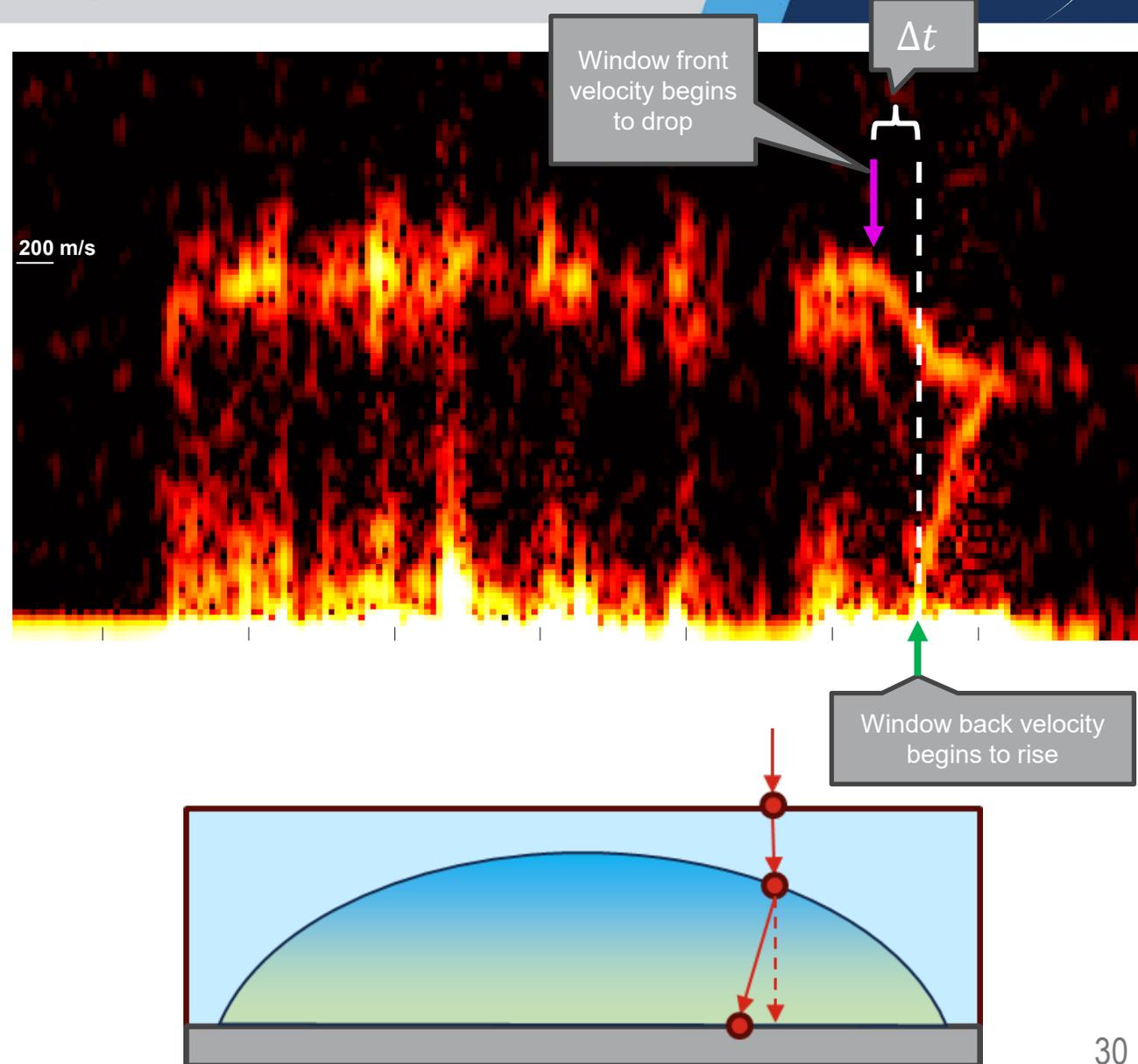
- ▶ On some foam-dome probes a time delta,  $\Delta t$ , is observed between the onset of window front velocity decrease, and the onset of window back velocity increase.
- ▶ The 1D theory indicates that these events should be synchronous:

$$v_{pdv}(t) = -0.2669 \cdot v_{back}(t) + 1.2669 \cdot v_{front}(t) - 4.438 \cdot 10^{-8} \cdot \frac{d}{dt} m_{beam}(t)$$

- ▶ What would happen if the beam deviates from a 1D path?
- ▶ The window back signal originates from the back-reflection of the beam entering the LiF window.
- ▶ If the beam bends inward due to the index of refraction change in the shocked LiF, the beam path could increase in length (resulting in apparent deceleration), independent from the  $m_{beam}(t)$  effects previously discussed.
- ▶ Notice that the  $v_{back}(t)$  term in the equation above actually originates in part from a path integral, and a separate part from the back reflection at the entry point.

$$v_{pdv}(t) = v_{back}(t) - \frac{d}{dt} \left[ \int_{x_{front}(t)}^{x_{back}(t)} n(x', t) dx' \right]$$

- ▶ This would seem to indicate that we could see asynchronous behavior if there are influences to the beam path??



# Forward Modeling Information

- ▶ Low complexity full 3D LSM forward simulations as well as 3D CTH forward simulations of the foam dome experiment
  - LSM: 10ns temporal resolution, 150micron spatial resolution, lower physics accuracy, explicit control of all LiF boundary conditions.
  - CTH: 100ns temporal resolution, 400micron spatial resolution, limited control of boundary conditions
  - Comparable run time (LSM=150s single GPU, CTH=1-2 minutes on a cluster).
- ▶ LSM uses a non-Hookean spring model based on the Birch-Murnaghan equation of state to accurately model linear shock-particle velocity relationship for LIF.
- ▶ Integration algorithm at right, based on details from (Grady, 2017), (Hockney, 2021), (Li et al., 2020)

Mass nodes  $N$ , Elastic element (edges)  $E$ .

The mass node positions

$$R_x, R_y, R_z \quad |R_*| = N \quad shape(R_*) = (N,)$$

Notation:

1. The Adjacency matrix of the LSM is given by *Adjacency*
2. Matrix multiplication:  $A \cdot B$
3. Elementwise multiplication:  $A \odot B$
4. All 2D matrices are sparse CSR, 1D matrices are dense
5. Addition and subtraction is performed only on non-zero entries.

While unsatisfied do:

1. Node displacements (for  $u = x, y, z$ )
  - $\Delta R_u = diag(R_u) \cdot Adjacency - Adjacency \cdot diag(R_u)$
  - $\Delta V_u = diag(V_u) \cdot Adjacency - Adjacency \cdot diag(V_u)$
1. Elastic element lengths (and inverse lengths)
  - $L = sqrt(\Delta R_x \odot \Delta R_x + \Delta R_y \odot \Delta R_y + \Delta R_z \odot \Delta R_z)$
  - $L^{-1} = power(L, -1)$  (elementwise)
2. Update LSM parameters
  - $K = K_0 \cdot \left(\frac{L_0}{L}\right)^{K'_0}$
3. Elastic element force magnitudes
  - $F_{mag} = K \odot (L_0 - L)$
4. Net acceleration vectors
  - $A_u = (F_{mag} \odot N_u - K_{damp} \odot \Delta V_u) \cdot ones(N, 1)$
5. Integration update
  - $V_u = V_u + \Delta t \cdot A_u$
  - $R_u = R_u + \Delta t \cdot V_u$

# How can we estimate $m_{beam}(t)$ and $\frac{d}{dt}m_{beam}(t)$ with experiment data?

- ▶ We seek a window front surface velocity estimate  $v^*(r, t)$  satisfying:

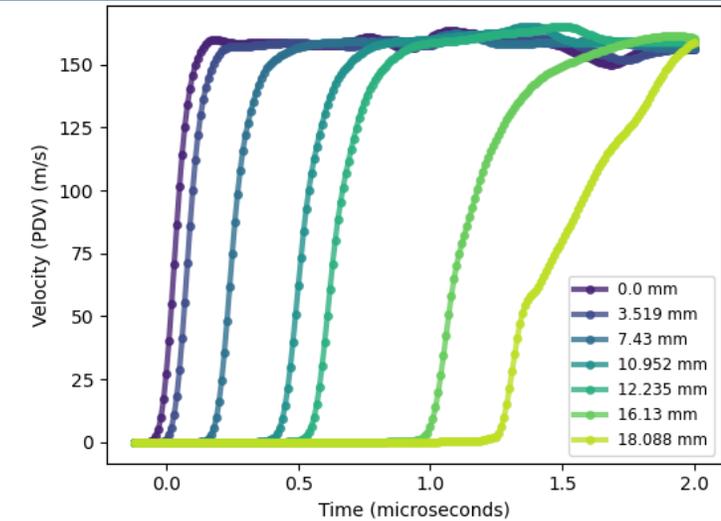
$$\mathcal{F}[v^*(r, t)] = v_{pdv}(r, t)$$

- ▶ An iterative process can be used to recover incremental improvements to  $v(r, t)$  resulting in satisfaction of the self-consistent condition described above.

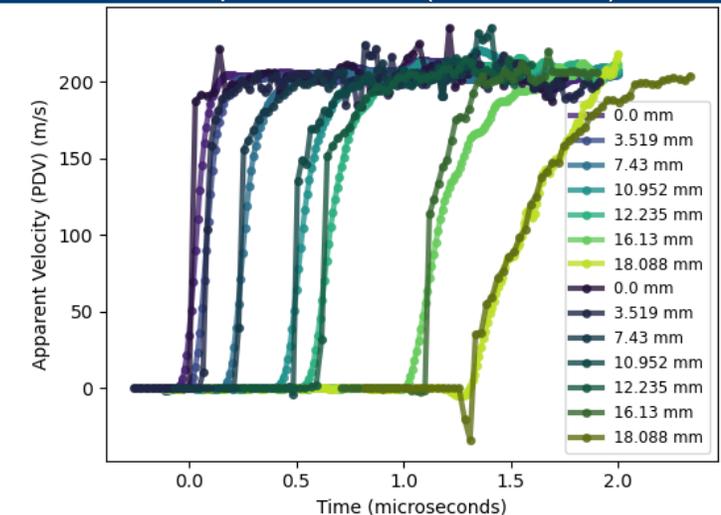
## ▶ Algorithm overview:

1. For a presumed radial-temporal velocity correction  $C'(r, t)$ , apply the correction to obtain a putative window front velocity function:  $v'(r, t) = \frac{1}{1.2669} \left( C'(r, t) + v_{pdv}(r, t) \right)$ .
2. Forward simulate using a window front simulator (3D) with boundary condition  $v'(r, t)$ .
3. Use the mass distribution from the forward simulation to compute  $\frac{d}{dt}m'_{beam}(r, t)$  for each PDV beam radius.
4. Update the velocity correction:  $C'(r, t) = \text{Interp} \left( \left\{ \left( r, \frac{d}{dt}m'_{beam}(r, t) \right) \right\} \right)$
5. Repeat steps 1-4 until the correction function converges.

Iterative recovery of embedded reflector velocity (18 iterations)



Iterative recovery of apparent velocities compared to experiment data (18 iterations)



# How can we estimate $m_{beam}(t)$ and $\frac{d}{dt}m_{beam}(t)$ with experiment data?

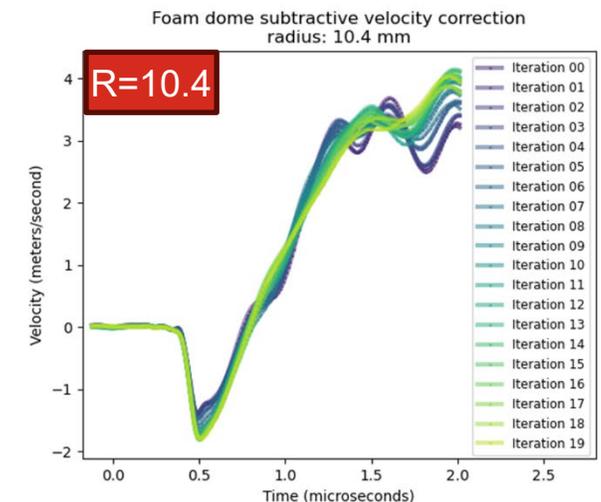
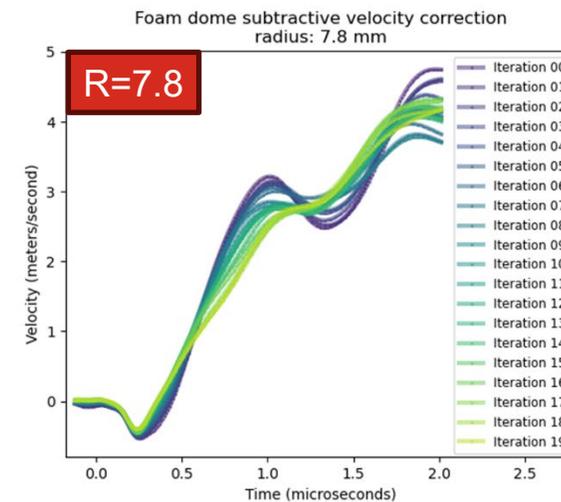
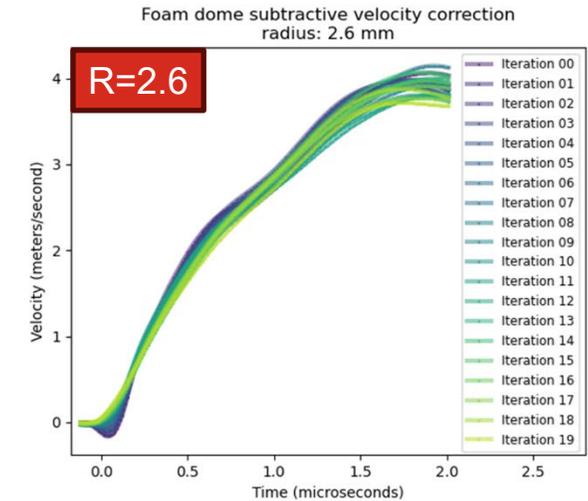
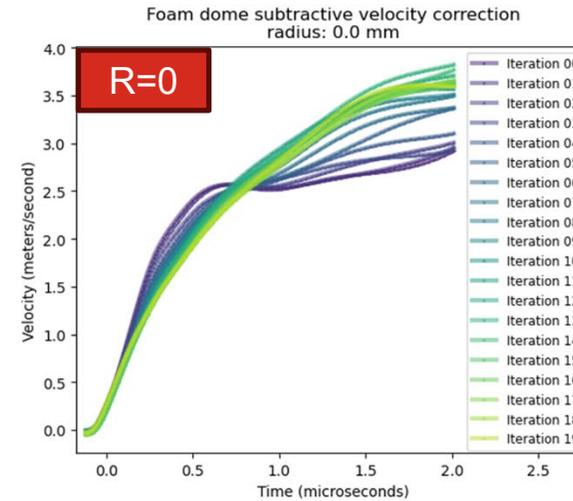
► Applying the iterative recovery algorithm to foam dome data was performed using a dynamic surface reconstruction (DSR) interpolator to transform the 8 (7) PDV points to the entire radial range of the window.

► The LSM forward model was used for these iterative tests and can easily be driven by boundary conditions in the dynamical system integration.

- The forward model consists of only the LiF window material, and evolves as a result of forcing the window front surface to evolve according to the velocity function.

► 20 iterations show good convergence properties using a half-stepping update procedure.

- Overcorrect if you take full-steps
- **Stability obtained with half-stepping**



# How can we estimate $m_{beam}(t)$ and $\frac{d}{dt}m_{beam}(t)$ with experiment data?

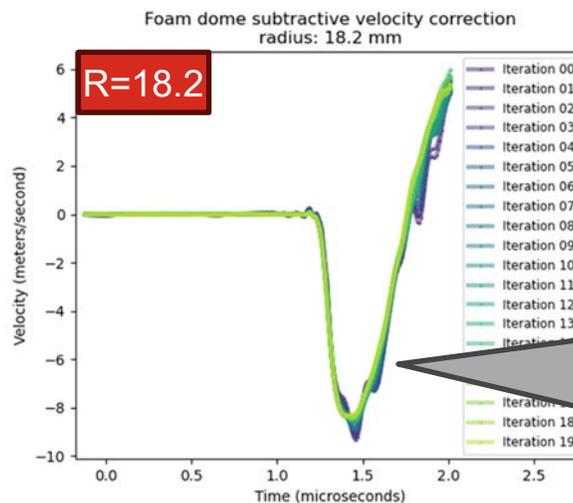
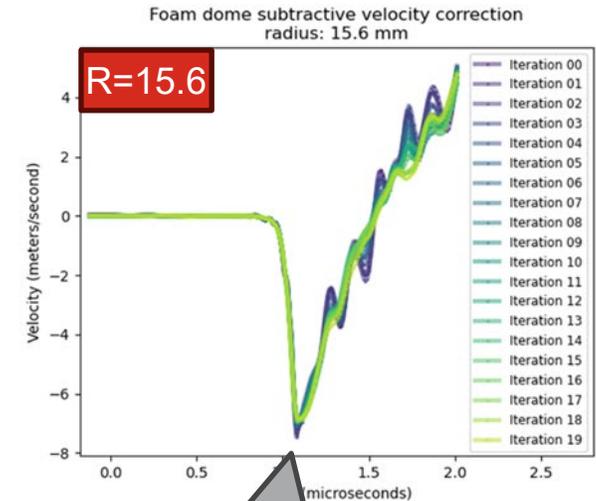
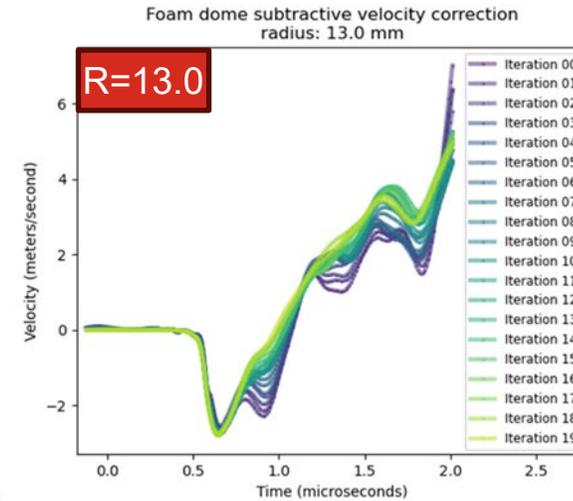
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The iterative algorithm results result from many operations that have a smoothing effect, namely:

1. Interpolating velocities
2. Coarse mesh size mollification

These effects can result in underestimates for velocity corrections