



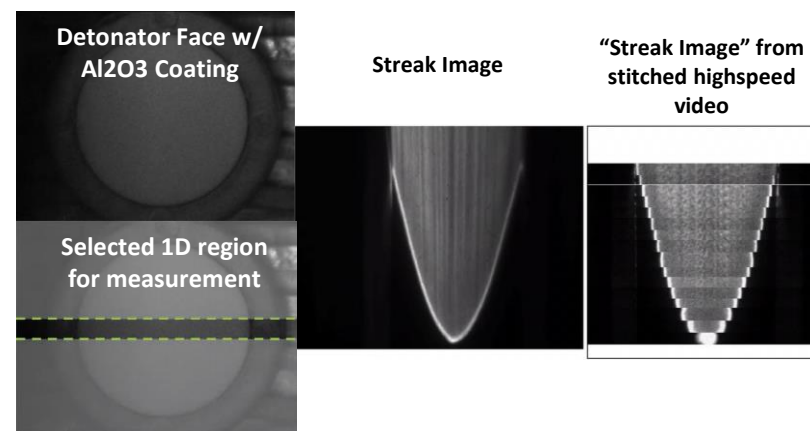
PDV Uncertainty Propagation

Colt Cagle, Rory Bigger, Steve Clarke,
Adrian Quintana

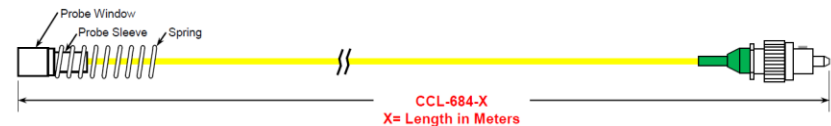
5_6_2026

Q-6 Measurement Objective: Detonator Breakout Time

- Often, detonator component testing requires *only* function time measurements
- Streak camera is golden goose
 - Difficult to maintain/procure
 - Symmetry information often ignored
- PDV as replacement
 - No spatial resolution
 - (Potentially) Increased time resolution
 - Quick test build/turnaround
 - ***Cannot fully replace streak until we can guarantee quality/integrity of data***
 - Uncertainty is still a big question



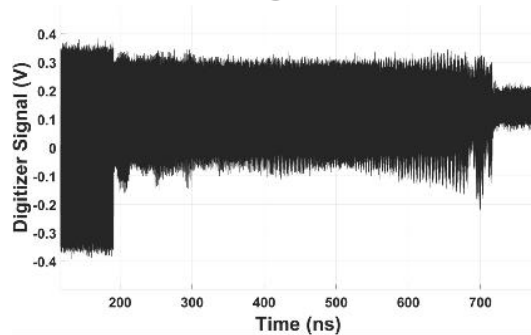
“Mousetrap” PDV fixture



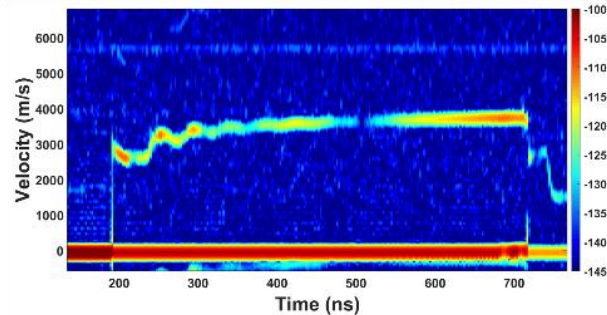
PDV Uncertainty: Taken Too Far

- PDV velocity extraction is a multi-stage process
- Can it go wrong?
 - Yes! But not easily
 - How wrong? **It depends...**

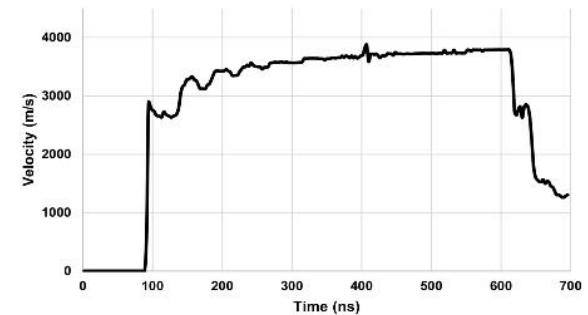
Digitizer



Spectrogram

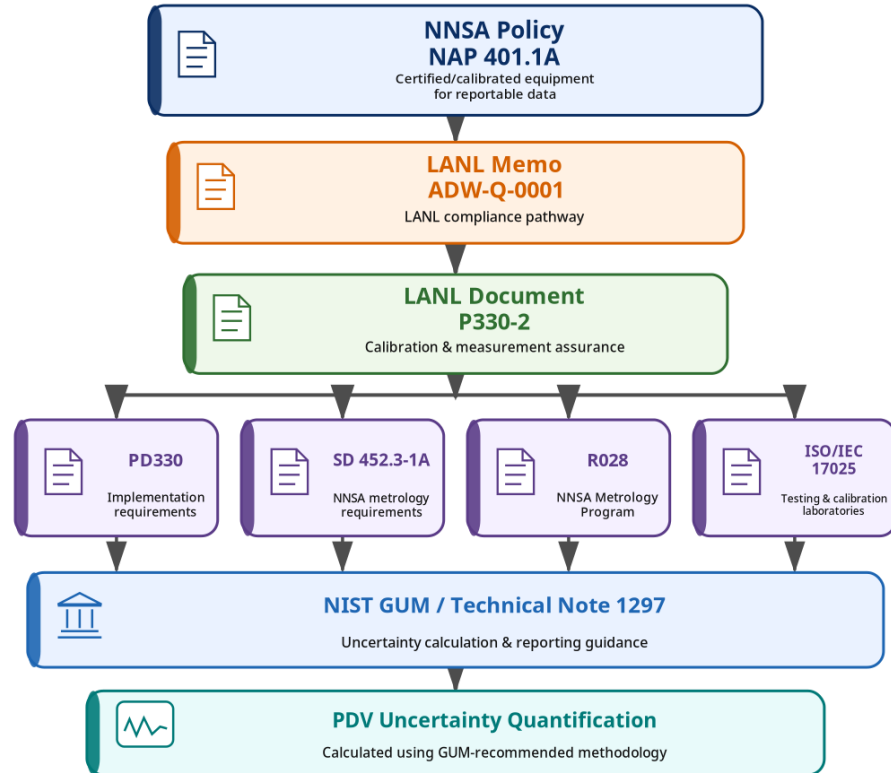


Velocity

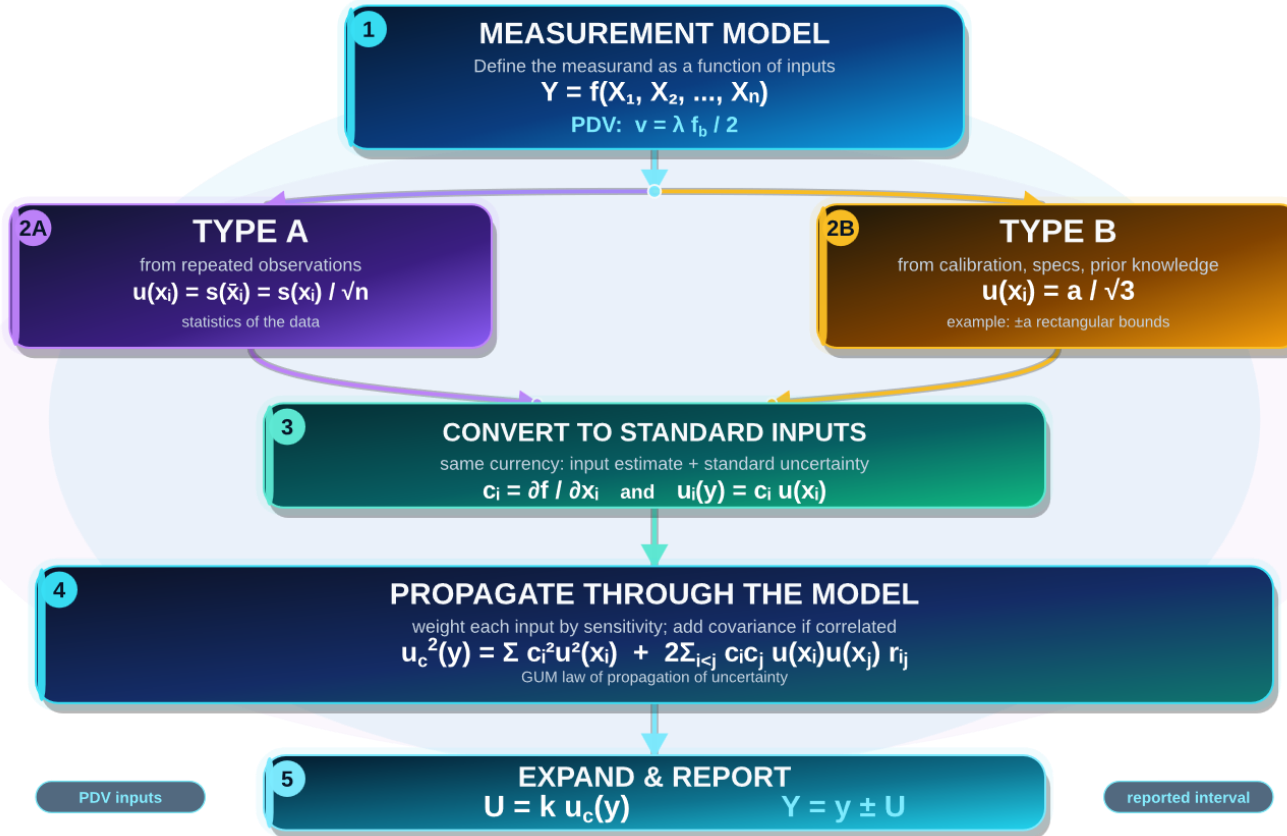


It Depends... Isn't Good Enough

- As a national laboratory, we overcomplicate things
- Q-6 is particularly subject to these protocols
 - Highly structured quality assurance standards
 - Measurement equipment should follow GUM methodology for uncertainty reporting

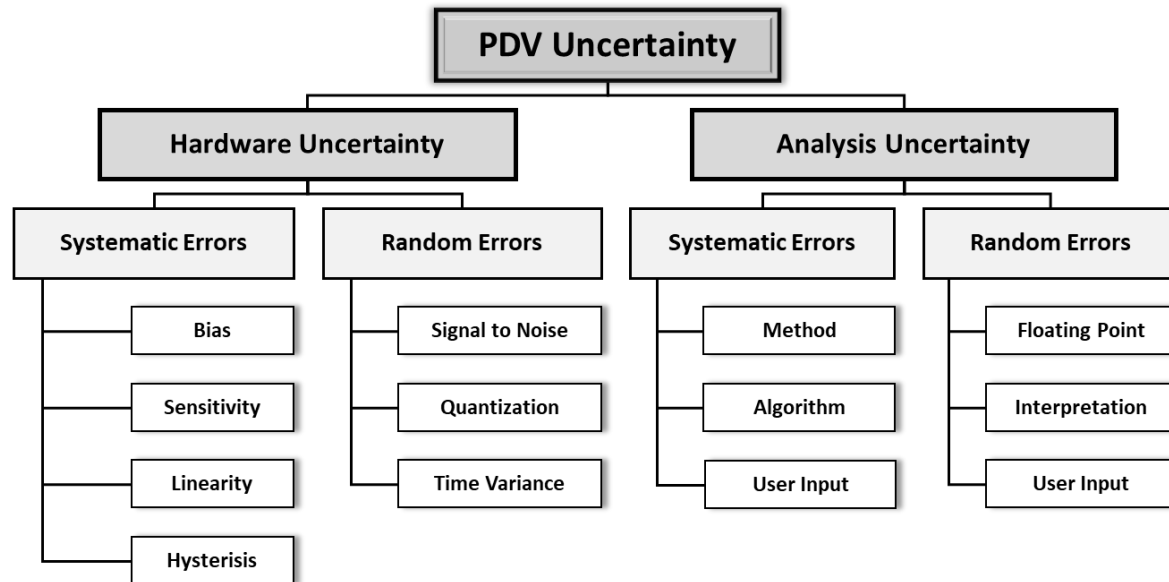


What is GUM-recommended Methodology?



PDV Uncertainty: What's So Difficult?

- Many independent and dependent sources of error
- Analysis uncertainty is particularly difficult to GUM



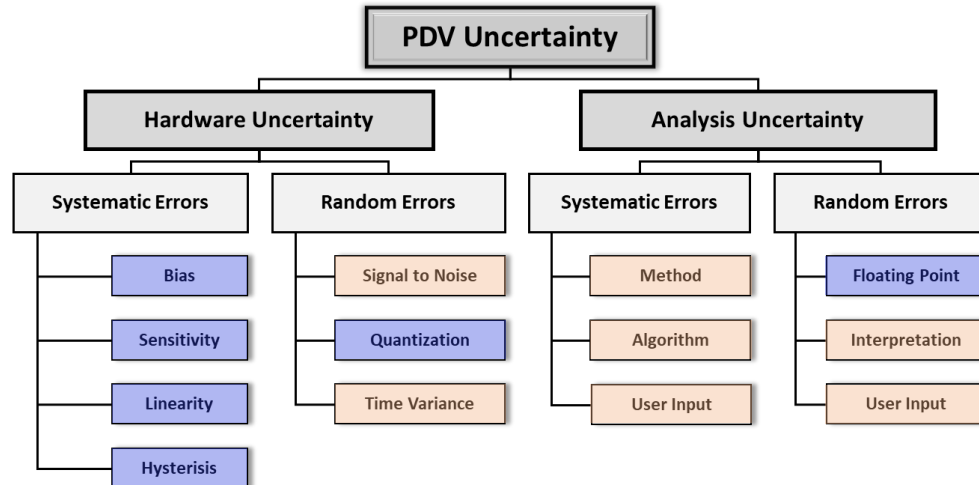
Uncertainty Calculation: Two-Pronged Approach

Characteristic Uncertainty

- Procedure to qualify precision and accuracy of a PDV system
- Minimum resolvable velocity possible by hardware via NIST

Shot Uncertainty

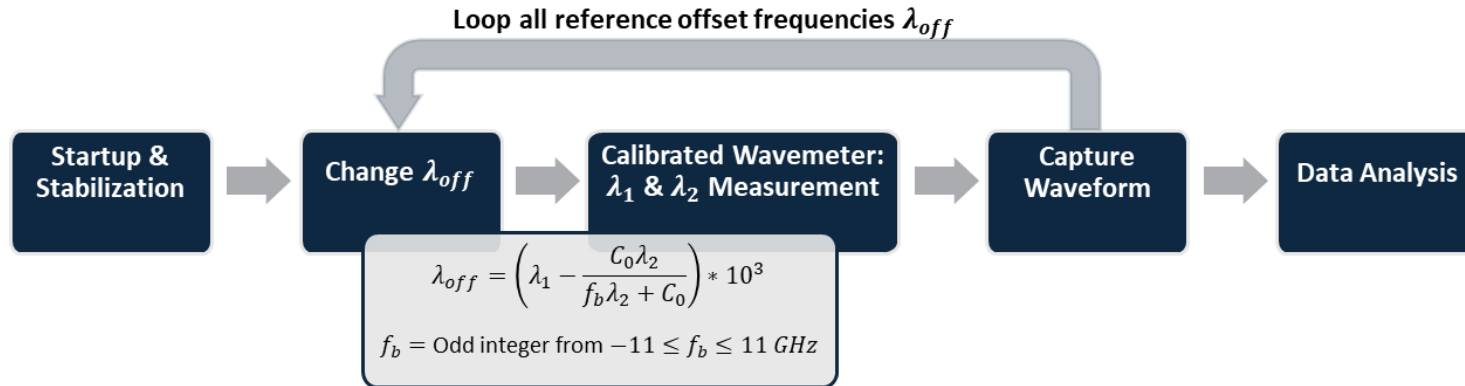
- Uncertainty generated from random variance of the test itself
- Uncertainty added via analysis routine



Characteristic Uncertainty

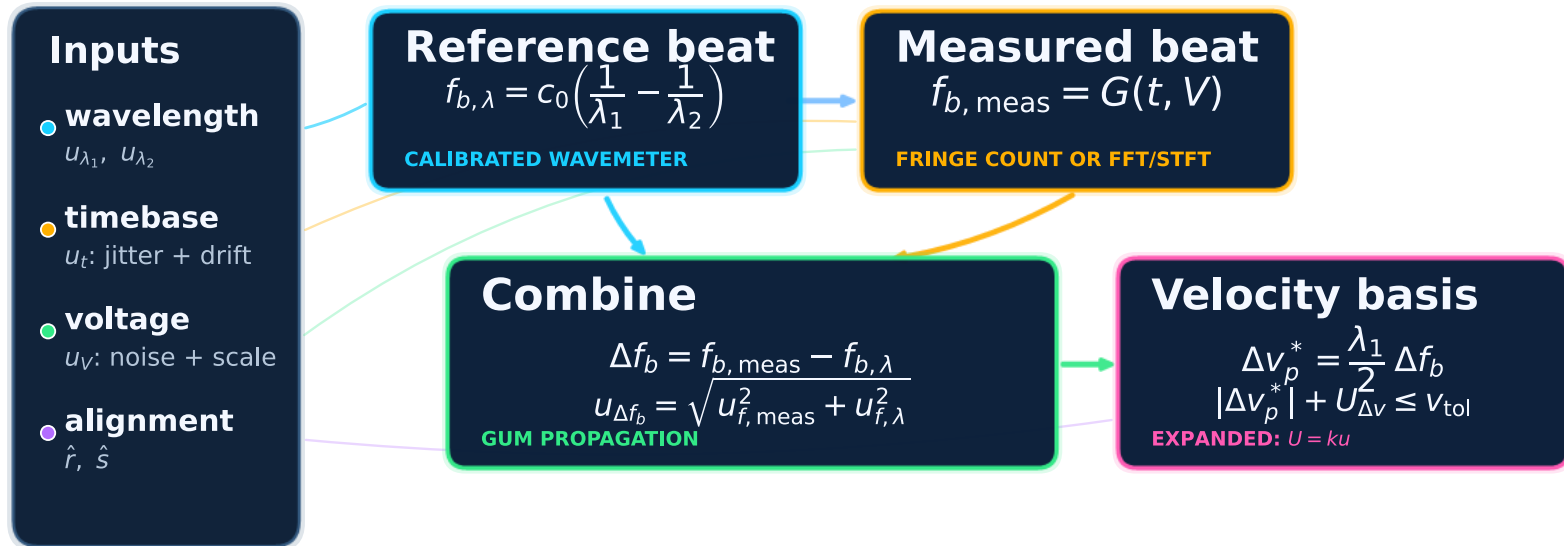
Characteristic Uncertainty: The Procedure

- Basic Idea: Create a known beat frequency, measure it, report deviations
 - Create know beat by measuring the reference and sample PDV lasers with a calibrated wavemeter
 - High accuracy wavemeter is necessary. Thorlabs wavemeter has $\pm 0.45 \text{ pm} \cong \pm 87 \frac{\text{m}}{\text{s}}$.
 - Sweep through multiple beat frequencies by changing reference laser offset
 - Build statistics and ensure no bias exists



Characteristic Uncertainty: Analysis Routine

- Many algorithms $G(t, V)$ exist for extracting the measured beat frequency
 - Two are examined in this work: Zero crossing and FFT



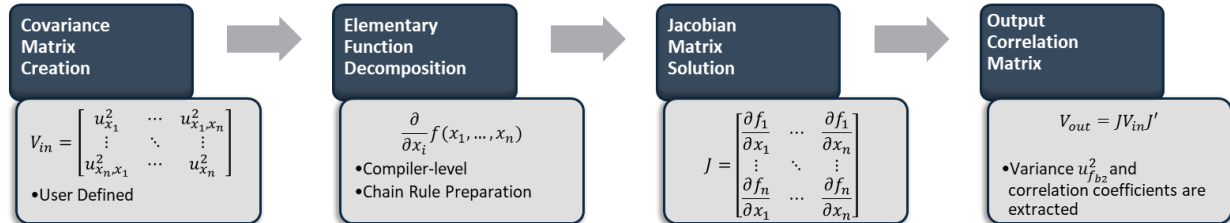
Zero Crossing vs. FFT Algorithm

Zero Crossing

- Simple fringe counting method
- $f_{b,meas} = \frac{\text{number of oscillations}}{\text{time span of oscillations}} = \frac{N_{osc}}{\Delta t_{osc}}$
- $u_{f_{b,meas}} = \sqrt{\left(\frac{\partial f_{b,meas}}{\partial \Delta t_{osc}}\right)^2} u_{\Delta t_{osc}}^2 = \frac{N_{osc} u_{\Delta t}}{\Delta t_{osc}^2}$
- Simple uncertainty propagation
- Very difficult to create robust algorithm
 - Noisy data, differing beat frequencies, etc. cause lots of problems

FFT

- High resilience
- *Difficult uncertainty propagation*
- Matlab extension handles uncertainty
 - METAS Unclib
 - Maintained by Federal Institute for Metrology in Switzerland
 - GUM Compliant



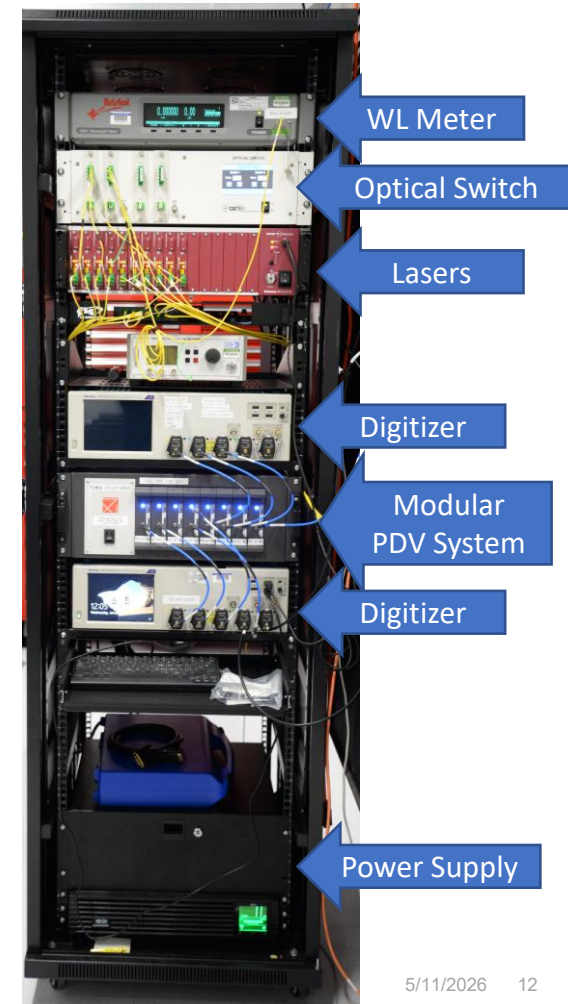
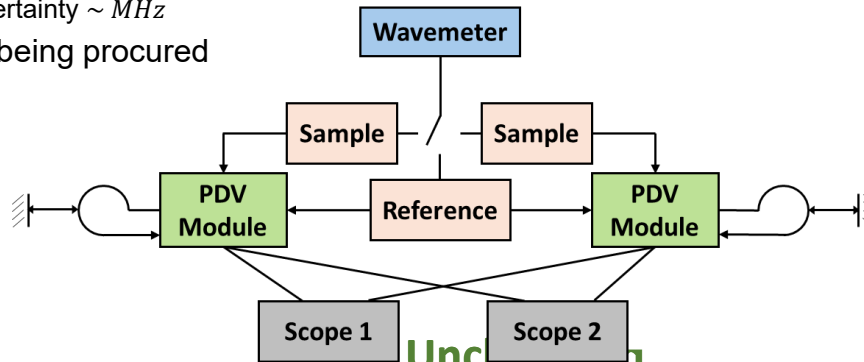
Characteristic Uncertainty At Q-6

Parameters

- Modular TS&I PDV cards
- O/E detectors Miteq SCMR-100K20G-30-15-10
- Tektronix oscilloscopes DPO72304SX
- Bristol 228A calibrated wavemeter
- NKT Basik Laser Cards
- $\{-11: 2: +11\}$ GHz frequency sweep

Results

- $U_{v_p^*} = 13.6 \frac{m}{s}$ for $k = 2$ coverage factor
- Wavemeter uncertainty dominant**
 - FFT uncertainty $\sim kHz$
 - Wavemeter uncertainty $\sim MHz$
- New wavemeter being procured



How Sensitive To Changes In Hardware?

- Purposefully vary parameters
 - New scopes, laser cards, bit depth, etc
- Parametric sweep conducted via one-factor-at-a-time approach
 - 475 waveforms captured

Parameter	Baseline Value
<i>Beat Frequency</i>	Configuration Set 1
<i>Power on O/E</i>	-5 dBm
<i>O/E Detector</i>	Miteq + Thorlabs
<i>Detector Gain</i>	50%
<i>Digital Filter</i>	None
<i>Scopes</i>	Configuration Set 1
<i>Bit Depth</i>	90%

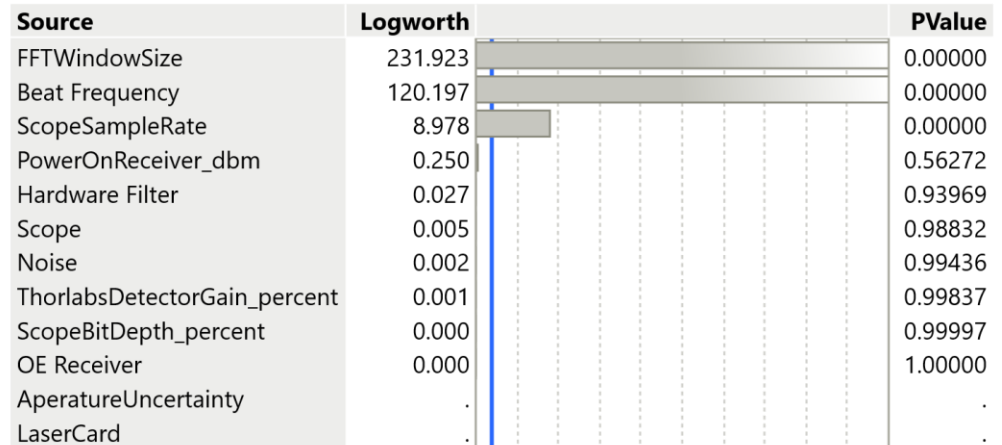
Factor	Configuration Set 1	Configuration Set 2
<i>Beat Frequency (GHz)</i>	-11, -5, -1, 1, 5, 11	-11 to 11 at fine resolution
<i>Scopes</i>	DPO72304SX, DPO72304SX	DPO714AX, DPO72304SX
<i>Scope Sample Rate (GS/s)</i>	50 GS/s, 50 GS/s	125 GS/s, 50 GS/s

Run	Parameter Varied	Value Tested	Other Conditions
1	Time After Startup	PDV Laser Wavelengths	N/A
2	Beat Frequency	Configuration Set 2	Baseline
3	Power on O/E (dBm)	10	Baseline
4	Power on O/E (dBm)	-5	Baseline
5	Power on O/E (dBm)	-15	Baseline
6	Power on O/E (dBm)	-25	Baseline
7	Bit Depth (%)	10	Baseline
8	Bit Depth (%)	50	Baseline
9	Digital Filter	VHF-3100+	Baseline
10	O/E Gain (%)	100	Baseline
11	O/E Gain (%)	75	Baseline
12	O/E Detector Repeatability	Miteq Only	Baseline
13	O/E Detector Repeatability	Thorlabs Only	Baseline
14	Beat Frequency	Configuration Set 2	Scope Configuration Set 2
15	SPC Calibration	Post-SPC	Scope Configuration Set 2
16	Scope Sample Rate (GS/s)	62.5, 25	Scope Configuration Set 2
17	Scope Sample Rate (GS/s)	12500, 2500	Scope Configuration Set 2

Only A Few Variables Are Significant

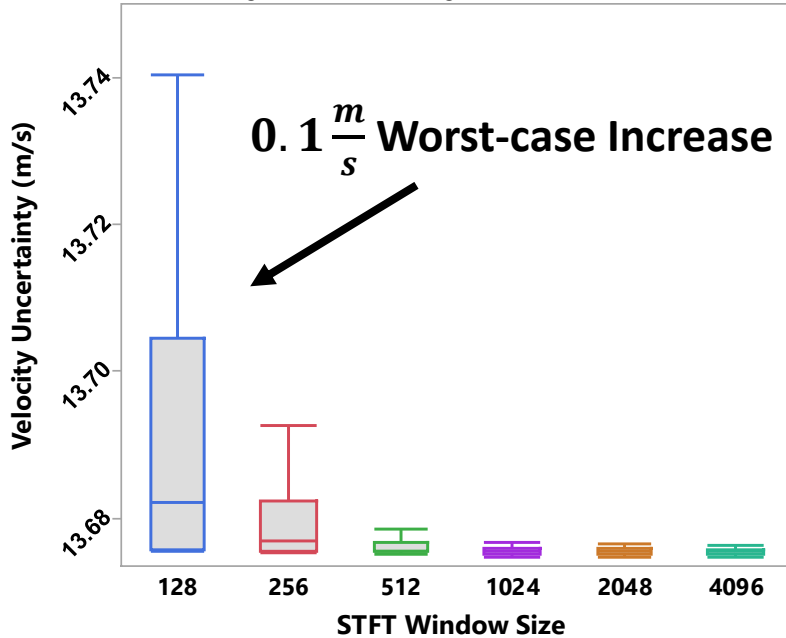
- Statistically significant variables:
Velocity Uncertainty
 - STFT Window Size
 - Beat Frequency
 - Scope Sample Rate

Velocity Uncertainty Regression Model

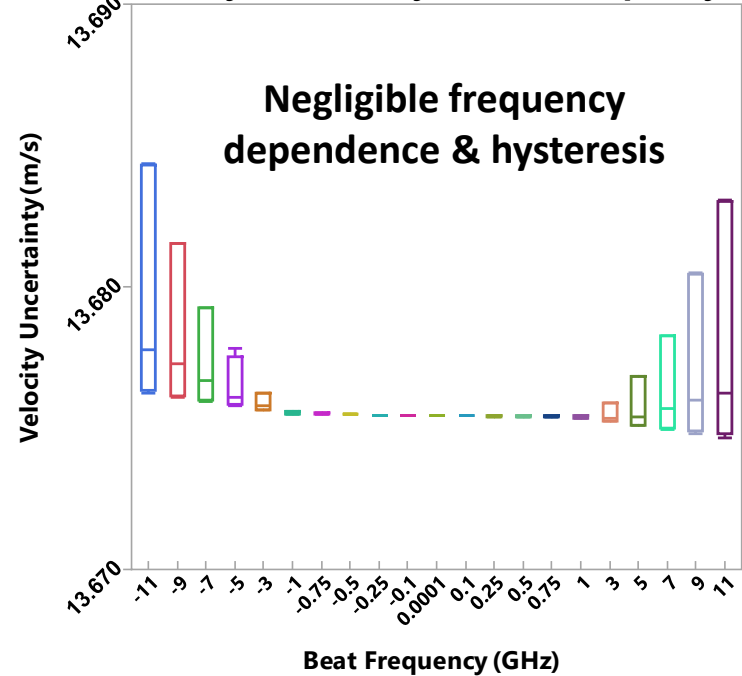


How Significant? Not At All

Velocity Uncertainty vs. STFT Window



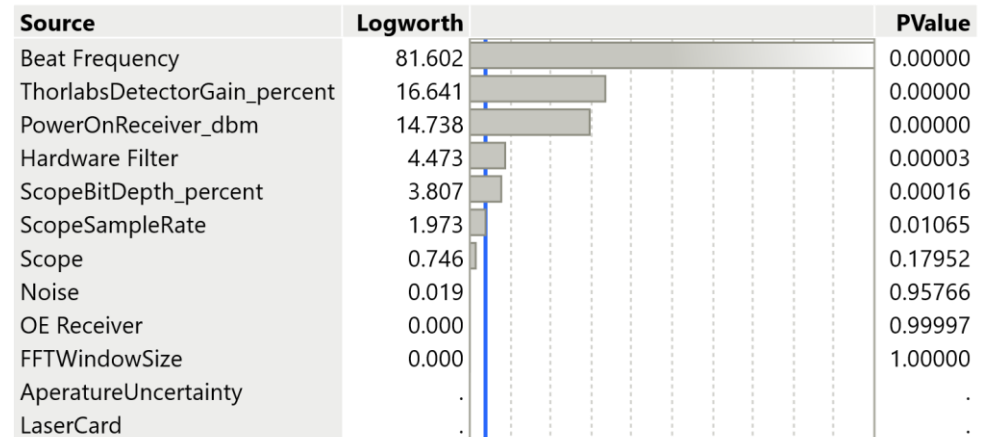
Velocity Uncertainty vs. Beat Frequency



How About Velocity Error?

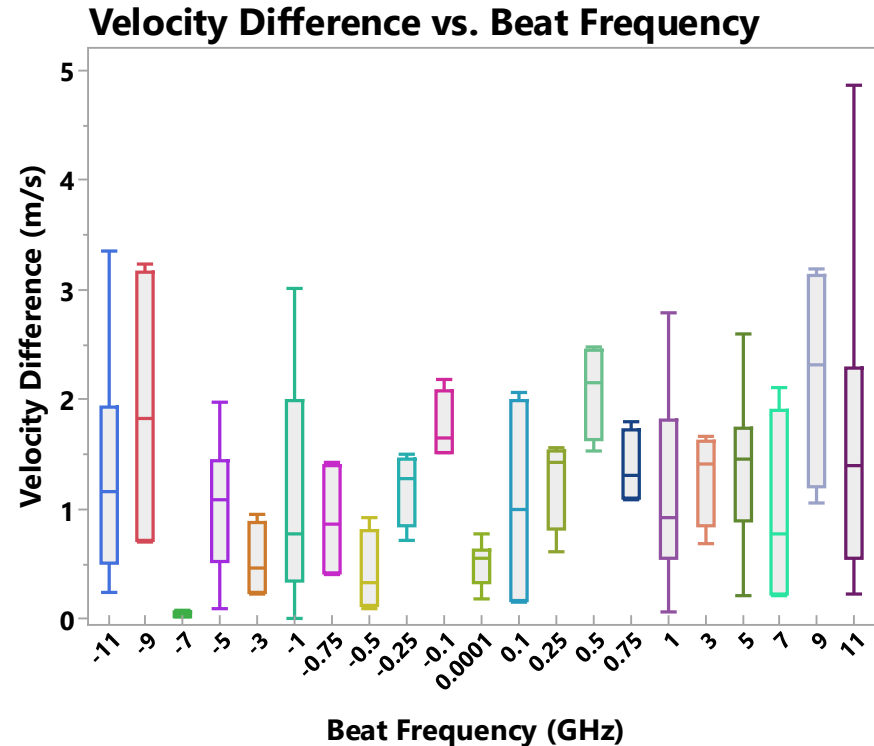
- Statistically significant variables:
Velocity Difference
 - Beat Frequency
 - Detector Gain
 - Power on Detector
 - Hardware Filter
 - Bit Depth

Velocity Difference Regression Model



Velocity Difference Hardware Independent Within $\sim \pm 5 \frac{m}{s}$

- All differences within accuracy of the wavemeter
- For now, no reportable correlations
- Same for all other variables
- Not true for EVERY PDV system, only the ones tested



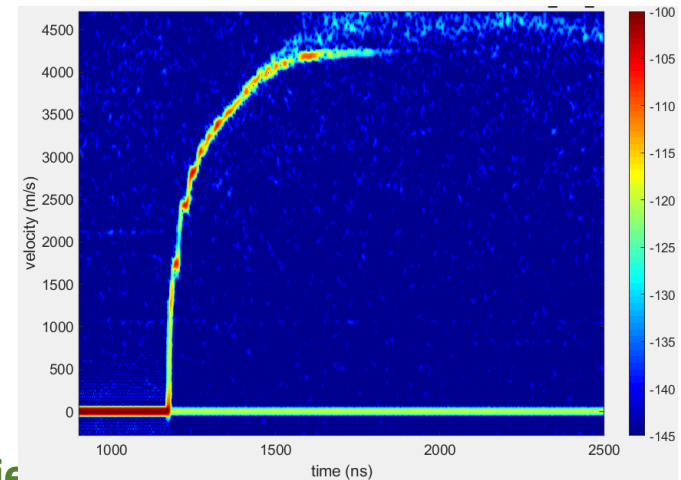
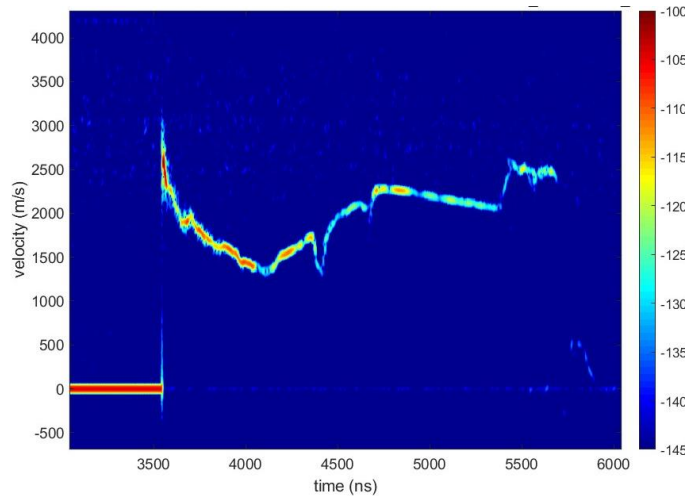
Unclassified

Shot Uncertainty

Unclassified

Shot Uncertainty Preliminary Study

- Scoping scale of the issue
- Useful estimates by Dolan, Ambrose, Strand, etc.
- Basic Goal: Understand max possible velocity deviations based on user input
 - Ran “Monte-Carlo” over a select range of spectrogram analysis parameters
 - STFT window size, Overlap Fraction, Windowing Function, Zero Padding, Velocity Extraction Method



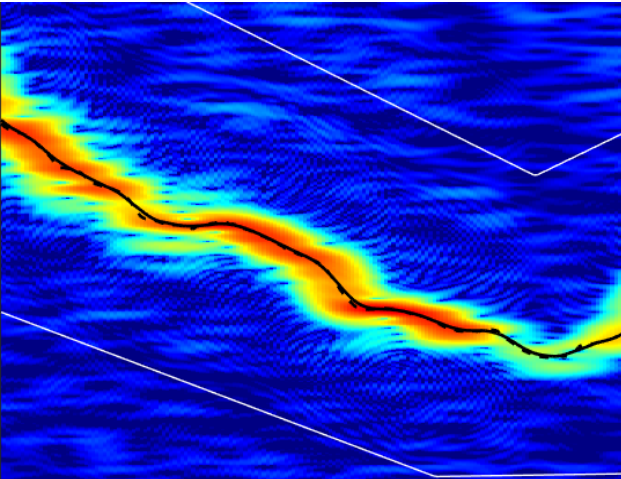
Monte Carlo Across STFT Analysis Parameters

Good Data

Worst case: $\pm 12 \frac{m}{s}$

Typical: $\pm 4 \frac{m}{s}$

No correlations

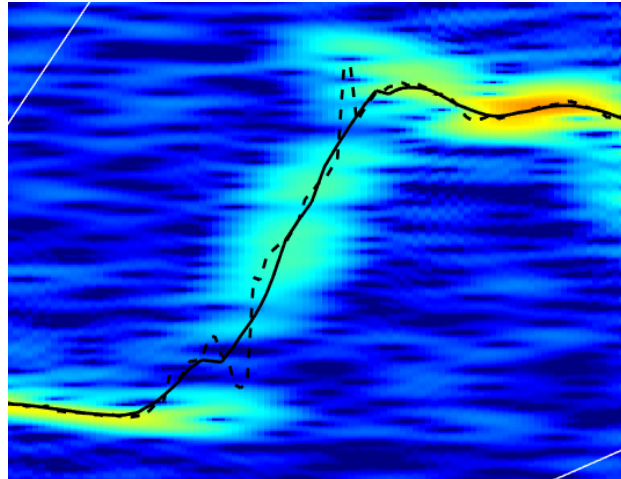


Okay Data

Worst case: $\pm 50 \frac{m}{s}$

Typical: $\pm 10 \frac{m}{s}$

Extraction method biased

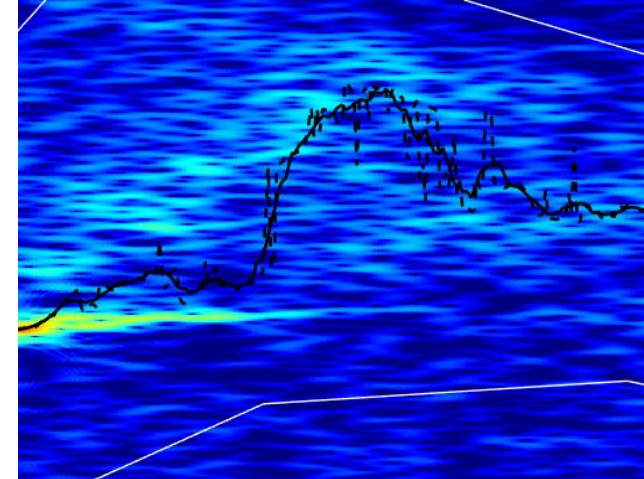


Tough Data

Worst case: $\pm 500 \frac{m}{s}$

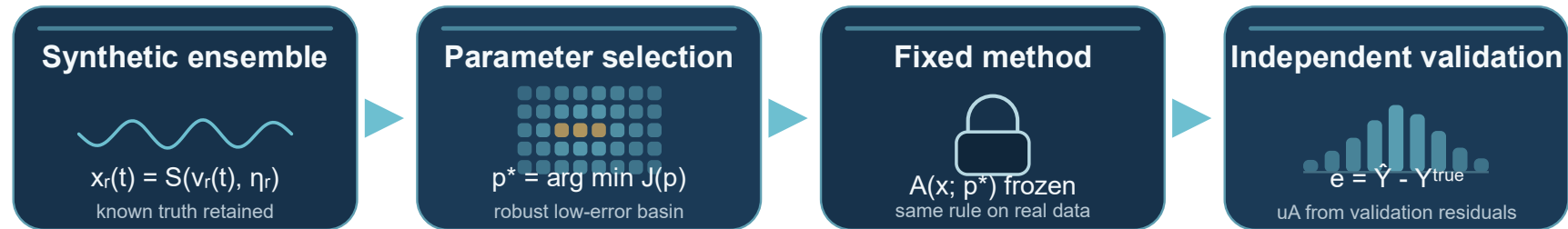
Typical: $\pm 50 \frac{m}{s}$

User interpretation biased



Shot Uncertainty: Can It GUM?

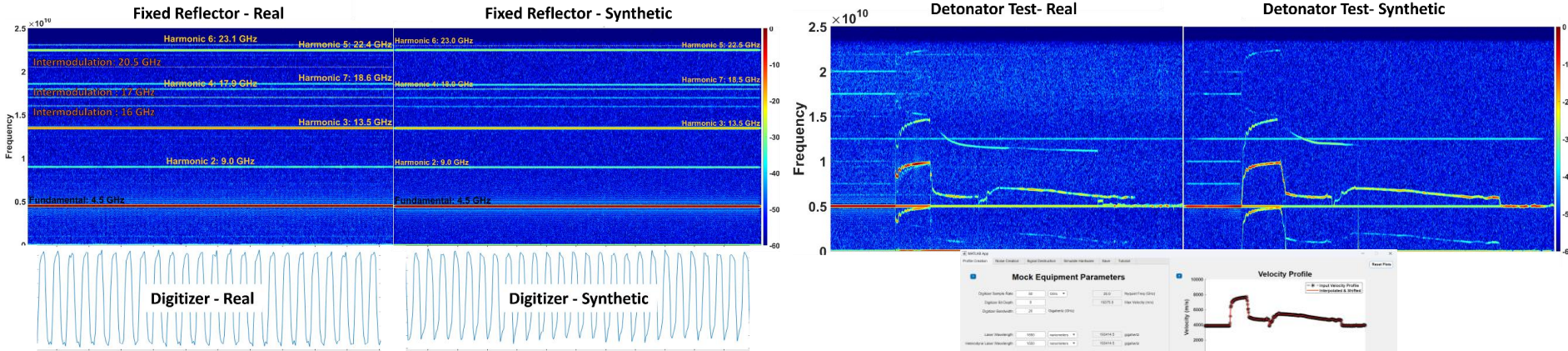
- Given the data variability and necessity for human input, its not easy
- Must rely on statistical methods
 - Under development
 - Basic structure similar to characteristic uncertainty procedure



selection set chooses the method; validation set assigns the uncertainty

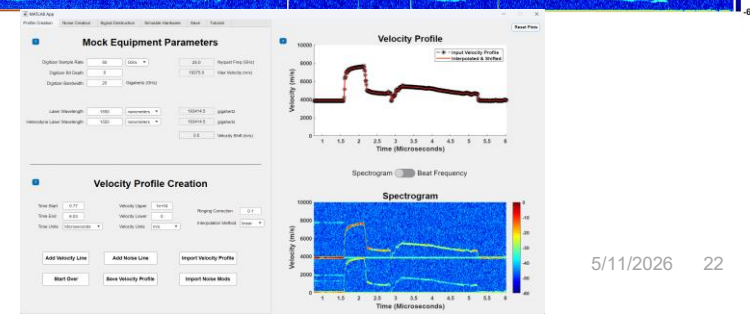
Synthetic PDV Data Creation (SPICE) Matlab GUI

- Goal: Generation of realistic synthetic PDV data
- Recent upgrades adding complex gaussian speckle w/ phase unwrapping
 - Properly implements physics of “multi-reflector” surface variations



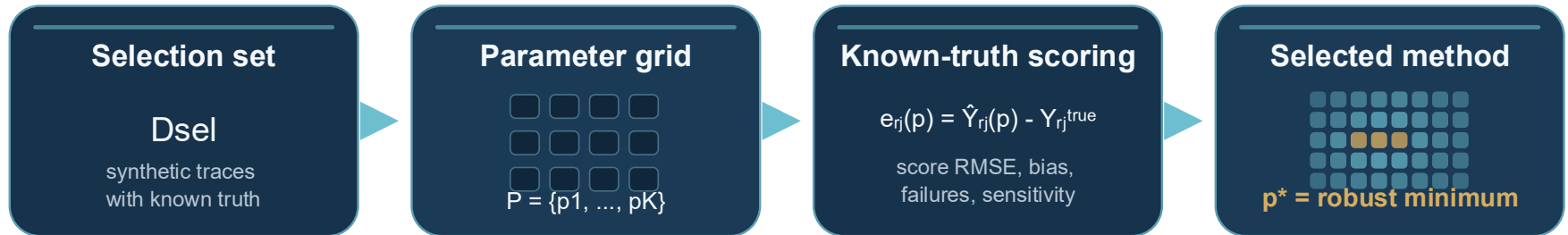
$$x(t) = \Re\{A_r S(t) e^{i\phi_{PDV}(t)}\}$$

$$S(t) \sim \sqrt{1 - d_k} e^{i\theta} + \sqrt{d_k} u_k(t)$$



Choose Analysis Method Set Based On Monte Carlo

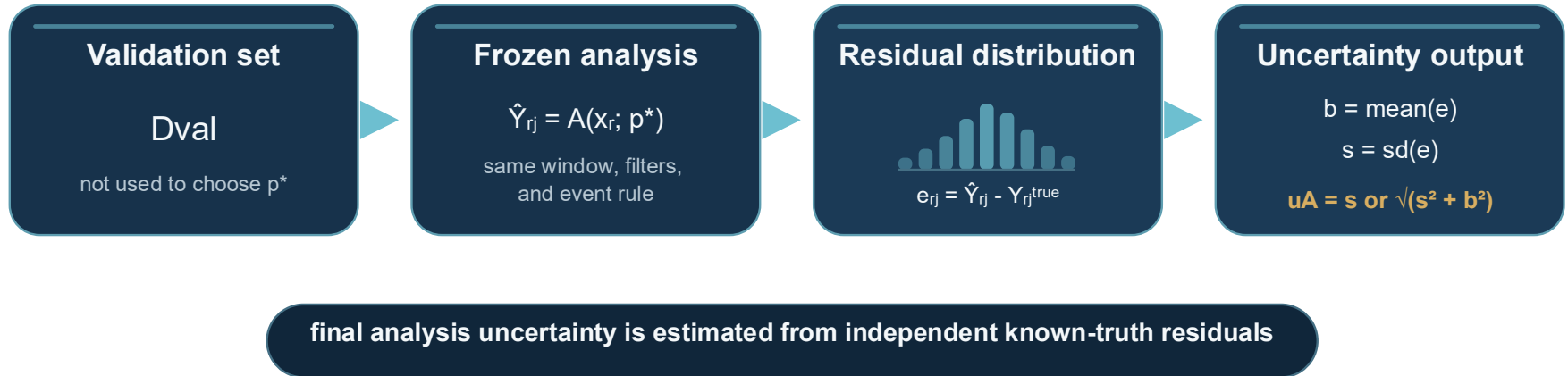
- Create a synthetic dataset w/ expected shot variations
- Run Monte-Carlo of STFT parameter set across all traces



$J(p)$ combines residual error, bias, failure rate, and local sensitivity
choose a broad low-error basin, not a single lucky pixel

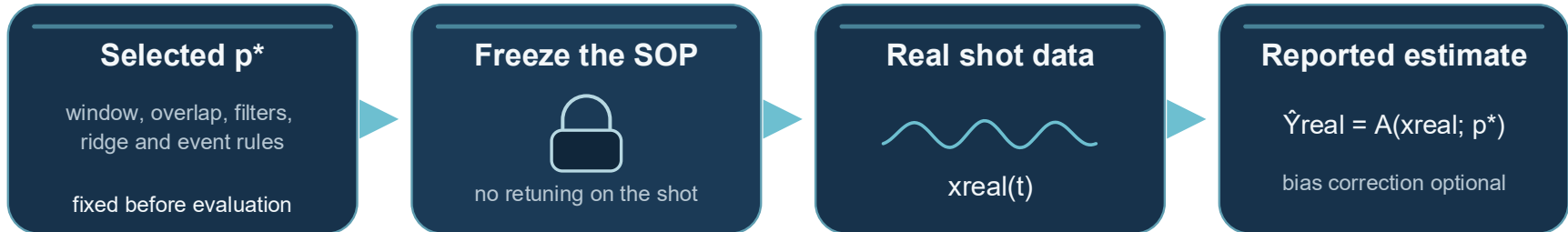
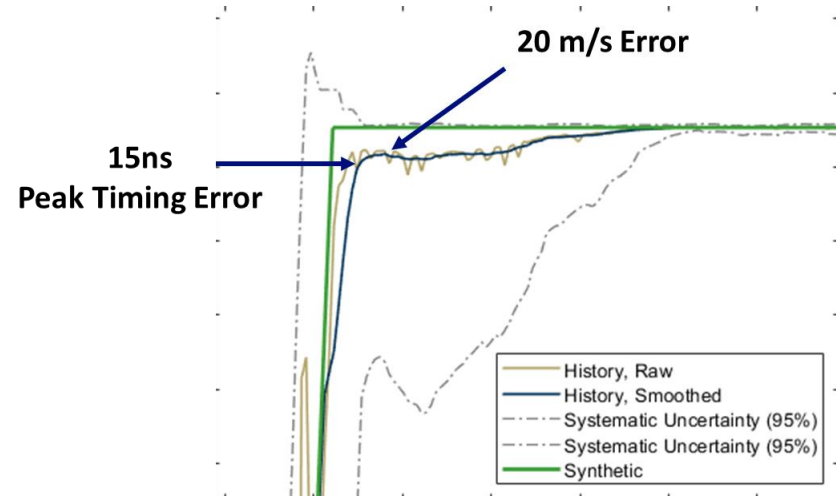
Generate Residuals From Selected Method

- Either statistically or parameter-wise generate residuals
- Still assuming synthetic data is accurate representation of real data



Apply Analysis & Corrections

- Once set, program will capture data and apply residual estimates after analysis
- Bias corrections (rolling off sharp edges, etc) possible



$Y_{reported} = \hat{Y}_{real} - b$ only if the synthetic bias correction is justified

Looking Forward: PDV At Q-6

- Still finalizing PDV shot uncertainty routine
 - Hope to finish within FY26 (so realistically by FY28...)
- Small test series of simultaneous streak + PDV
 - Prove timing congruency
- Triature PDV Setup (i.e. Dolan)
 - Hardware procured, finding time to run comparison
- Dual-Core probe testing
 - Isolate launch and return fibers, hopefully removing more baseline noise
 - Parts in procurement

Backup

Reference Beat Uncertainty, $f_{b,\lambda}$

REFERENCE BEAT

calibrated wavelengths define the expected beat frequency

Calibrated inputs

wavemeter measurements

$$\lambda_1 \pm u_{\lambda_1}, \lambda_2 \pm u_{\lambda_2}$$

Interference signal

reference-sample term

$$I(t) = \sum_{n=1}^{N_R(t)} A_R A_n \cos \Phi_n(t)$$

Type B

λ

Expected beat frequency

known frequency target

$$f_{b,\lambda} = c_0 \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

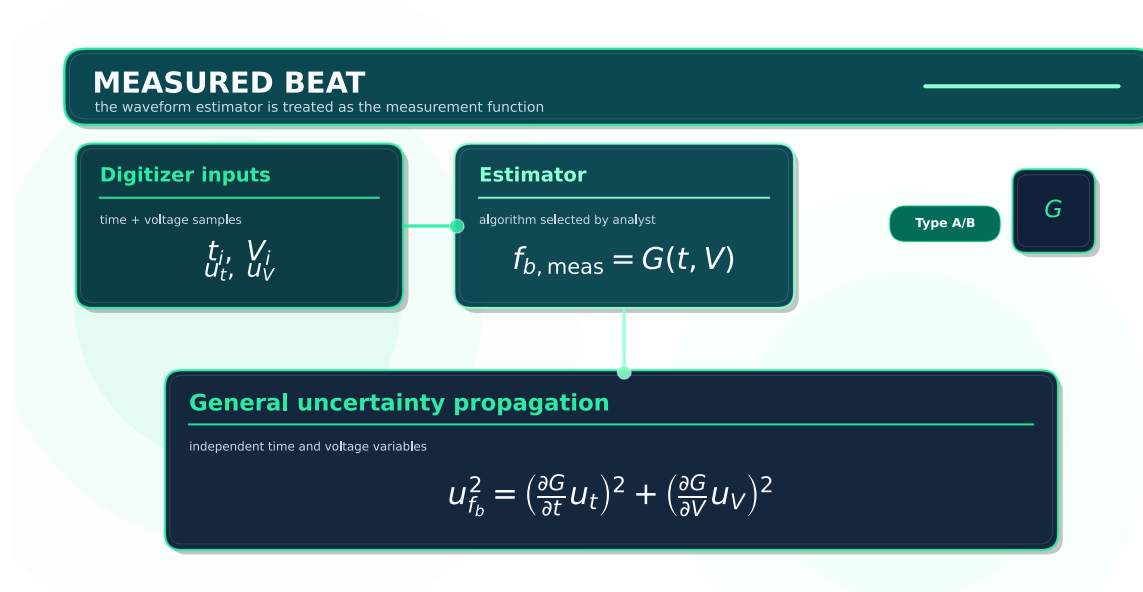
Type B propagation

wavelength uncertainty to beat uncertainty

$$u_{f_{b,\lambda}}^2 = \left(\frac{c_0}{\lambda_1^2} u_{\lambda_1} \right)^2 + \left(\frac{c_0}{\lambda_2^2} u_{\lambda_2} \right)^2$$

Sample Beat Uncertainty, $f_{b,meas}$

- Digitizer time uncertainty is primary culprit
 - Random noise and clock drift



Analysis Function Uncertainty

ZERO-CROSSING METHOD

fringe-counting reduces the problem to a time-interval uncertainty

Frequency estimator

count oscillations

$$f_{b, \text{meas}} = \frac{N_{\text{osc}}}{\Delta t_{\text{osc}}}$$

Estimator uncertainty

count exact; interval uncertain

fringe counting

$$u_{f_{b, \text{meas}}} = \frac{N_{\text{osc}}}{\Delta t_{\text{osc}}^2} u_{\Delta t}$$

Noise term

datasheet + acquisition settings

$$N_{\text{rms}} = \sqrt{u_{\text{rand}}^2 + \left(\frac{BW}{25 \text{ GHz}} DNF VS \right)^2}$$

Time interval uncertainty

slew-rate noise + jitter + clock drift

$$u_{\Delta t} = \sqrt{\left(\frac{N_{\text{rms}}}{SR_1} \right)^2 + \left(\frac{N_{\text{rms}}}{SR_2} \right)^2 + u_{\text{rand}}^2 + \left(\frac{TBA \Delta t}{\sqrt{3}} \right)^2}$$

Analysis Function Uncertainty

FFT / STFT METHOD

windowed Fourier analysis can be propagated through covariance

Windowed frame

local analysis buffer

$$\tilde{x}_m^w(n) = x(n + mR)w(n)$$

STFT bin

zero-padded windowed frame

$$\tilde{X}_m(\omega_k) = \sum_{n=-N_z/2}^{N_z/2-1} \tilde{x}_m^{w,z}(n)e^{-j\omega_k nT}$$

FFT/STFT

Frequency grid

bin locations

$$\omega_k = \frac{2\pi k}{NT}$$

Covariance propagation

GUM / METAS style

$$\mathbf{V}_{\text{out}} = \mathbf{J}\mathbf{V}_{\text{in}}\mathbf{J}^T$$

Estimator output

selected peak

$$\tilde{X}_m \rightarrow f_b, u_{f_b}$$

Comparison Uncertainty

COMPARISON BASIS

hardware characterization compares measured and reference beat frequency

Difference

measured minus reference

$$\Delta f_b = f_{b, \text{meas}} - f_{b, \lambda}$$

Combined standard uncertainty

independent contributions

beat-frequency basis

$$u_{\Delta f_b} = \sqrt{u_{f_{b, \text{meas}}}^2 + u_{f_{b, \lambda}}^2}$$

Expanded uncertainty

coverage factor

$$U_{\Delta f_b} = k u_{\Delta f_b}$$

Agreement check

frequency-domain pass criterion

$$|\Delta f_b| \leq U_{\Delta f_b}$$

Velocity Uncertainty

VELOCITY BASIS

frequency error is converted to the acceptance metric

Velocity error

PDV velocity relation

$$\Delta v_p^i = \frac{\lambda_1}{2} \Delta f_b$$

Velocity uncertainty

frequency + wavelength terms

k = 2 typical

$$u_{\Delta v_p^i} = \sqrt{\left(\frac{\lambda_1}{2} u_{\Delta f_b}\right)^2 + \left(\frac{\Delta f_b}{2} u_{\lambda_1}\right)^2}$$

Expanded velocity

reported bound

$$U_{\Delta v_p^i} = k u_{\Delta v_p^i}$$

Acceptance criterion

velocity-domain qualification

$$|\Delta v_p^i| + U_{\Delta v_p^i} \leq v_{\text{tol}}$$