

Evolution of Strength During Beta to Gamma (bct) Polymorphic Transition in Pure Tin: Combined Compression & Shear Plate Impact Experiments

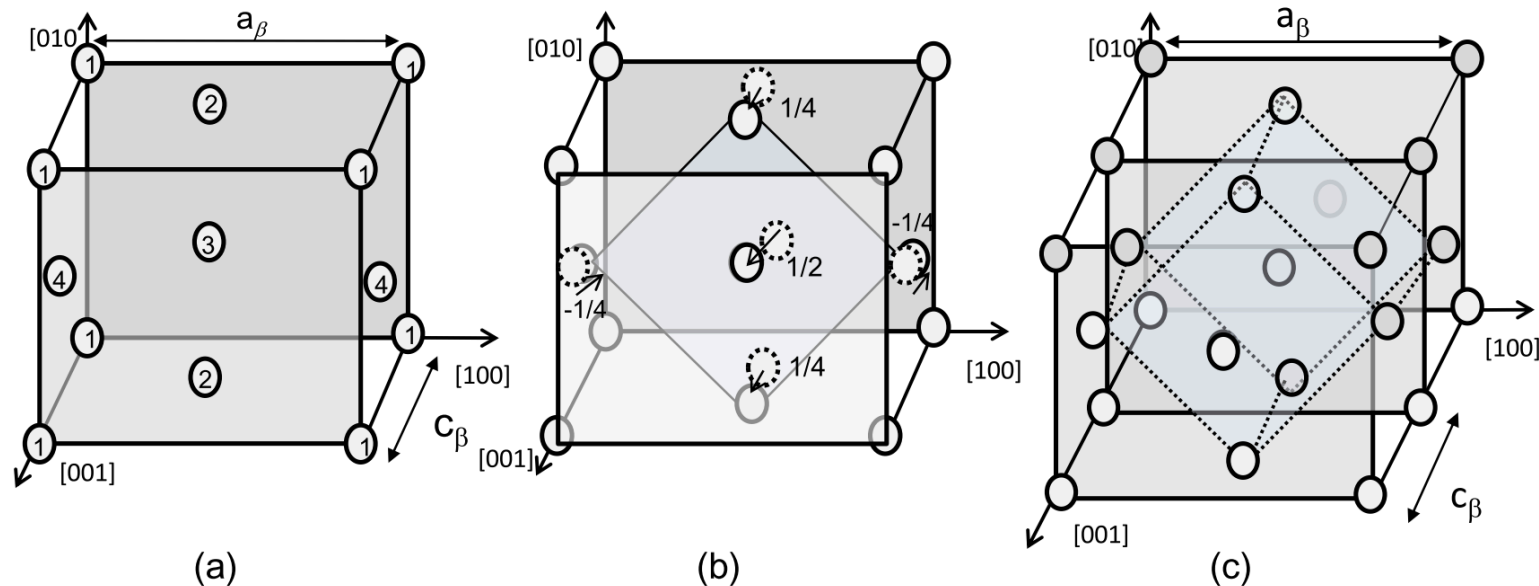
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Acknowledgment: US DOE LLNL (David Bober)


PDV Workshop
Washington State University
May 2026

Introduction

At room temperature pure tin undergoes a solid-state structural (polymorphic) transition from β -Sn (distorted bct) to γ -Sn (bct) at $\sim 9\text{-}10$ GPa and then from bct to bcc phase at ~ 44 GPa.



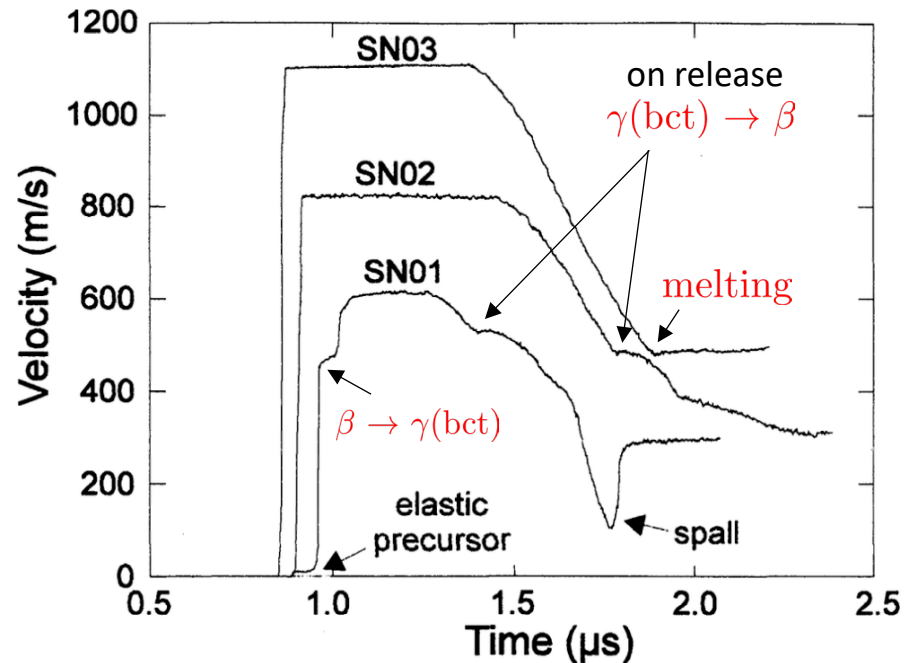
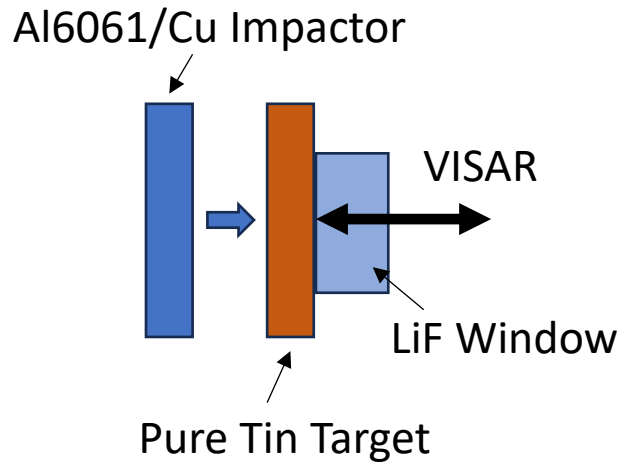
Distorted bct Structure
 β – (white) Sn
 $a = 5.833 \text{ \AA}$; $c/a = 0.545$

~ 9.5 to 10 GPa

Atomic Rearrangement

More symmetric bct Structure, γ -Sn
 $a = 3.81 \text{ \AA}$; $c/a = 0.910$
 Volume decrease $\sim 3\text{-}5\%$

Previous Work: Shock Induced Polymorphic Transitions in Tin – Mabire *et al.* (1999)

- At room temperature, pure Sn undergoes $\beta \rightarrow \gamma(\text{bct})$ transition at ~ 10 GPa.
- Melting of tin at ambient pressure occurs at ~ 505 K.



SN01 (Al6061: 1235 m/s; 12.3 GPa)

- Elastic Precursor
- Plastic wave in the β Phase
- $\beta \rightarrow \gamma(\text{bct})$ transformation
- Plastic Wave in the γ -Sn (bct) Phase
- Release – Reverse Transformation $\gamma(\text{bct}) \rightarrow \beta$
- Spall

SN02 (Cu: 1131 m/s ; 17.9 GPa)

- Overdriven Shock Wave
- $\gamma(\text{bct}) \rightarrow \beta$ Reverse Transformation

SN03 (Cu: 1518 m/s; 26.2 GPa)

- Overdriven shock wave
- Solid to Liquid Transition (melting) during release

Motivation

Make direct measurements of the dynamic strength of β -Sn and γ -Sn (bct) phases during the low-pressure polymorphic transition in pure (Tri-lab 99.8 %) tin.

Avg. grain size $\sim 65 \mu\text{m}$. (Impurities: O ~ 0.0025 , N < 0.0005 , C 0.0019, S < 0.0005 , Pb 0.11, Cu 0.054, In 0.0099)



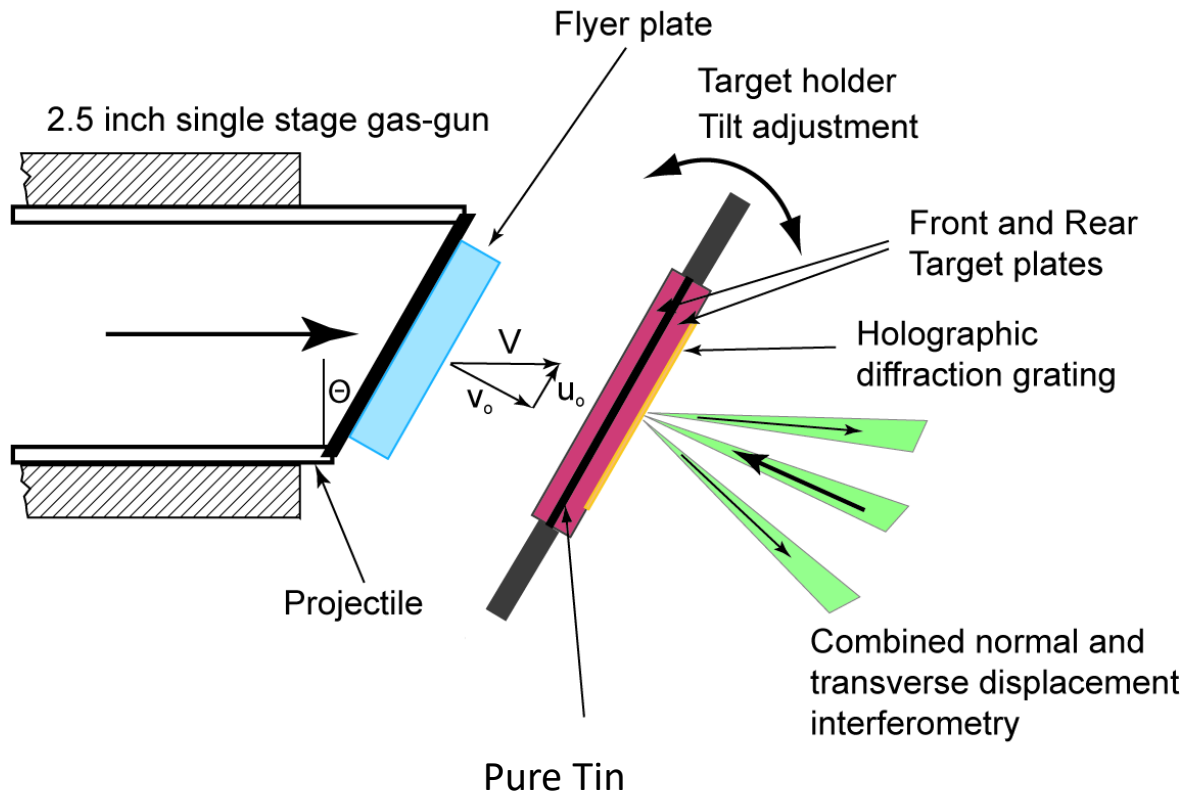
Room temperature, Pressure $\sim 10 \text{ GPa}$

Approach:

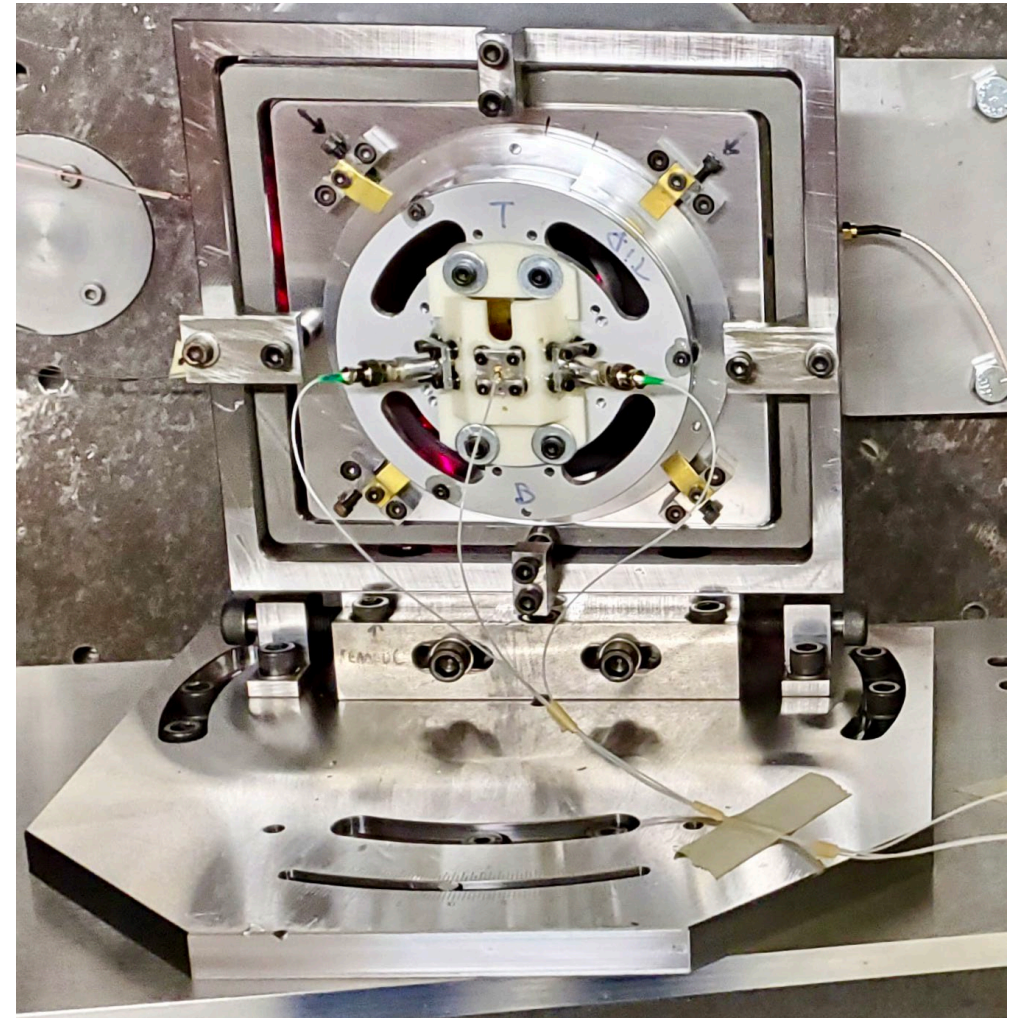
Utilize the combined compression-and-shear plate impact experimental configuration.

Use pressure (compression) to drive the transformation and shear to determine the dynamic strength

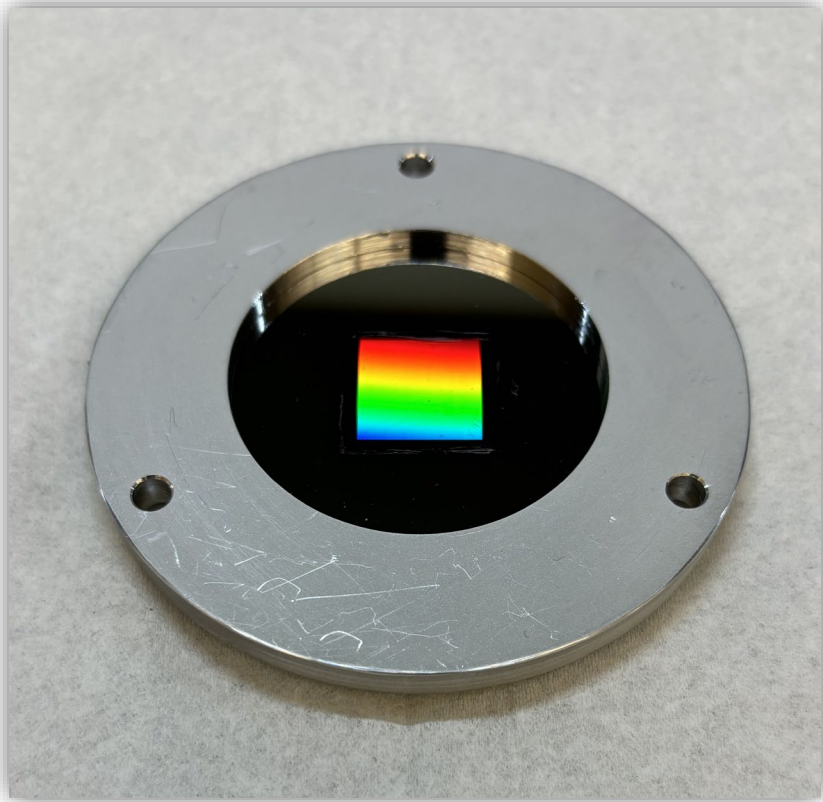
Experimental Configuration: Combined Compression & Shear Plate Impact



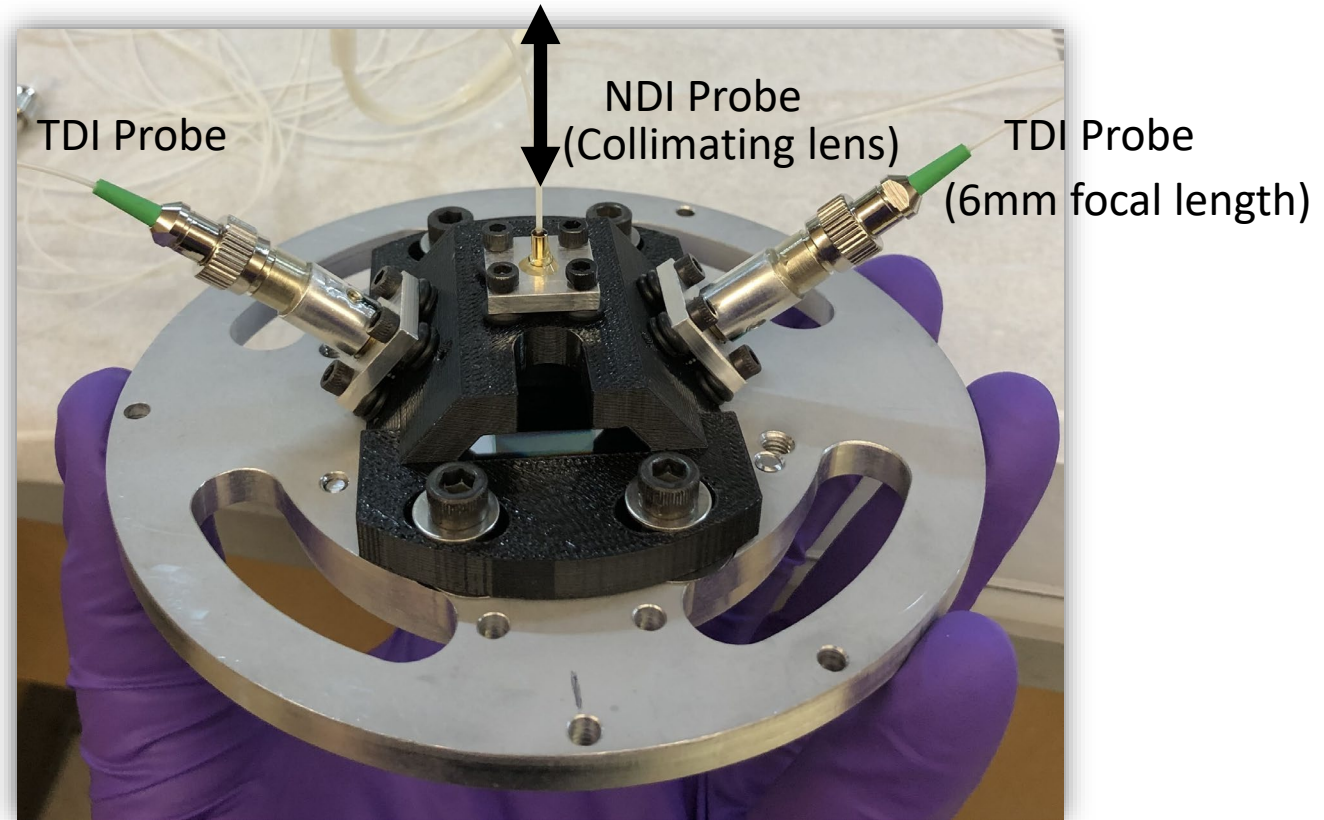
Impactor and the Target plates are either D3 Tool Steel or WC 3 wt. % Co binder



Combined Transverse and Normal Velocimetry Probes

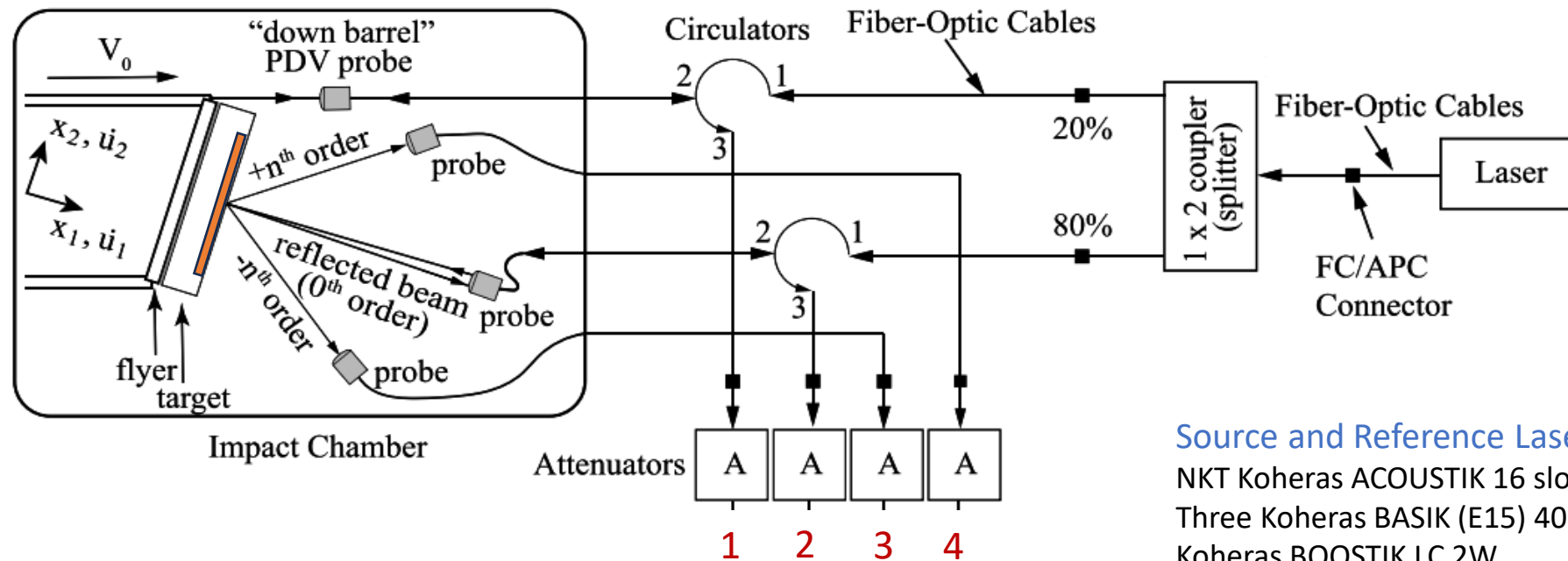


Holographic Grating – 500 lines/mm



Laser beam impinges on grating using the NDI probe.
 0^{th} order and $\pm 1^{\text{st}}$ order diffracted beams are generated

Schematic of PDV System for Combined Measurement of Normal and Transverse Particle Velocities



Source and Reference Lasers:
 NKT Koheras ACOUSTIK 16 slot rack
 Three Koheras BASIK (E15) 40mW
 Koheras BOOSTIK LC 2W

Fiber Optics Hardware:
 Thorlabs and Agiltron

Detectors and Digitizer:
 MITEQ 20 GHz
 Tektronix MSO64B 6-BW-6000 upgraded to 8GHZ

Mix 3 + 4 \Rightarrow Transverse Particle Velocity

Mix 2 + Ref \Rightarrow Normal Particle Velocity

Mix 1 + Ref \Rightarrow Projectile Velocity

Mix 3 and 4 beams separately with Ref \Rightarrow Normal and Transverse Particle Velocities

Normal and Transverse Particle Velocities

$$I^\pm = I_S^\pm + I_R^\pm + 2\sqrt{I_S^\pm I_R^\pm} \cos \left[\underbrace{\frac{2\pi}{\lambda_S} \{u_1(t)(1 + \cos \theta_n)\}}_{\text{Normal motion}} \pm \underbrace{u_2(t) \sin \theta_n}_{\text{Transverse motion}} + \underbrace{2\pi(f_S - f_R)t - \phi^\pm}_{\text{Heterodyne}} \right]$$

Applying STFT to recorded PDV fringes -- signal frequencies for the +/- diffracted beams

$$f^+(t) = \frac{1}{\lambda_S} [\dot{u}_1(1 + \cos \theta_n) + \dot{u}_2(t) \sin \theta_n] + (f_S - f_R)$$

$$f^-(t) = \frac{1}{\lambda_S} [\dot{u}_1(1 + \cos \theta_n) - \dot{u}_2(t) \sin \theta_n] + (f_S - f_R)$$

$$d \sin \theta_n = n \lambda_S$$

n – diffraction order

d – pitch of diffraction grating (500 lines/mm)

λ_S wavelenth of source laser

f_S frequency of source laser

f_R frequency of reference laser

Then,

Normal particle velocity

$$\dot{u}_1(t) = \frac{\lambda_S}{2(1 + \cos \theta_n)} [f^+(t) + f^-(t) - 2(f_S - f_R)]$$

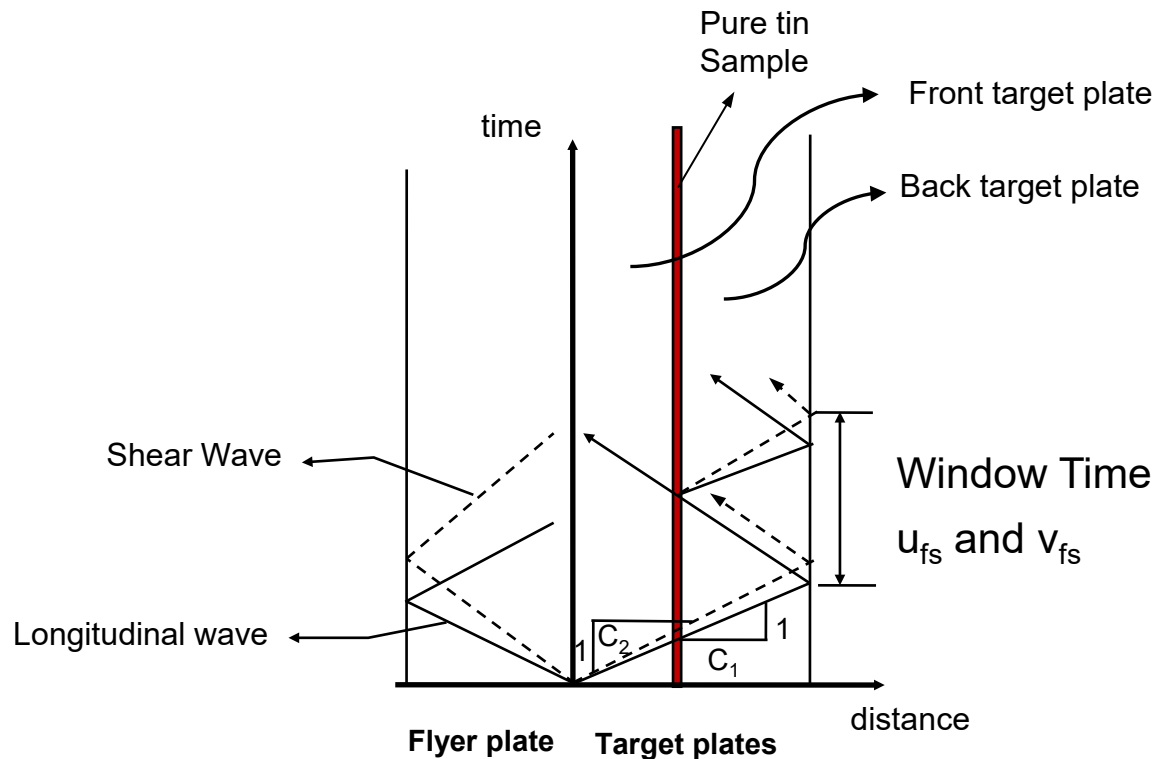
Transverse particle velocity

$$\dot{u}_2(t) = \frac{\lambda_S}{2 \sin \theta_n} [f^+(t) - f^-(t)]$$

Sensitivity of normal and transverse motion

Normal: $\frac{\lambda_S}{2} \frac{1}{(1 + \cos \theta_n)}$; Transverse: $\frac{d}{2n}$

Stress and Strain States in the Tin Sample



Lagrangian Time-Distance Diagram

Normal stress

$$\sigma(t) = \frac{1}{2} Z_{Lt} u_{fs}(t)$$

↑ Longitudinal Impedance (elastic)
↑ Normal particle velocity

Shear stress (strength)

$$\tau(t) = \frac{1}{2} Z_{Tt} v_{fs}(t)$$

↑ Transverse Impedance (elastic)
↑ Transverse particle velocity

Shear strain rate

$$\dot{\gamma}(t) = \frac{1}{h_o} \left[V_o \sin \theta - \frac{1}{2} \left(\frac{Z_{Tf} + Z_{Tt}}{Z_{Tf}} \right) v_{fs}(t) \right]$$

← Sample thickness ↑ Oblique angle

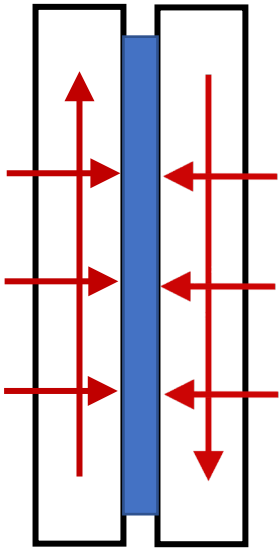
Shear strain

$$\gamma(t) = \int_0^t \dot{\gamma}(t) dt$$

Using Method of Characteristics – Elastic Anvil Plates

Experimental Considerations

Anvil Plates: D3 Steel and WC 3 wt.% Co



Anvil	ρ_o	Z_L	Z_T	k (shear)	θ	V_{max}	σ	τ
	Kg/m^3	$GPa/(mm/\mu s)$	$GPa/(mm/\mu s)$	GPa	deg	m/s	GPa	GPa
D3 CH Steel	7635.0	45.58	24.63	1.6	18	194.1	4.21	0.74
WC-3% Co	15130.0	105.45	63.99	2.9	18	121.2	6.09	1.20

von Mises Yield Criterion:

$$f = \sqrt{\frac{1}{2} S_{ij} S_{ij}} - k = 0 \quad k - \text{yield stress in shear}$$

D3 CH Steel – Strength of the β -Sn phase

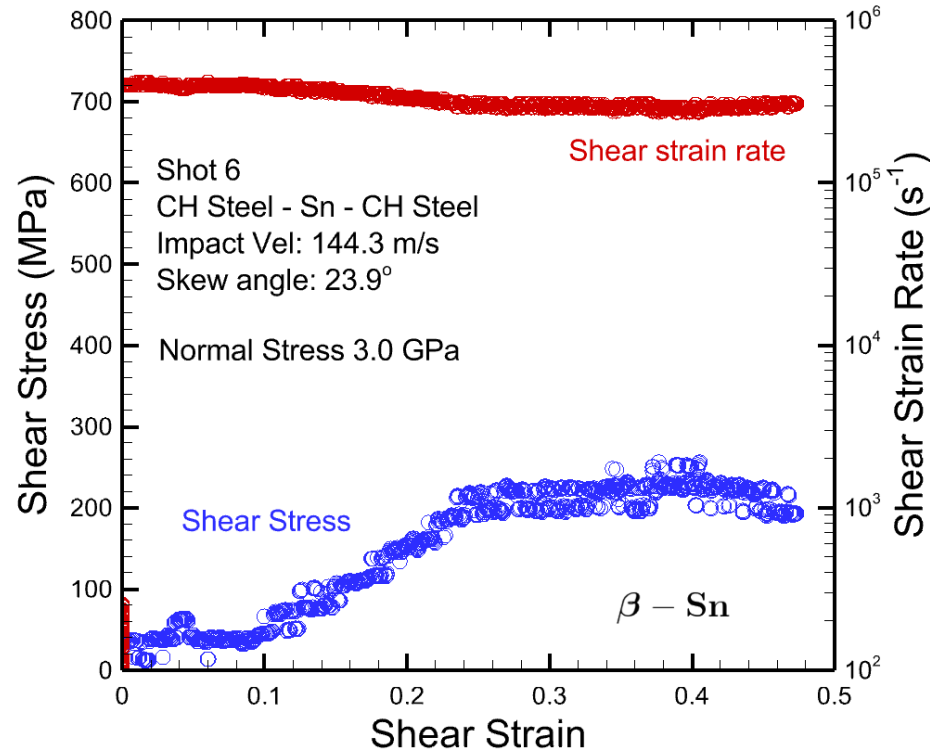
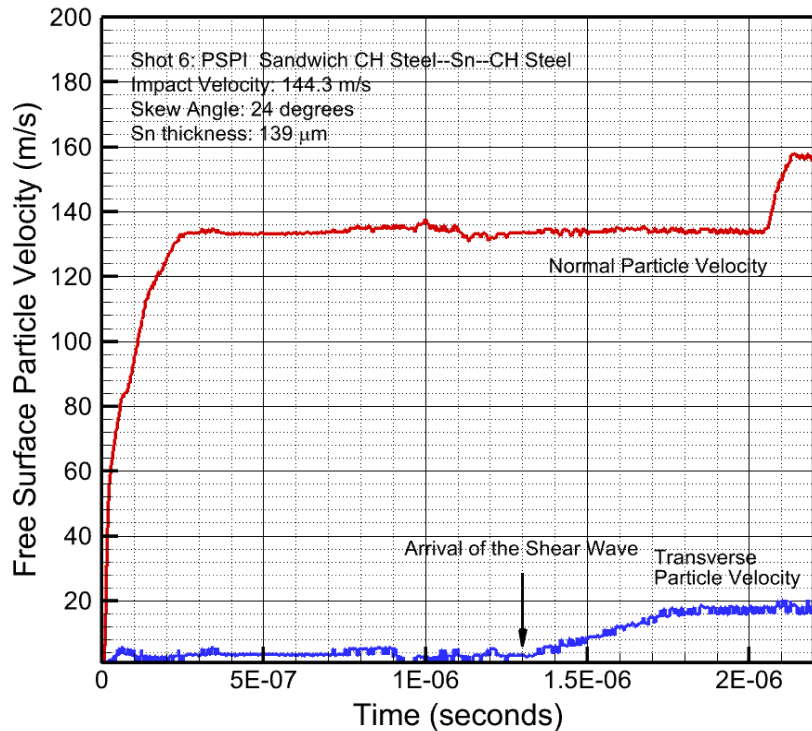
WC-Co binder – Strength of the transformed γ -Sn (bct) phase

Experiment (Shot) Matrix

Shot #	Experimental Config.	Target Config.	Impactor	Impact Velocity (m/s)	Skew Angle θ (degrees)	Sn-thickness (μm)	Normal Pressure (GPa)
Shot 1	PSPI	WC-WC	WC	165.0	18.0	N/A	8.27
Shot 2	PSPI	WC-WC	WC	238.4	18.0	N/A	11.96
Shot 12	PSPI	Steel-Steel	Steel	210.3	24.1	N/A	4.38
Shot 3	PSPI	WC-Sn-WC	WC	171.7	18.0	125.0	8.61 (β -Sn)
Shot 4	PSPI	WC-Sn-WC	WC	181.4	17.9	180.0	9.10 (Mixed)
Shot 5	PSPI	WC-Sn-WC	WC	259.5	17.9	132.0	13.02 (γ -Sn)
Shot 6	PSPI	Steel-Sn-Steel	Steel	144.3	24.0	139.0	3.00 (β -Sn)
Shot 9	NPI	WC-Sn-WC	WC	267.0	0.0	133.0	14.08 (γ -Sn)

Strength of β - Sn: Steel-Sn-Steel (Shot 6)

Impact vel 144.3 m/s, Skew angle 23.9°, Sn thickness 139 μm



Elastic stress analysis of impacting steel plates:

Normal stress: 3.0 GPa
 Shear stress: 227 MPa
 Equivalent stress (q): 1.04 GPa

von Mises Yield Criterion:

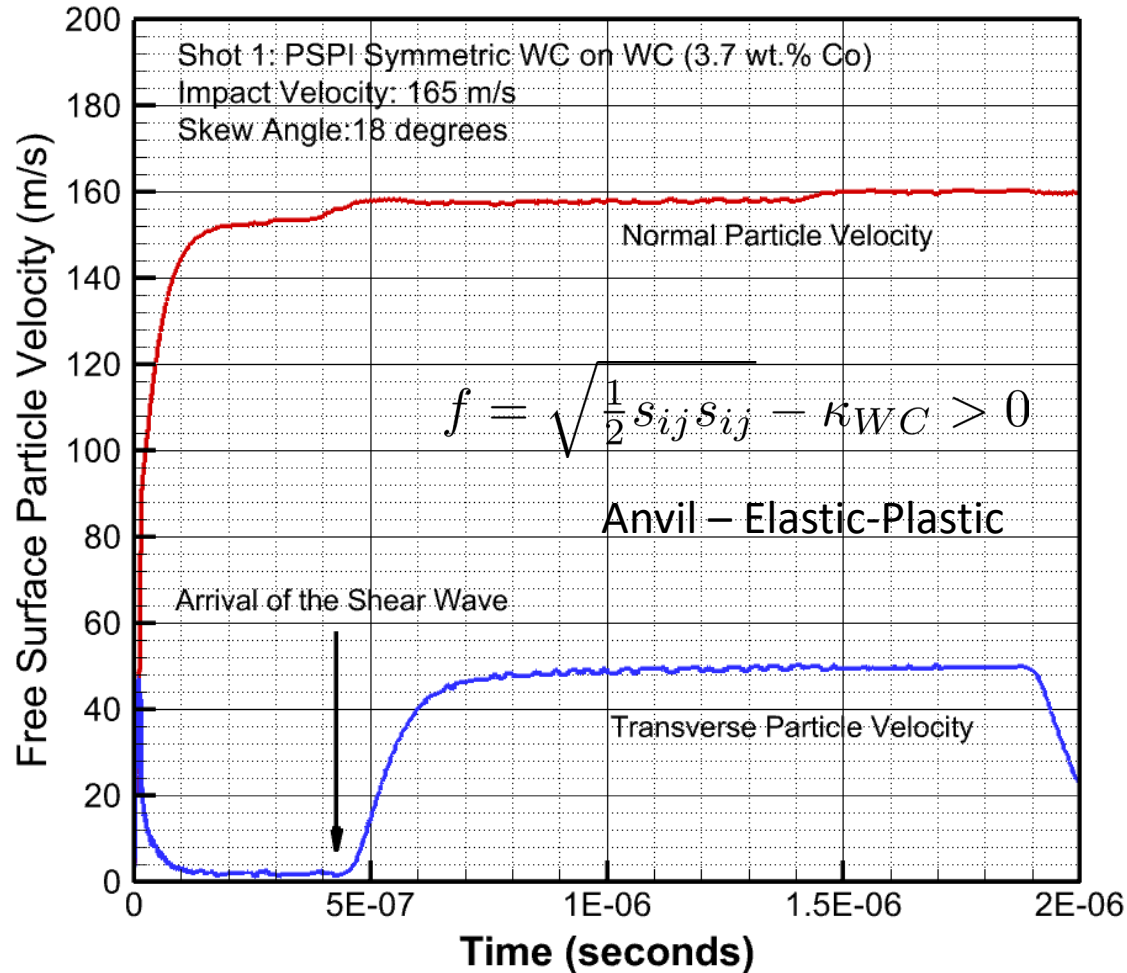
$$f = q - \kappa_{steel} < 0$$

$\kappa_{steel} = 1.6$ GPa in shear

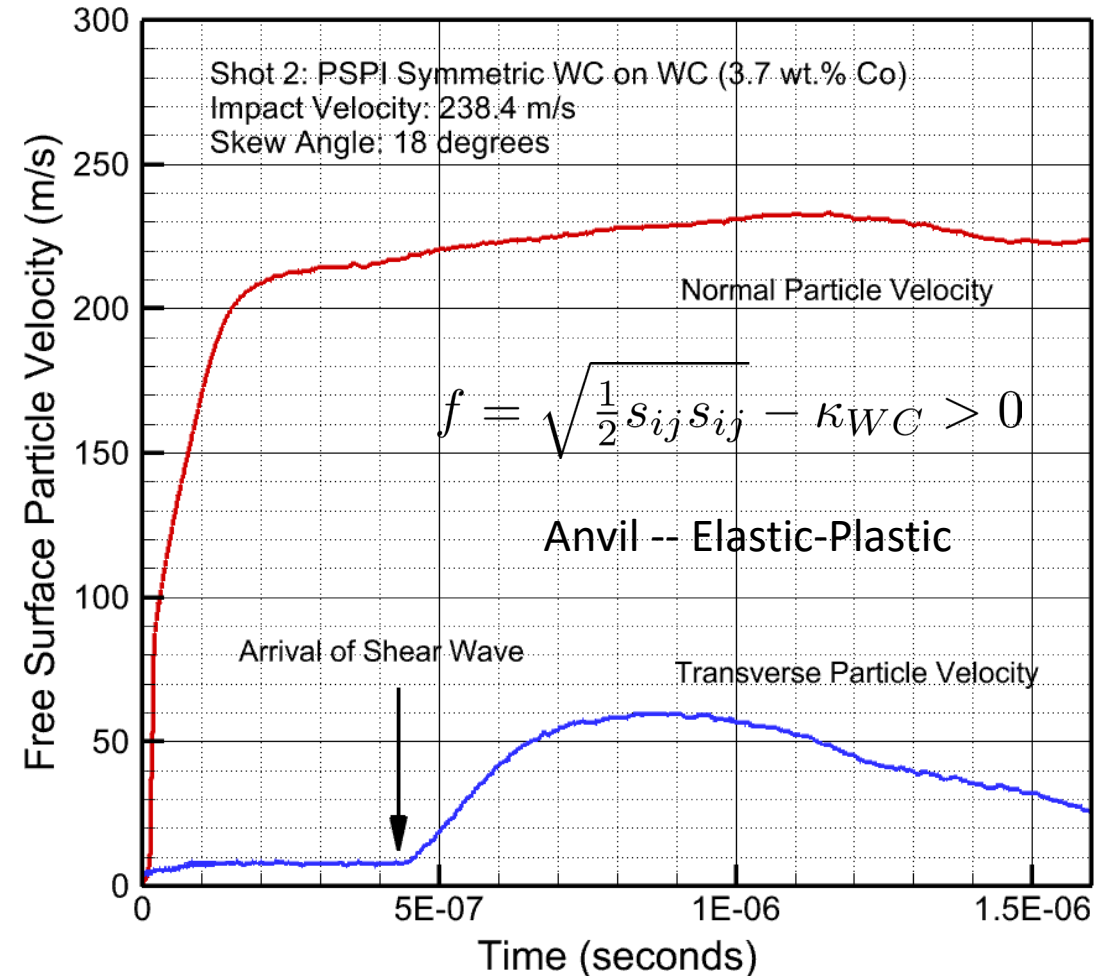
Shear strength of β - Sn at 3.0 GPa Normal Stress and shear strain rate of $3 \times 10^5 \text{ s}^{-1}$ increases (strain hardens) from 25 MPa at yield to 250 MPa

Strength of γ -Sn (bct) – Calibration of WC Anvil Plates

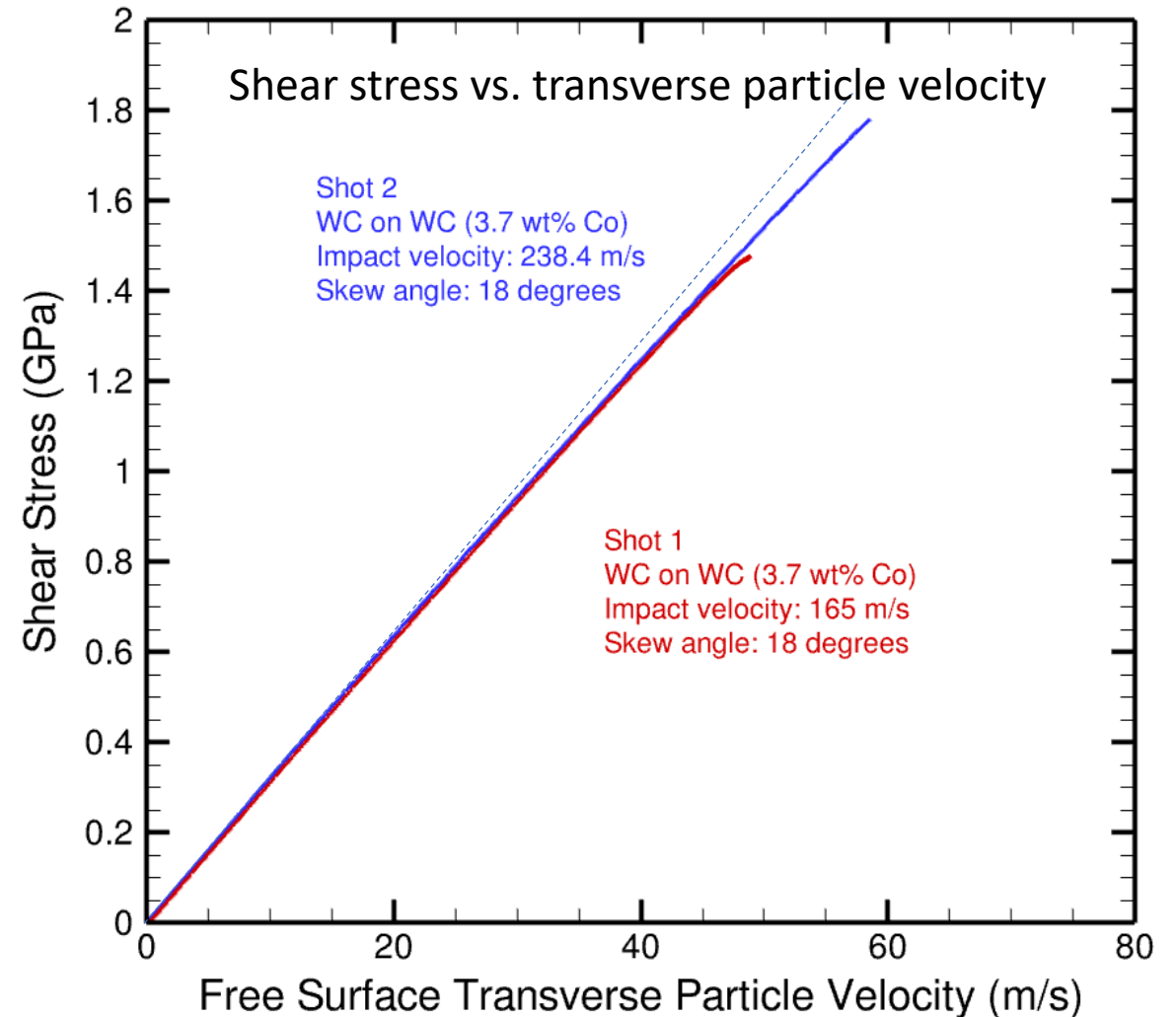
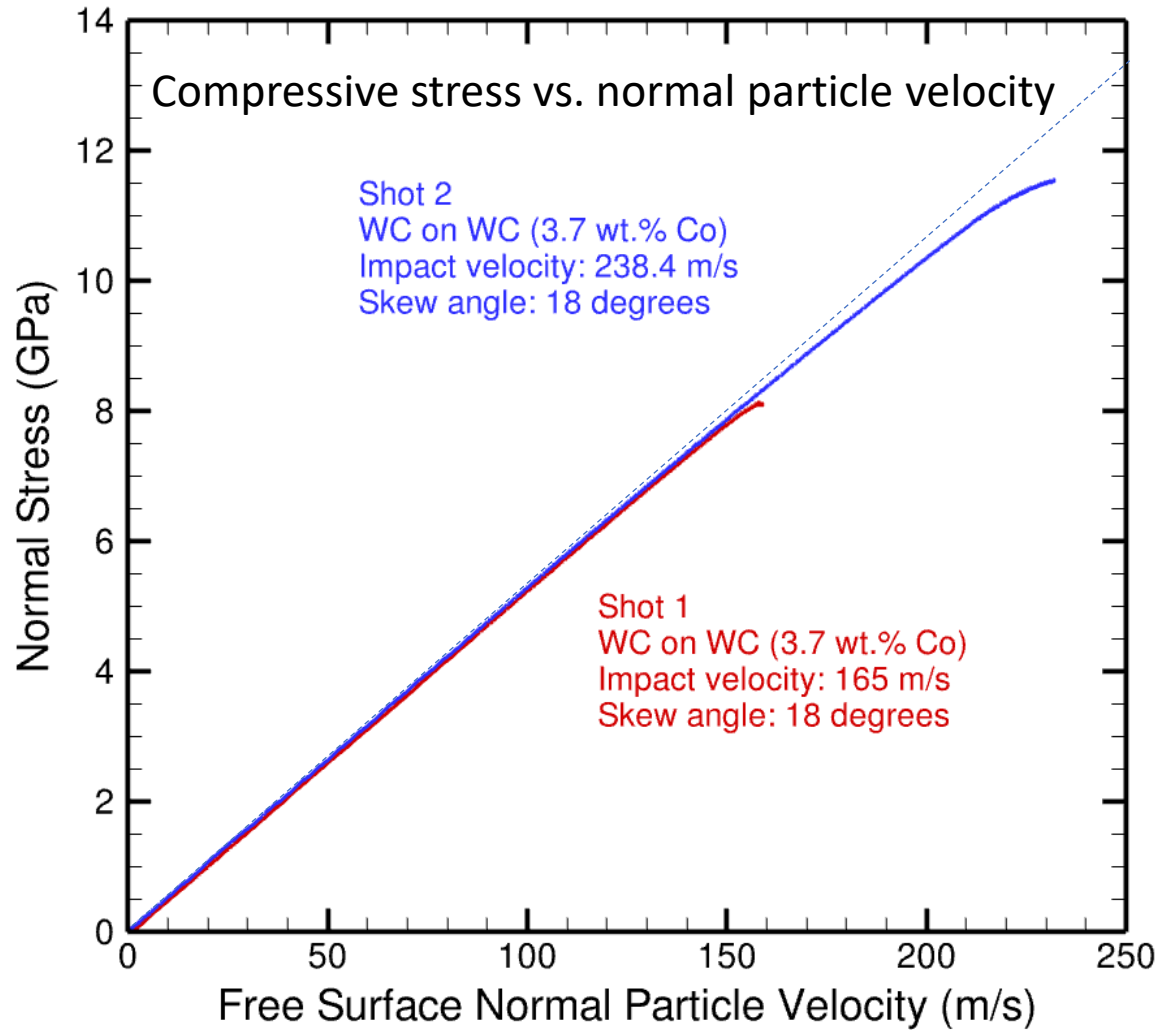
Impact velocity 165 m/s



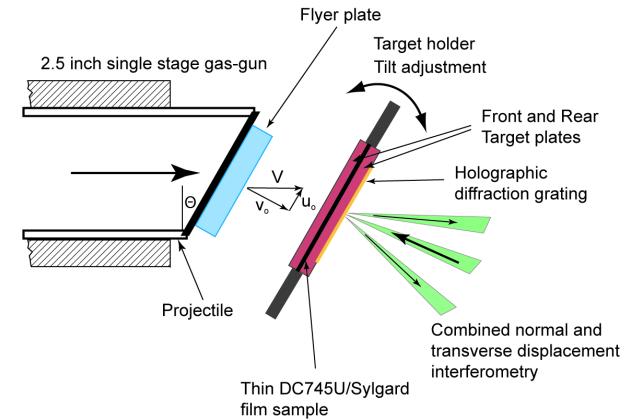
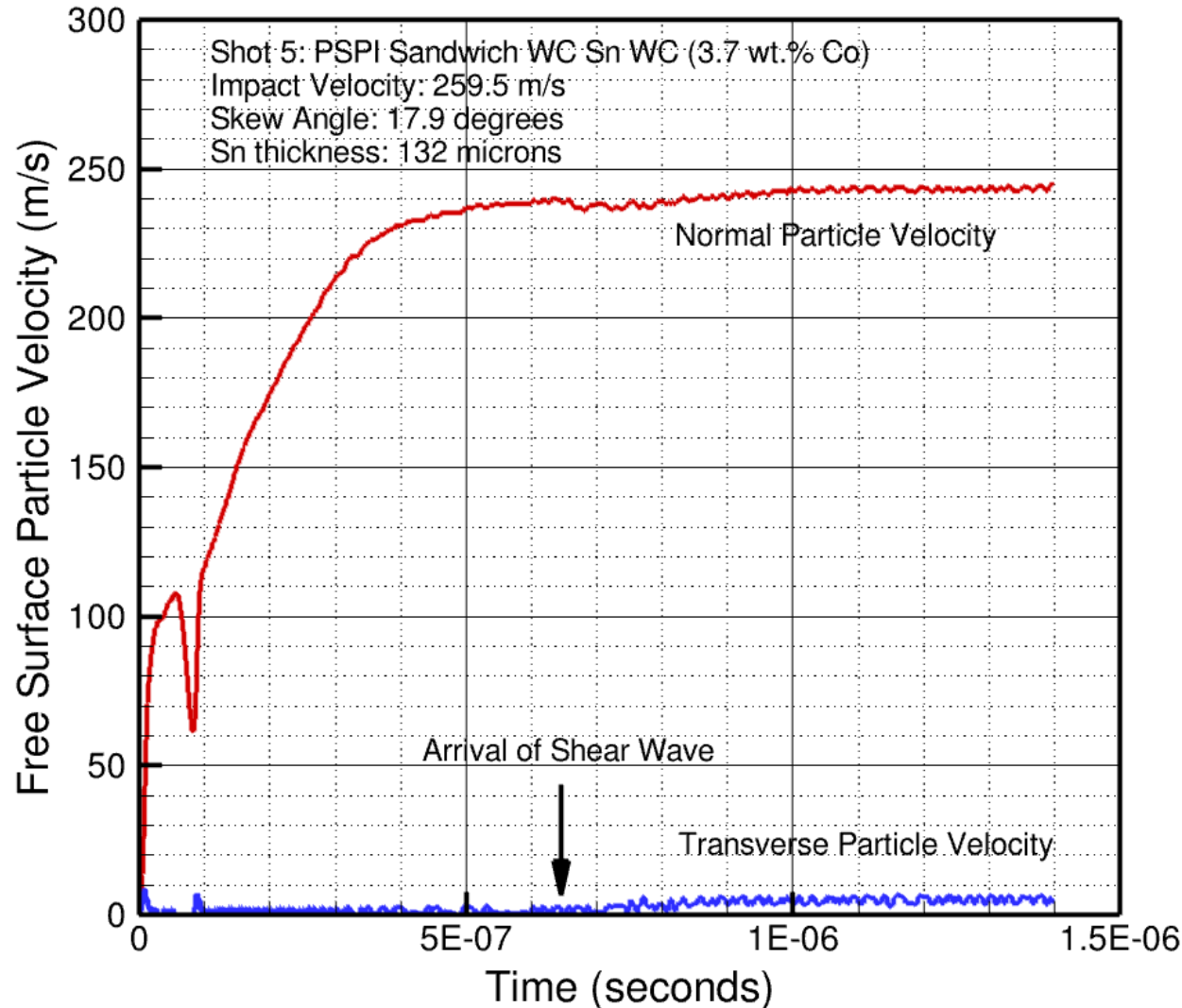
Impact velocity 238.4 m/s



Calibration Curves for WC : Normal and Shear Stress vs Free Surface Particle Velocity



Strength of γ -Sn (bct) – WC-Sn-WC configuration



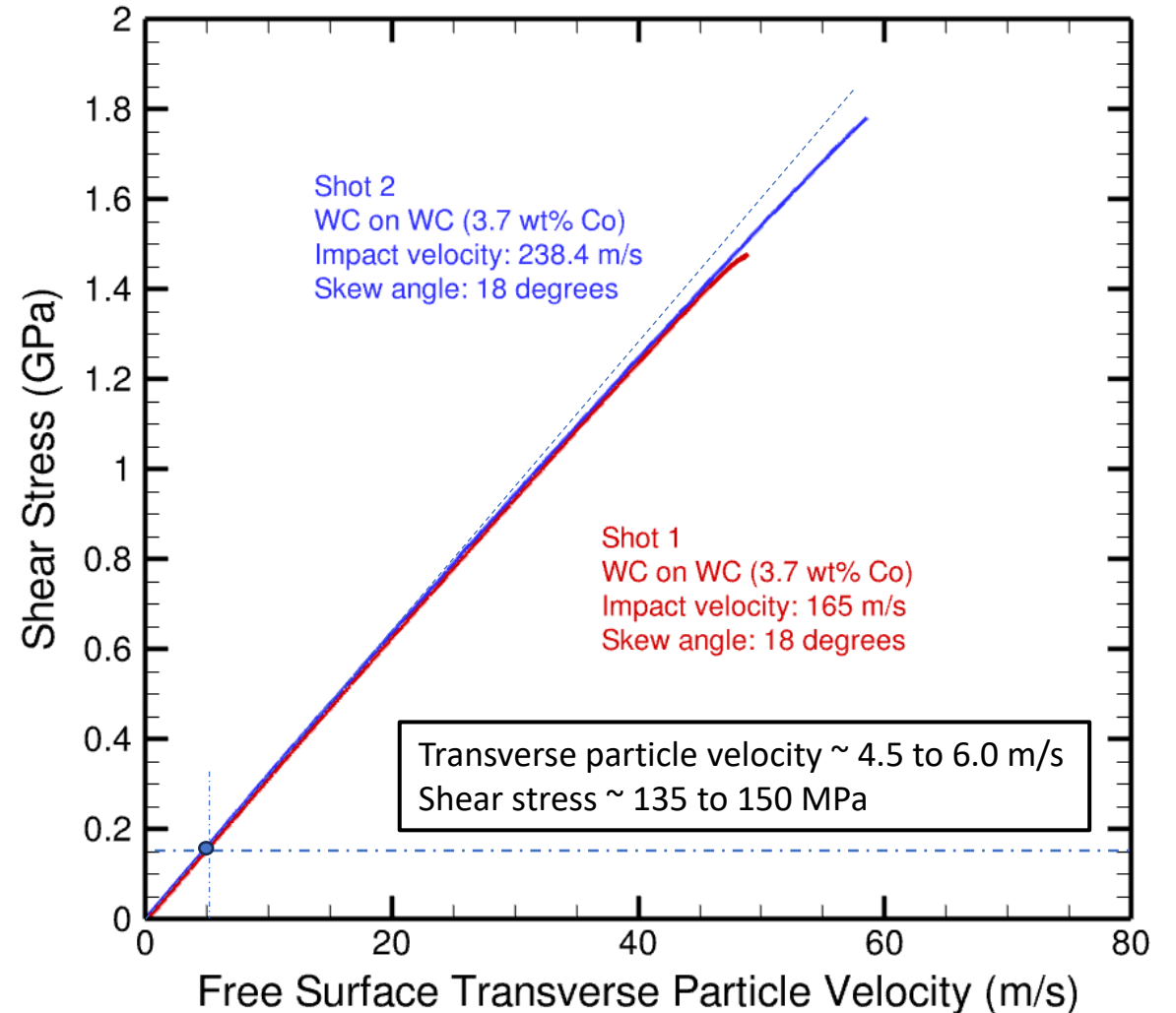
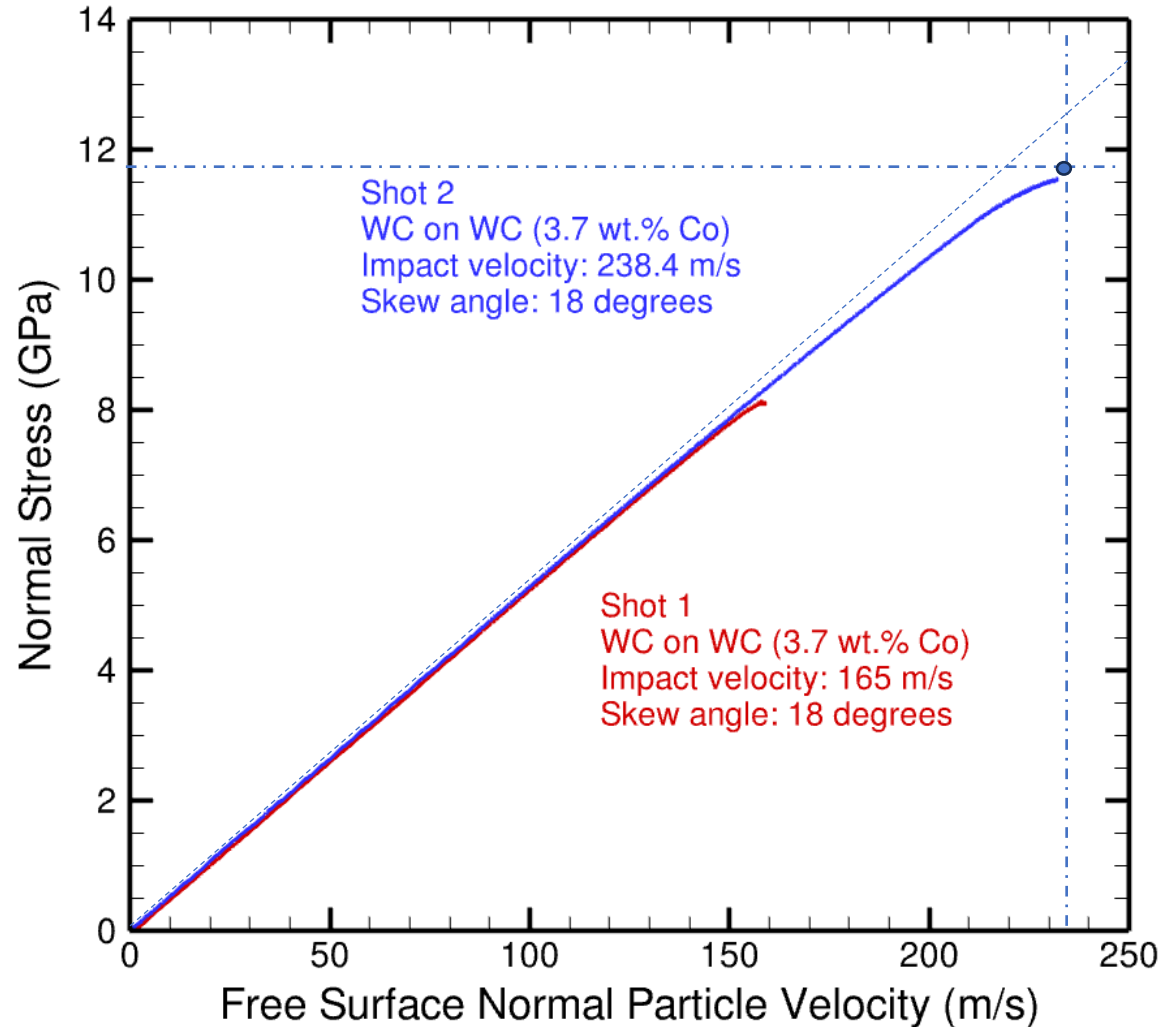
Challenge:

Measurement of very low levels of transverse particle velocity (< 10 m/s).

Exacerbated by the high impedance of the WC anvil plates.

PDV is the **ONLY** way to make these Measurements!

Normal and Shear Stress for γ -Sn (bct)



New Oblique Plate Capability at ISP



- *Large bore – 40mm bore*
- *Velocities 0.1 to 2.5 km/s*
- *Compression shear capability*
- *Resistive Heating ~750 C*
- *Inductive heating > 2000 C*
- *Material Strength*
- *Hypersonic studies*
- *Multiphase EOS studies*

Summary

Investigated the dynamic strength of beta and gamma (bct) tin using high-strain-rate combined compression-shear plate impact experiments.

- Flow strength of β -Sn was evaluated using the CH Steel-Sn-CH Steel target configuration at normal stress ~ 3 GPa. The flow stress was determined to increase from 25 to 225 MPa in shear at shear strain rates $\sim 3 \times 10^5 \text{ s}^{-1}$.
- To determine the flow stress of γ -Sn, WC-Sn-WC target configuration was used at normal stress > 10 GPa. The WC plates were calibrated to provide normal and shear stress vs free surface particle velocity profiles.

The flow strength of γ -Sn was ~ 120 to 130 GPa at shear strain rates of $\sim 1 \times 10^5 \text{ s}^{-1}$.